EXAMPLE 3.1

A tension test for a steel alloy results in the stress–strain diagram shown in Fig. 3–18. Calculate the modulus of elasticity and the yield strength based on a 0.2% offset. Identify on the graph the ultimate stress and the fracture stress.

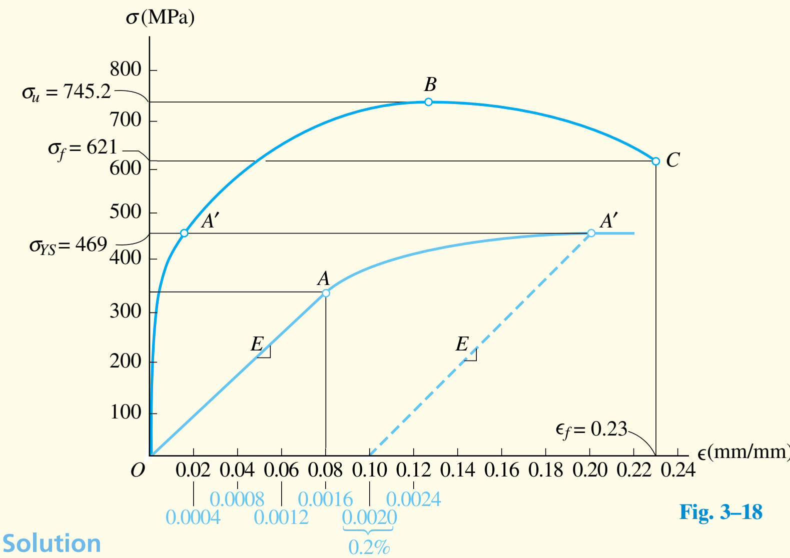


Fig. 3–18

Solution

Modulus of Elasticity. We must calculate the *slope* of the initial straight-line portion of the graph. Using the magnified curve and scale shown in color, this line extends from point O to an estimated point A, which has coordinates of approximately (0.0016 mm/mm, 345 MPa). Therefore,

$$E = \frac{345 \text{ MPa}}{0.0016 \text{ mm/mm}} = 215 \text{ GPa} \qquad \textit{Ans.}$$

Note that the equation of the line OA is thus $\sigma = 215(10^3)\epsilon$.

Yield Strength. For a 0.2% offset, we begin at a strain of 0.2% or 0.0020 mm/mm and graphically extend a (dashed) line parallel to OA until it intersects the $\sigma - \epsilon$ curve at A'. The yield strength is approximately

$$\sigma_{YS} = 469 \text{ MPa} \qquad \textit{Ans.}$$

Ultimate Stress. This is defined by the peak of the $\sigma - \epsilon$ graph, point B in Fig. 3–18.

$$\sigma_u = 745.2 \text{ MPa} \qquad \textit{Ans.}$$

Fracture Stress. When the specimen is strained to its maximum of $\epsilon_f = 0.23$ mm/mm, it fractures at point C. Thus,

$$\sigma_f = 621 \text{ MPa} \qquad \textit{Ans.}$$

EXAMPLE 3.2

The stress–strain diagram for an aluminum alloy that is used for making aircraft parts is shown in Fig. 3–19. If a specimen of this material is stressed to 600 MPa, determine the permanent strain that remains in the specimen when the load is released. Also, compute the modulus of resilience both before and after the load application.

Solution

Permanent Strain. When the specimen is subjected to the load, it strain-hardens until point B is reached on the $\sigma - \epsilon$ diagram, Fig. 3–19. The strain at this point is approximately 0.023 mm/mm. When the load is released, the material behaves by following the straight line BC, which is parallel to line OA. Since both lines have the same slope, the strain at point C can be determined analytically. The slope of line OA is the modulus of elasticity, i.e.,

$$E = \frac{450\text{ MPa}}{0.006\text{ mm/mm}} = 75.0\text{ GPa}$$

From triangle CBD, we require

$$E = \frac{BD}{CD} = \frac{600(10^6)\text{ Pa}}{CD} = 75.0(10^9)\text{ Pa}$$

$$CD = 0.008\text{ mm/mm}$$

This strain represents the amount of *recovered elastic strain*. The permanent strain, ϵ_{OC}, is thus

$$\epsilon_{OC} = 0.023\text{ mm/mm} - 0.008\text{ mm/mm}$$
$$= 0.0150\text{ mm/mm} \qquad \textit{Ans.}$$

Note: If gauge marks on the specimen were originally 50 mm apart, then after the load is *released* these marks will be 50 mm + (0.0150)(50 mm) = 50.75 mm apart.

Modulus of Resilience. Applying Eq. 3–8, we have*

$$(u_r)_{\text{initial}} = \frac{1}{2}\sigma_{pl}\epsilon_{pl} = \frac{1}{2}(450\text{ MPa})(0.006\text{ mm/mm})$$
$$= 1.35\text{ MJ/m}^3 \qquad \textit{Ans.}$$

$$(u_r)_{\text{final}} = \frac{1}{2}\sigma_{pl}\epsilon_{pl} = \frac{1}{2}(600\text{ MPa})(0.008\text{ mm/mm})$$
$$= 2.40\text{ MJ/m}^3 \qquad \textit{Ans.}$$

By comparison, the effect of strain-hardening the material has caused an increase in the modulus of resilience; however, note that the modulus of toughness for the material has decreased since the area under the original curve, $OABF$, is larger than the area under curve CBF.

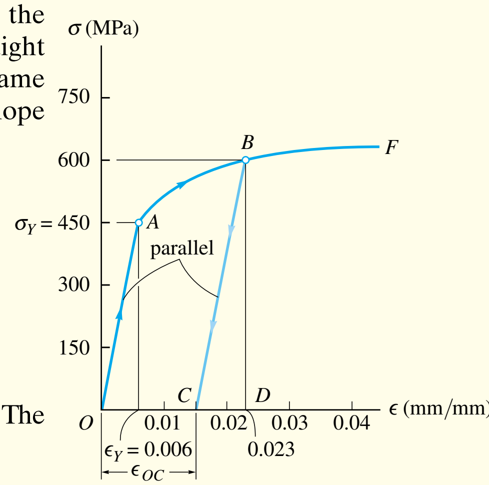
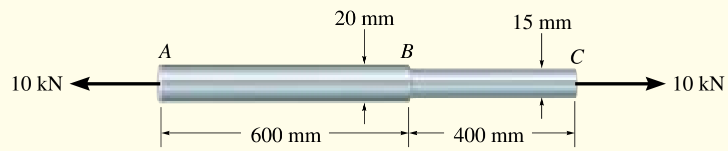


Fig. 3–19

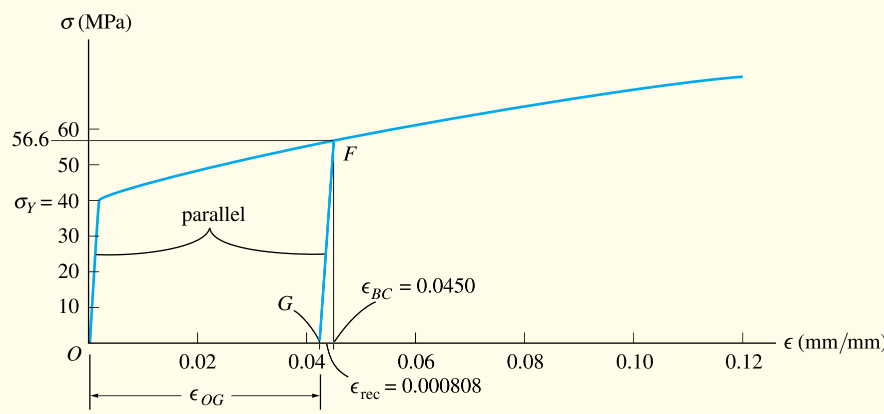
*Work in the SI system of units is measured in joules, where 1 J = 1 N · m.

EXAMPLE 3.3

An aluminum rod shown in Fig. 3–20*a* has a circular cross section and is subjected to an axial load of 10 kN. If a portion of the stress–strain diagram for the material is shown in Fig. 3–20*b*, determine the approximate elongation of the rod when the load is applied. If the load is removed, what is the permanent elongation of the rod? Take $E_{al} = 70$ GPa.



(a)



(b)

Fig. 3–20

Solution

For the analysis we will neglect the *localized deformations* at the point of load application and where the rod's cross-sectional area suddenly changes. (These effects will be discussed in Sections 4.1 and 4.7.) Throughout the midsection of each segment the normal stress and deformation are uniform.

In order to study the deformation of the rod, we must obtain the strain. This is done by first calculating the stress, then using the stress–strain diagram to obtain the strain. The normal stress within each segment is

$$\sigma_{AB} = \frac{P}{A} = \frac{10(10^3)\ \text{N}}{\pi\ (0.01\ \text{m})^2} = 31.83\ \text{MPa}$$

$$\sigma_{BC} = \frac{P}{A} = \frac{10(10^3)\ \text{N}}{\pi\ (0.0075\ \text{m})^2} = 56.59\ \text{MPa}$$

From the stress–strain diagram, the material in region AB is strained *elastically* since $\sigma_Y = 40\ \text{MPa} > 31.83\ \text{MPa}$. Using Hooke's law,

$$\epsilon_{AB} = \frac{\sigma_{AB}}{E_{\text{al}}} = \frac{31.83(10^6)\ \text{Pa}}{70(10^9)\ \text{Pa}} = 0.0004547\ \text{mm/mm}$$

The material within region BC is strained plastically, since $\sigma_Y = 40\ \text{MPa} < 56.59\ \text{MPa}$. From the graph, for $\sigma_{BC} = 56.59\ \text{MPa}$,

$$\epsilon_{BC} \approx 0.045\ \text{mm/mm}$$

The approximate elongation of the rod is therefore

$$\begin{aligned}\delta &= \Sigma\epsilon L = 0.0004547\ (600\ \text{mm}) + 0.045\ (400\ \text{mm}) \\ &= 18.3\ \text{mm}\end{aligned}$$ *Ans.*

When the 10-kN load is removed, segment AB of the rod will be restored to its original length. Why? On the other hand, the material in segment BC will recover elastically along line FG, Fig. 3–20*b*. Since the slope of FG is E_{al}, the elastic strain recovery is

$$\epsilon_{\text{rec}} = \frac{\sigma_{BC}}{E_{\text{al}}} = \frac{56.59(10^6)\ \text{Pa}}{70(10^9)\ \text{Pa}} = 0.000808\ \text{mm/mm}$$

The remaining plastic strain in segment BC is then

$$\epsilon_{OG} = 0.0450 - 0.000808 = 0.0442\ \text{mm/mm}$$

Therefore, when the load is removed the rod remains elongated by an amount

$$\delta' = \epsilon_{OG} L_{BC} = 0.0442\ (400\ \text{mm}) = 17.7\ \text{mm}$$ *Ans.*

EXAMPLE 3.4

A bar made of A-36 steel has the dimensions shown in Fig. 3–22. If an axial force of $P = 80$ kN is applied to the bar, determine the change in its length and the change in the dimensions of its cross section after applying the load. The material behaves elastically.

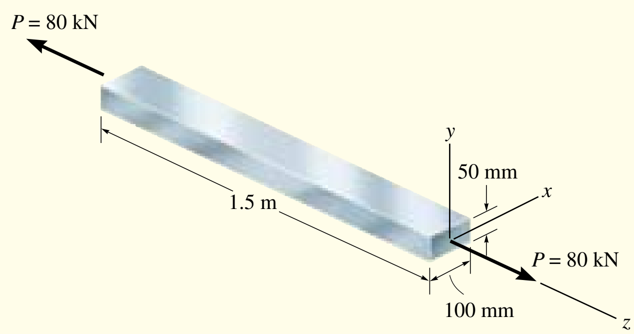


Fig. 3–22

Solution

The normal stress in the bar is

$$\sigma_z = \frac{P}{A} = \frac{80(10^3)\text{ N}}{(0.1\text{ m})(0.05\text{ m})} = 16.0(10^6)\text{ Pa}$$

From the table on the inside back cover for A-36 steel, $E_{st} = 200$ GPa, and so the strain in the z direction is

$$\epsilon_z = \frac{\sigma_z}{E_{st}} = \frac{16.0(10^6)\text{ Pa}}{200(10^9)\text{ Pa}} = 80(10^{-6})\text{ mm/mm}$$

The axial elongation of the bar is therefore

$$\delta_z = \epsilon_z L_z = [80(10^{-6})](1.5\text{ m}) = 120\ \mu\text{m} \qquad \textit{Ans.}$$

Using Eq. 3–9, where $\nu_{st} = 0.32$ as found from the inside back cover, the contraction strains in *both* the x and y directions are

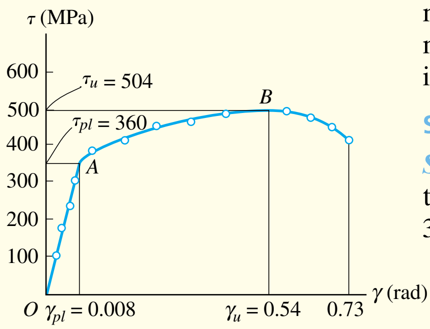
$$\epsilon_x = \epsilon_y = -\nu_{st}\epsilon_z = -0.32[80(10^{-6})] = -25.6\ \mu\text{m/m}$$

Thus the changes in the dimensions of the cross section are

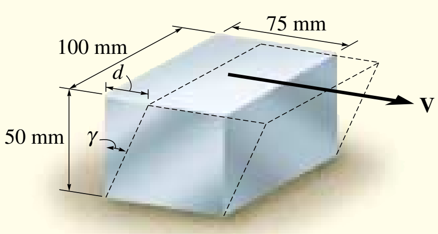
$$\delta_x = \epsilon_x L_x = -[25.6(10^{-6})](0.1\text{ m}) = -2.56\ \mu\text{m} \qquad \textit{Ans.}$$

$$\delta_y = \epsilon_y L_y = -[25.6(10^{-6})](0.05\text{ m}) = -1.28\ \mu\text{m} \qquad \textit{Ans.}$$

EXAMPLE 3.5



(a)



(b)

Fig. 3–25

A specimen of titanium alloy is tested in torsion and the shear stress–strain diagram is shown in Fig. 3–25*a*. Determine the shear modulus G, the proportional limit, and the ultimate shear stress. Also, determine the maximum distance d that the top of a block of this material, shown in Fig. 3–25*b*, could be displaced horizontally if the material behaves elastically when acted upon by a shear force **V**. What is the magnitude of **V** necessary to cause this displacement?

Solution

Shear Modulus. This value represents the slope of the straight-line portion OA of the τ–γ diagram. The coordinates of point A are (0.008 rad, 360 MPa). Thus,

$$G = \frac{360 \text{ MPa}}{0.008 \text{ rad}} = 45(10^3) \text{ MPa} \qquad \textit{Ans.}$$

The equation of line OA is therefore $\tau = 45(10^3)\gamma$, which is Hooke's law for shear.

Proportional Limit. By inspection, the graph ceases to be linear at point A. Thus,

$$\tau_{pl} = 360 \text{ MPa} \qquad \textit{Ans.}$$

Ultimate Stress.

This value represents the maximum shear stress, point B. From the graph,

$$\tau_u = 504 \text{ MPa} \qquad \textit{Ans.}$$

Maximum Elastic Displacement and Shear Force. Since the maximum elastic shear strain is 0.008 rad, a very small angle, the top of the block in Fig. 3–25*b* will be displaced horizontally:

$$\tan(0.008 \text{ rad}) \approx 0.008 \text{ rad} = \frac{d}{50 \text{ mm}}$$

$$d = 0.4 \text{ mm} \qquad \textit{Ans.}$$

The corresponding *average* shear stress in the block is $\tau_{pl} = 360$ MPa. Thus, the shear force V needed to cause the displacement is

$$\tau_{\text{avg}} = \frac{V}{A}; \qquad 360 \text{ MPa} = \frac{V}{(75 \text{ mm})(100 \text{ mm})}$$

$$V = 2700 \text{ kN} \qquad \textit{Ans.}$$

EXAMPLE 3.6

An aluminum specimen shown in Fig. 3–26 has a diameter of $d_0 = 25$ mm and a gauge length of $L_0 = 250$ mm. If a force of 165 kN elongates the gauge length 1.20 mm, determine the modulus of elasticity. Also, determine by how much the force causes the diameter of the specimen to contract. Take $G_{al} = 26$ GPa and $\sigma_Y = 440$ MPa.

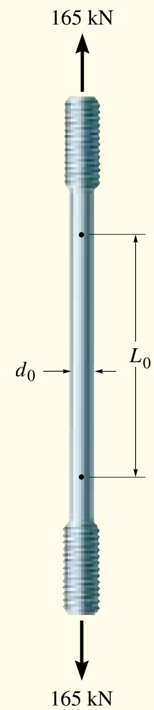


Fig. 3–26

Solution

Modulus of Elasticity. The average normal stress in the specimen is

$$\sigma = \frac{P}{A} = \frac{165(10^3)\text{ N}}{(\pi/4)(0.025\text{ m})^2} = 336.1\text{ MPa}$$

and the average normal strain is

$$\epsilon = \frac{\delta}{L} = \frac{1.20\text{ mm}}{250\text{ mm}} = 0.00480\text{ mm/mm}$$

Since $\sigma < \sigma_Y = 440$ MPa, the material behaves elastically. The modulus of elasticity is

$$E_{al} = \frac{\sigma}{\epsilon} = \frac{336.1(10^6)\text{ Pa}}{0.00480} = 70.0\text{ GPa} \qquad \textit{Ans.}$$

Contraction of Diameter. First, we will determine Poisson's ratio for the material using Eq. 3–11.

$$G = \frac{E}{2(1+\nu)}$$

$$26\text{ GPa} = \frac{70.0\text{ GPa}}{2(1+\nu)}$$

$$\nu = 0.346$$

Since $\epsilon_{long} = 0.00480$ mm/mm, then by Eq. 3–9,

$$\nu = \frac{\epsilon_{lat}}{\epsilon_{long}}$$

$$0.346 = \frac{\epsilon_{lat}}{0.00480\text{ mm/mm}}$$

$$\epsilon_{lat} = -0.00166\text{ mm/mm}$$

The contraction of the diameter is therefore

$$\delta' = (0.00166)(25\text{ mm})$$

$$= 0.0415\text{ mm} \qquad \textit{Ans.}$$