EXAMPLE 2.1

The slender rod shown in Fig. 2–4 is subjected to an increase of temperature along its axis, which creates a normal strain in the rod of $\epsilon_z = 40(10^{-3})z^{1/2}$, where z is given in meters. Determine (a) the displacement of the end B of the rod due to the temperature increase, and (b) the average normal strain in the rod.

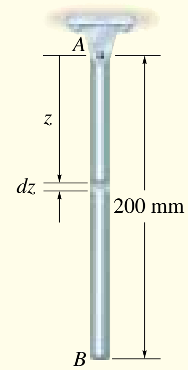


Fig. 2–4

Solution

Part (a). Since the normal strain is reported at each point along the rod, a differential segment dz, located at position z, Fig. 2–4, has a deformed length that can be determined from Eq. 2–3; that is,

$$dz' = [1 + 40(10^{-3})z^{1/2}]\,dz$$

The sum total of these segments along the axis yields the *deformed length* of the rod, i.e.,

$$\begin{aligned} z' &= \int_0^{0.2\text{ m}} [1 + 40(10^{-3})z^{1/2}]\,dz \\ &= z + 40(10^{-3})(\tfrac{2}{3}z^{3/2})\Big|_0^{0.2\text{ m}} \\ &= 0.20239\text{ m} \end{aligned}$$

The displacement of the end of the rod is therefore

$$\Delta_B = 0.20239\text{ m} - 0.2\text{ m} = 0.00239\text{ m} = 2.39\text{ mm}\downarrow \qquad \textit{Ans.}$$

Part (b). The average normal strain in the rod is determined from Eq. 2–1, which assumes that the rod or "line segment" has an original length of 200 mm and a change in length of 2.39 mm. Hence,

$$\epsilon_{avg} = \frac{\Delta s' - \Delta s}{\Delta s} = \frac{2.39\text{ mm}}{200\text{ mm}} = 0.0119\text{ mm/mm} \qquad \textit{Ans.}$$

EXAMPLE 2.2

A force acting on the grip of the lever arm shown in Fig. 2–5*a* causes the arm to rotate clockwise through an angle of $\theta = 0.002$ rad. Determine the average normal strain developed in the wire BC.

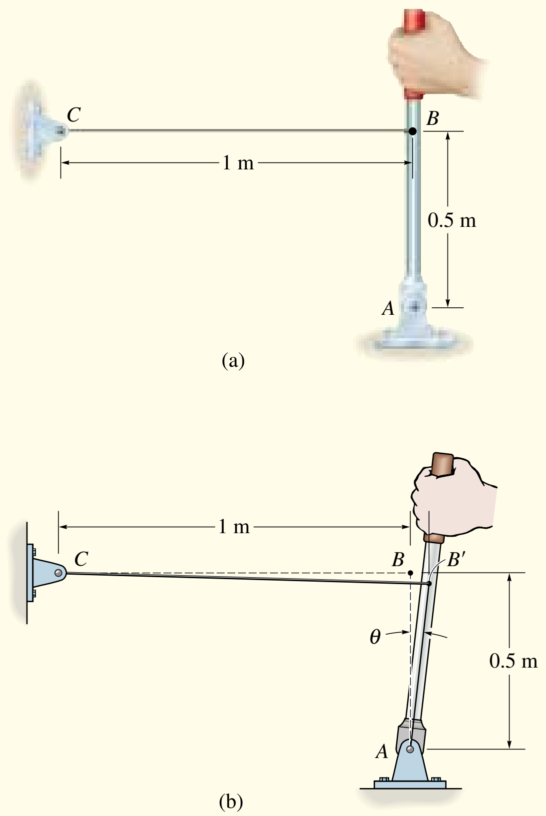


Fig. 2–5

Solution

Since $\theta = 0.002$ rad is small, the stretch in the wire CB, Fig. 2–5*b*, is $BB' = \theta(0.5\text{ m}) = (0.002\text{ rad})(0.5\text{ m}) = 0.001$ m. The average normal strain in the wire is therefore,

$$\epsilon_{avg} = \frac{BB'}{CB} = \frac{0.001}{1\text{ m}} = 0.001\text{ m/m} \qquad \textit{Ans.}$$

EXAMPLE 2.3

The plate is deformed into the dashed shape shown in Fig. 2–6*a*. If in this deformed shape horizontal lines on the plate remain horizontal and do not change their length, determine (a) the average normal strain along the side AB, and (b) the average shear strain in the plate relative to the x and y axes.

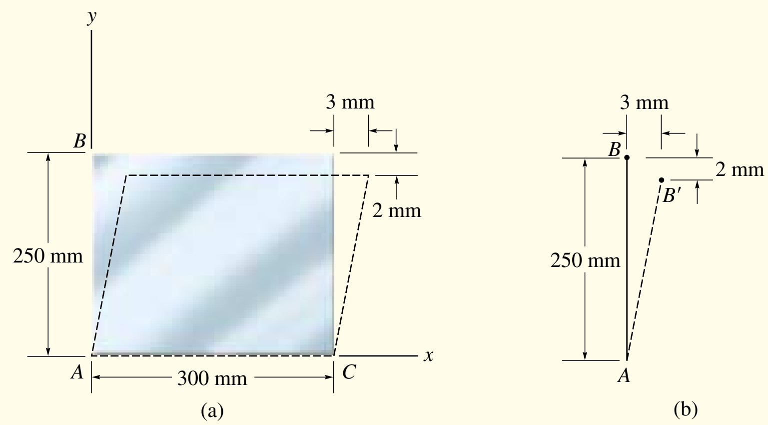
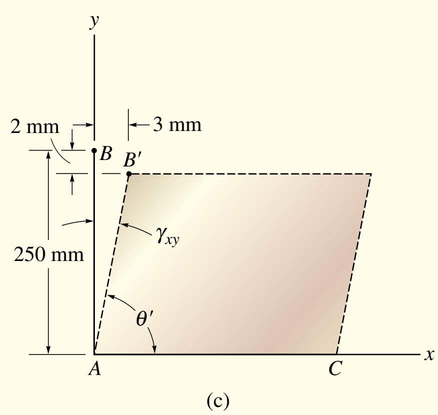


Fig. 2–6



Solution

Part (a). Line AB, coincident with the y axis, becomes line AB' after deformation, as shown in Fig. 2–6*b*. The length of this line is

$$AB' = \sqrt{(250 - 2)^2 + (3)^2} = 248.018 \text{ mm}$$

The average normal strain for AB is therefore

$$(\epsilon_{AB})_{avg} = \frac{AB' - AB}{AB} = \frac{248.018 \text{ mm} - 250 \text{ mm}}{250 \text{ mm}}$$

$$= -7.93(10^{-3}) \text{ mm/mm} \qquad \textit{Ans.}$$

The negative sign indicates the strain causes a contraction of AB.

Part (b). As noted in Fig. 2–6*c*, the once 90° angle BAC between the sides of the plate, referenced from the x, y axes, changes to θ' due to the displacement of B to B'. Since $\gamma_{xy} = \pi/2 - \theta'$, then γ_{xy} is the angle shown in the figure. Thus,

$$\gamma_{xy} = \tan^{-1}\left(\frac{3 \text{ mm}}{250 \text{ mm} - 2 \text{ mm}}\right) = 0.0121 \text{ rad} \qquad \textit{Ans.}$$

EXAMPLE 2.4

The plate shown in Fig. 2–7*a* is fixed connected along *AB* and held in the rigid horizontal guides at its top and bottom, *AD* and *BC*. If its right side *CD* is given a uniform horizontal displacement of 2 mm, determine (a) the average normal strain along the diagonal *AC*, and (b) the shear strain at *E* relative to the *x*, *y* axes.

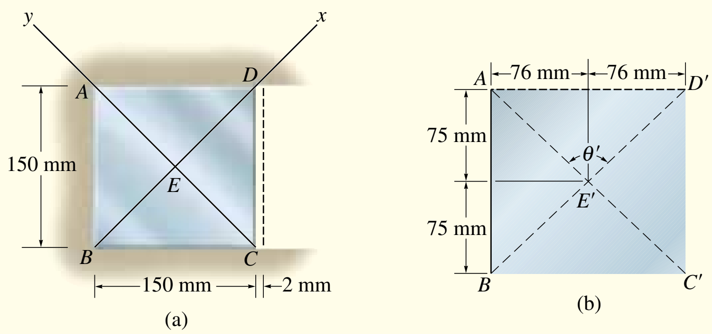


Fig. 2–7

Solution

Part (a). When the plate is deformed, the diagonal *AC* becomes *AC*′, Fig. 2–7*b*. The length of diagonals *AC* and *AC*′ can be found from the Pythagorean theorem. We have

$$AC = \sqrt{(0.150)^2 + (0.150)^2} = 0.21213 \text{ m}$$
$$AC' = \sqrt{(0.150)^2 + (0.152)^2} = 0.21355 \text{ m}$$

Therefore the average normal strain along the diagonal is

$$(\epsilon_{AC})_{\text{avg}} = \frac{AC' - AC}{AC} = \frac{0.21355 \text{ m} - 0.21213 \text{ m}}{0.21213 \text{ m}}$$
$$= 0.00669 \text{ mm/mm} \qquad \textit{Ans.}$$

Part (b). To find the shear strain at *E* relative to the *x* and *y* axes, it is first necessary to find the angle θ', which specifies the angle between these axes after deformation, Fig. 2–7*b*. We have

$$\tan\left(\frac{\theta'}{2}\right) = \frac{76 \text{ mm}}{75 \text{ mm}}$$
$$\theta' = 90.759^\circ = \frac{\pi}{180^\circ}(90.759^\circ) = 1.58404 \text{ rad}$$

Applying Eq. 2–4, the shear strain at *E* is therefore

$$\gamma_{xy} = \frac{\pi}{2} - 1.58404 \text{ rad} = -0.0132 \text{ rad} \qquad \textit{Ans.}$$

According to the sign convention, the *negative sign* indicates that the angle θ' is *greater than* 90°. Note that if the *x* and *y* axes were horizontal and vertical, then due to the deformation $\gamma_{xy} = 0$ at point *E*.