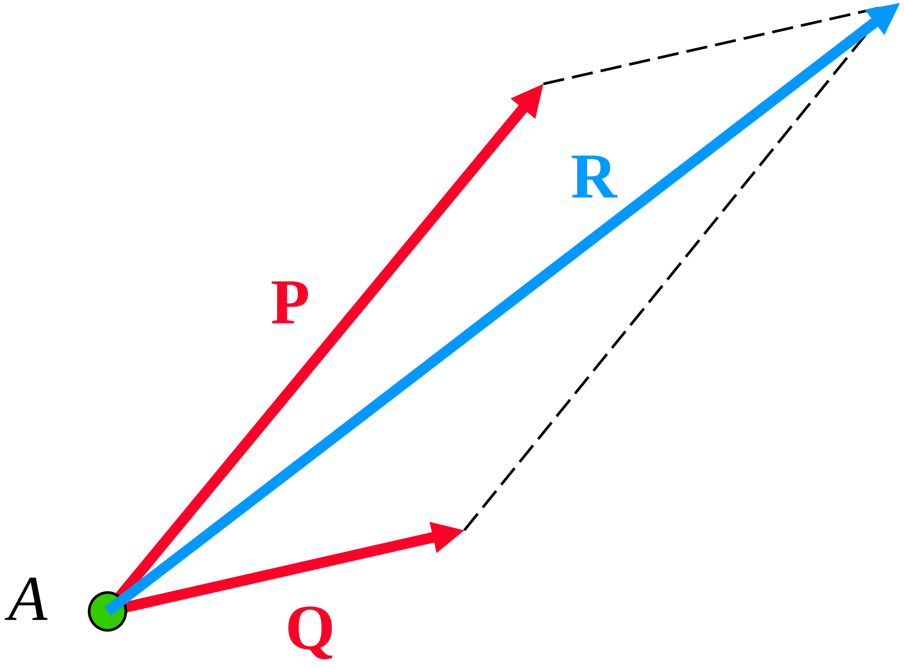
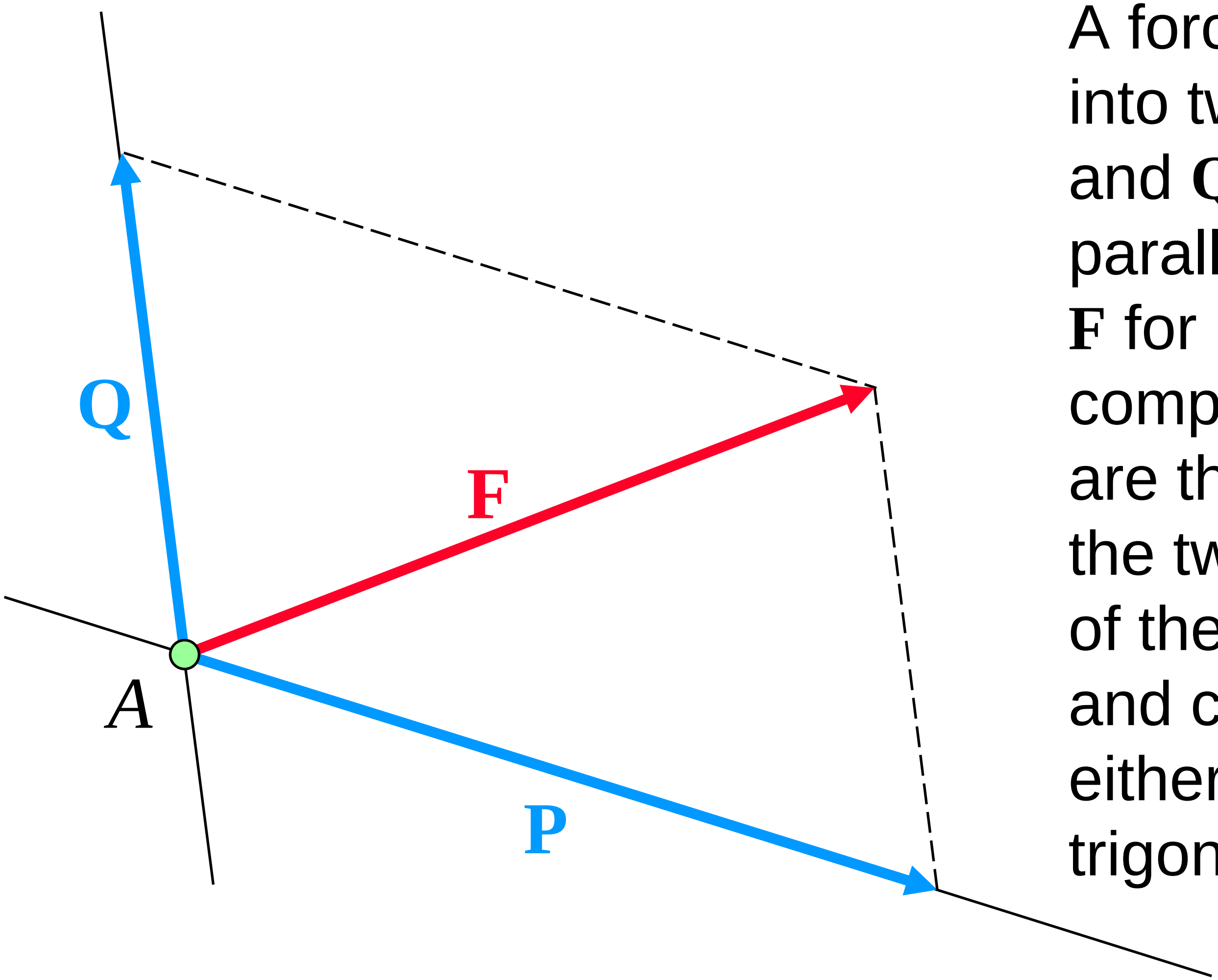
Chapter 2   STATICS OF PARTICLES

Forces are ***vector quantities***; they add according to the ***parallelogram law. The*** magnitude and direction of the resultant **R** of two forces **P** and **Q** can be determined either graphically or by trigonometry.



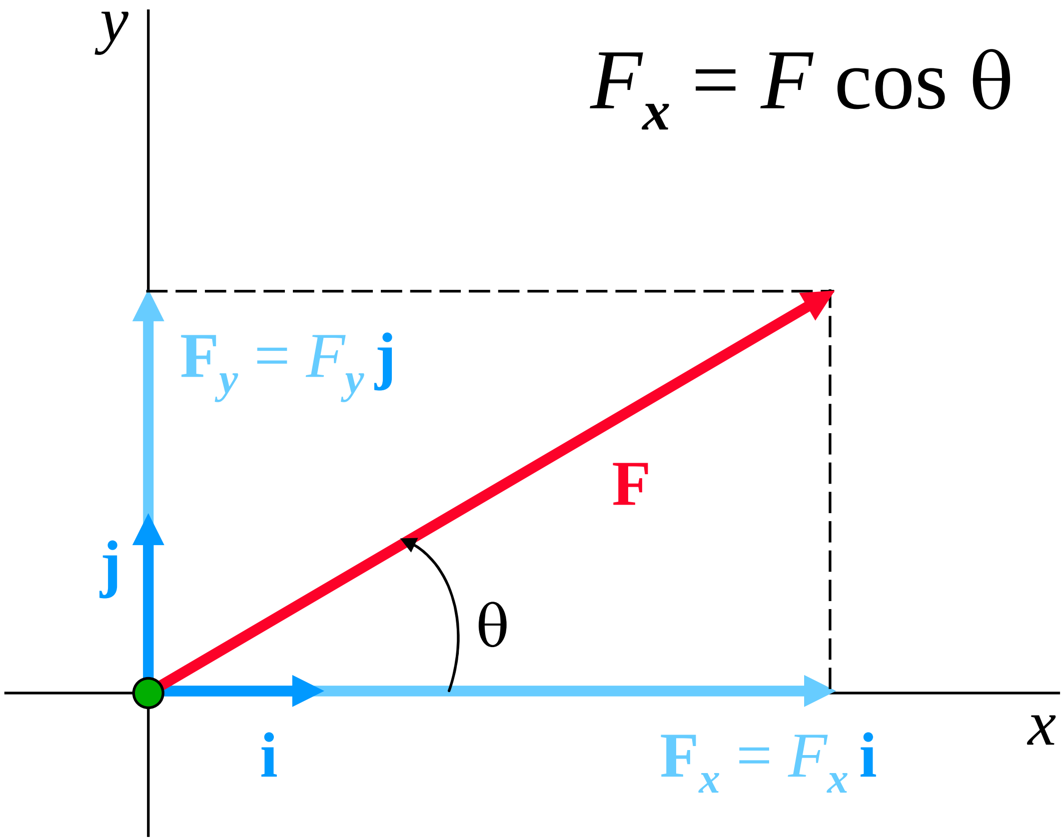
Any given force acting on a particle can be resolved into two or more ***components***, i.e.., it can be replaced by two or more forces which have the same effect on the particle.



A force **F** can be resolved into two components **P** and **Q** by drawing a parallelogram which has **F** for its diagonal; the components **P** and **Q** are then represented by the two adjacent sides of the parallelogram and can be determined either graphically or by trigonometry.

A force $\mathbf{F}$ is said to have been resolved into two ***rectangular components*** if its components are directed along the coordinate axes. Introducing the ***unit vectors*** **i** and **j** along the x and y axes,

$$\mathbf{F} = F_x\,\mathbf{i} + F_y\,\mathbf{j}$$



$$F_x = F\cos\theta \qquad F_y = F\sin\theta$$

$$\tan\theta = \frac{F_y}{F_x}$$

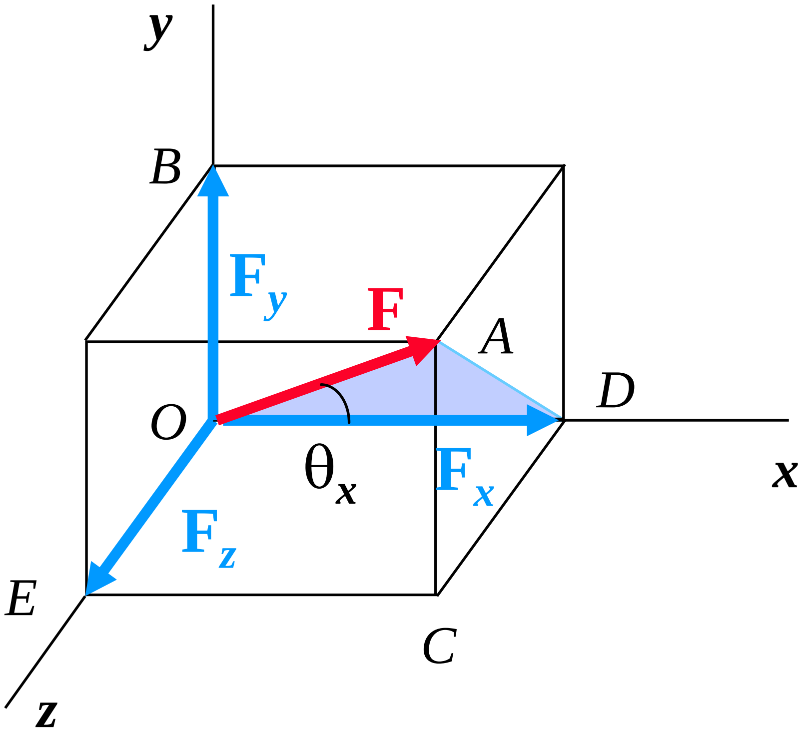
$$F = \sqrt{F_x^2 + F_y^2}$$

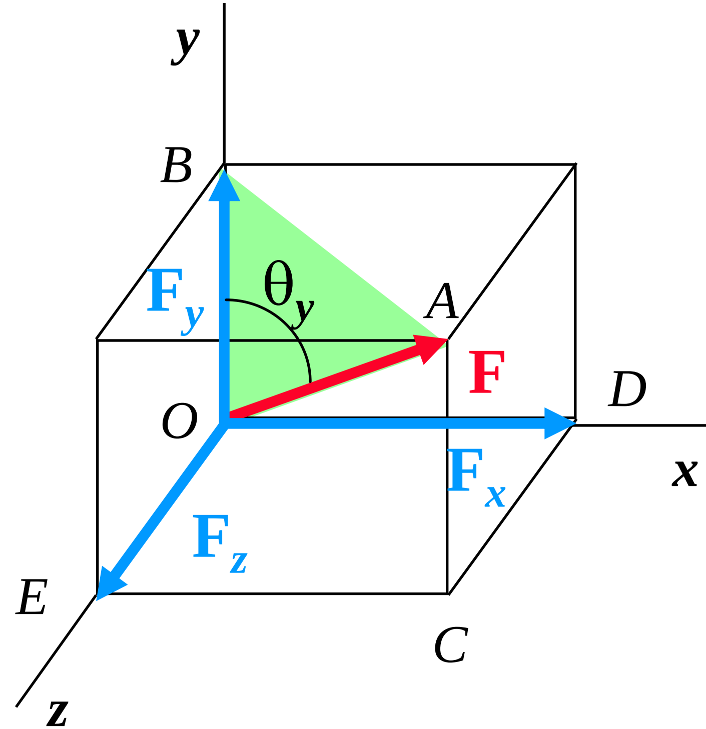
When ***three or more coplanar forces*** act on a particle, the rectangular components of their resultant **R** can be obtained by adding algebraically the corresponding components of the given forces.

$$R_x = \Sigma R_x \qquad R_y = \Sigma R_y$$

The magnitude and direction of **R** can be determined from

$$\tan\theta = \frac{R_y}{R_x} \qquad R = \sqrt{R_x^2 + R_y^2}$$

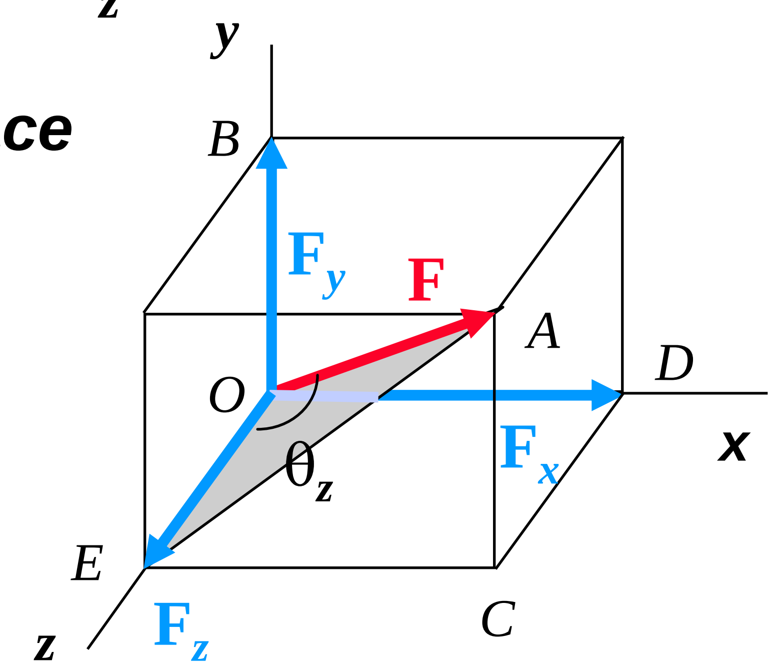


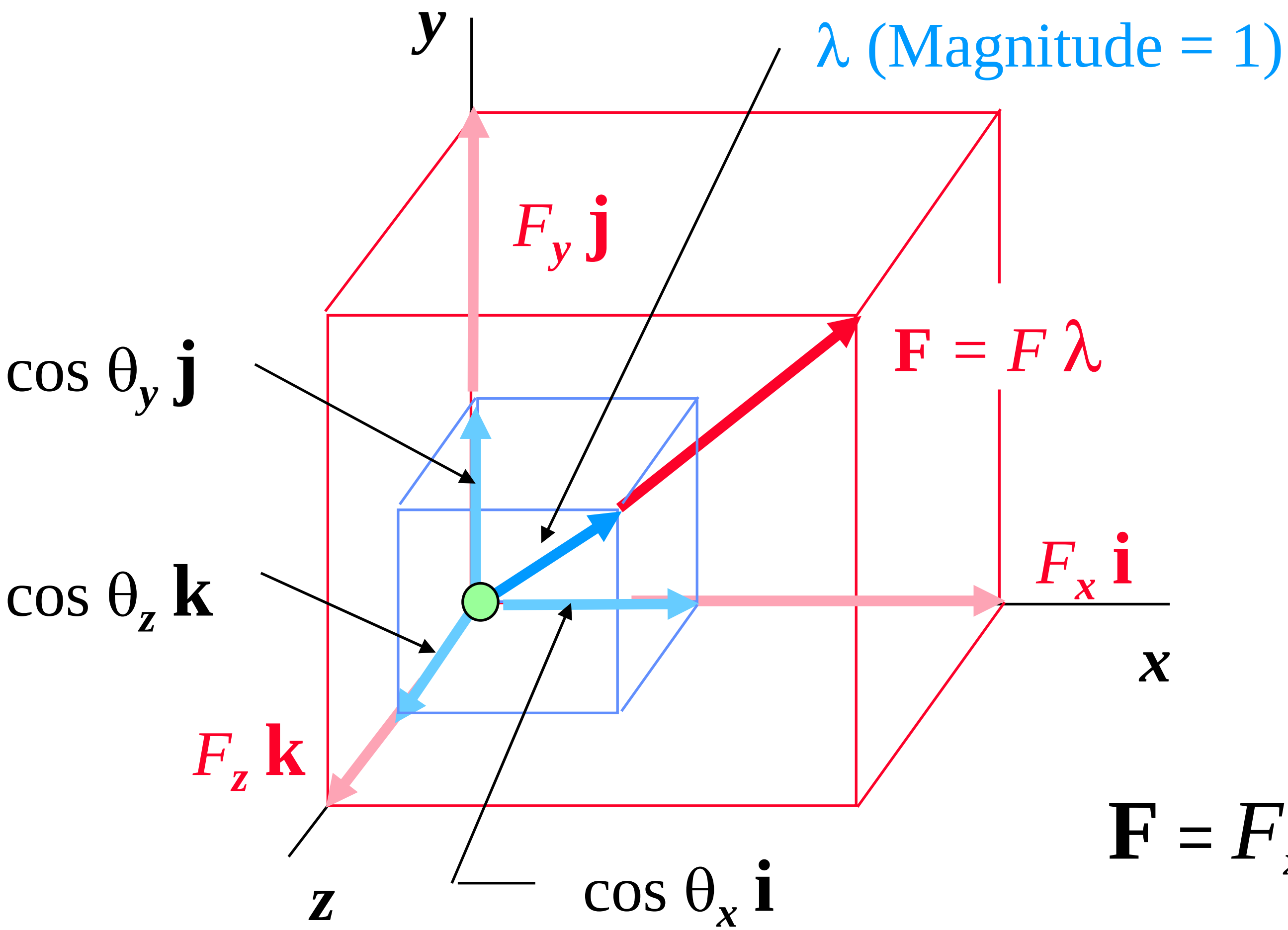


A force **F** in ***three-dimensional space*** can be resolved into components

$$F_x = F \cos \theta_x \quad F_y = F \cos \theta_y$$

$$F_z = F \cos \theta_z$$



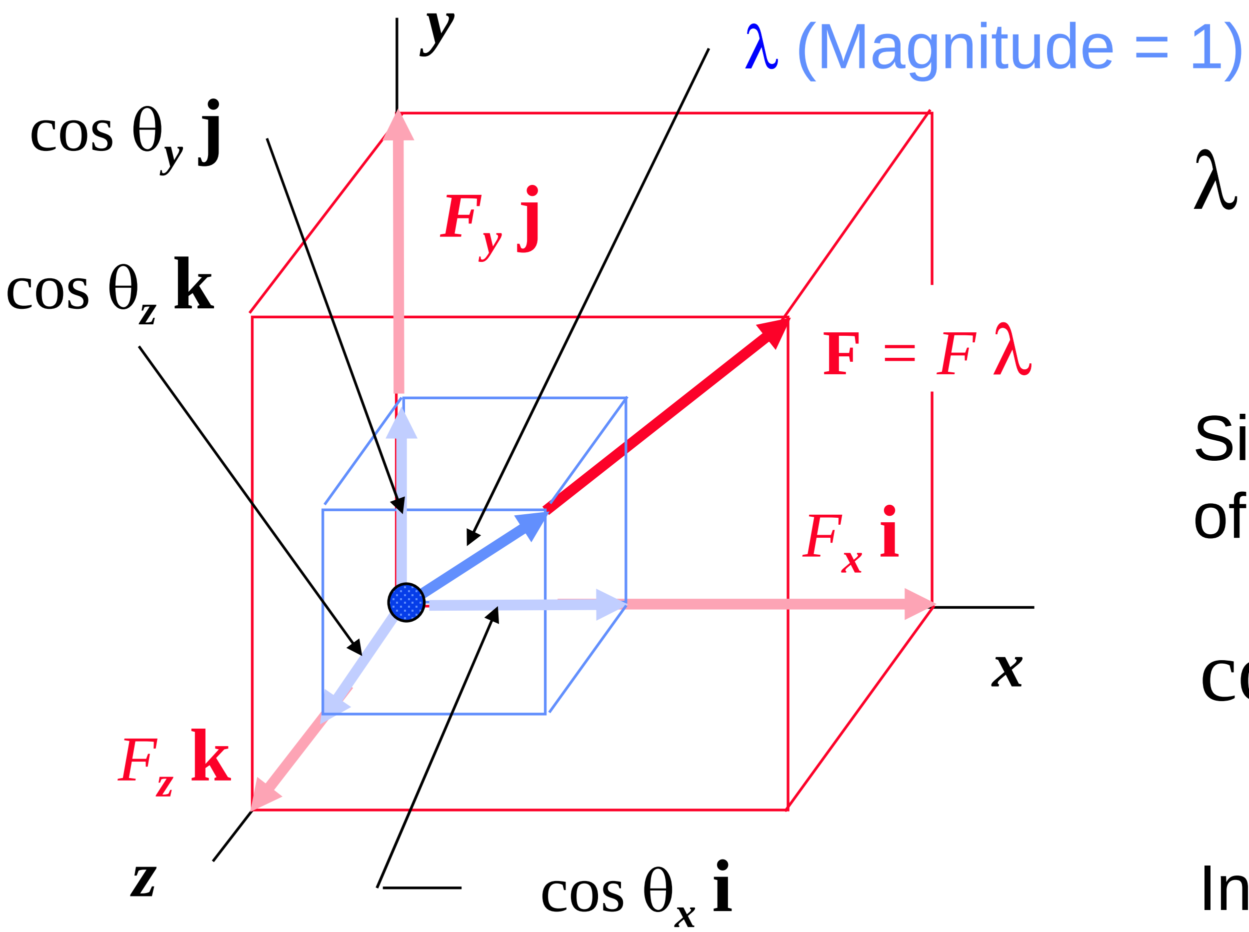


The cosines of θ_x , θ_y , and θ_z are known as the *direction cosines* of the force **F**. Using the unit vectors **i** , **j**, and **k**, we write

$$\mathbf{F} = F_x\,\mathbf{i} + F_y\,\mathbf{j} + F_z\,\mathbf{k}$$

or

$$\mathbf{F} = F\,(\cos\theta_x\,\mathbf{i} + \cos\theta_y\,\mathbf{j} + \cos\theta_z\,\mathbf{k}\,)$$



$$\lambda = \cos\theta_x\,\mathbf{i} + \cos\theta_y\,\mathbf{j} + \cos\theta_z\,\mathbf{k}$$

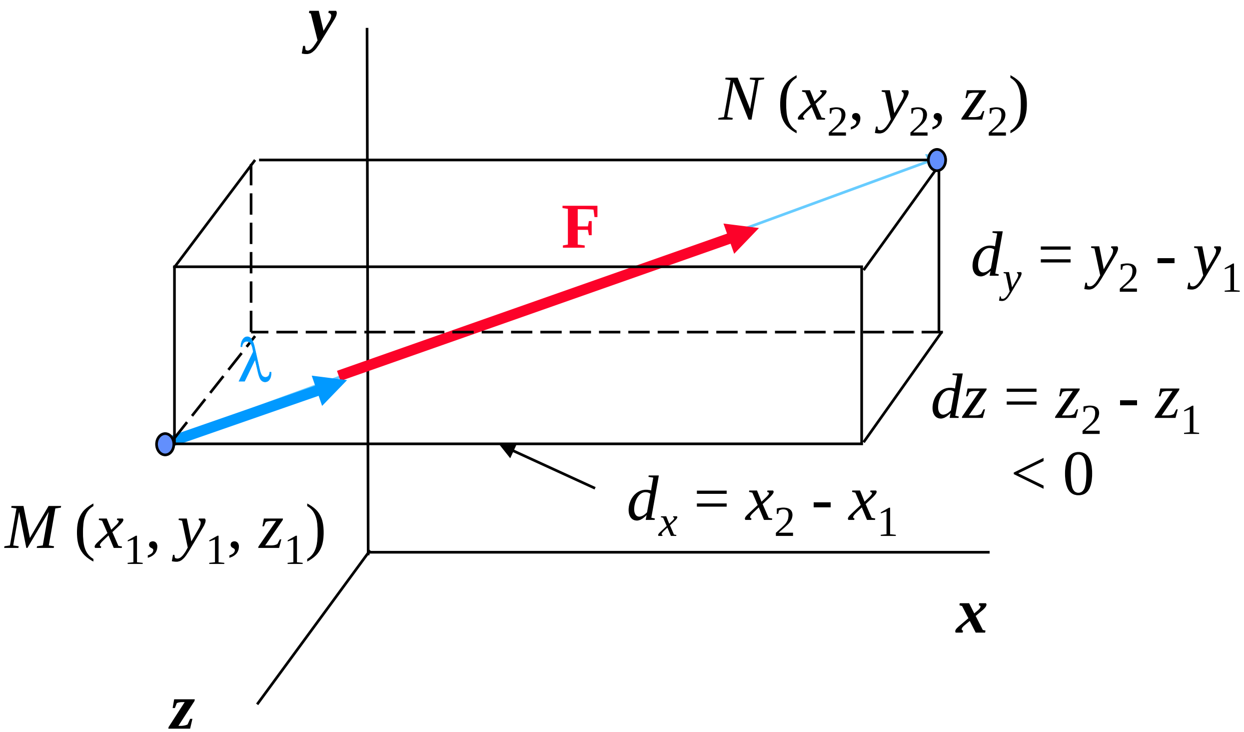
Since the magnitude of λ is unity, we have

$$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$$

In addition,

$$F = \sqrt{F_x^{\,2} + F_y^{\,2} + F_z^{\,2}}$$

$$\cos\theta_x = \frac{F_x}{F} \qquad \cos\theta_y = \frac{F_y}{F} \qquad \cos\theta_z = \frac{F_z}{F}$$

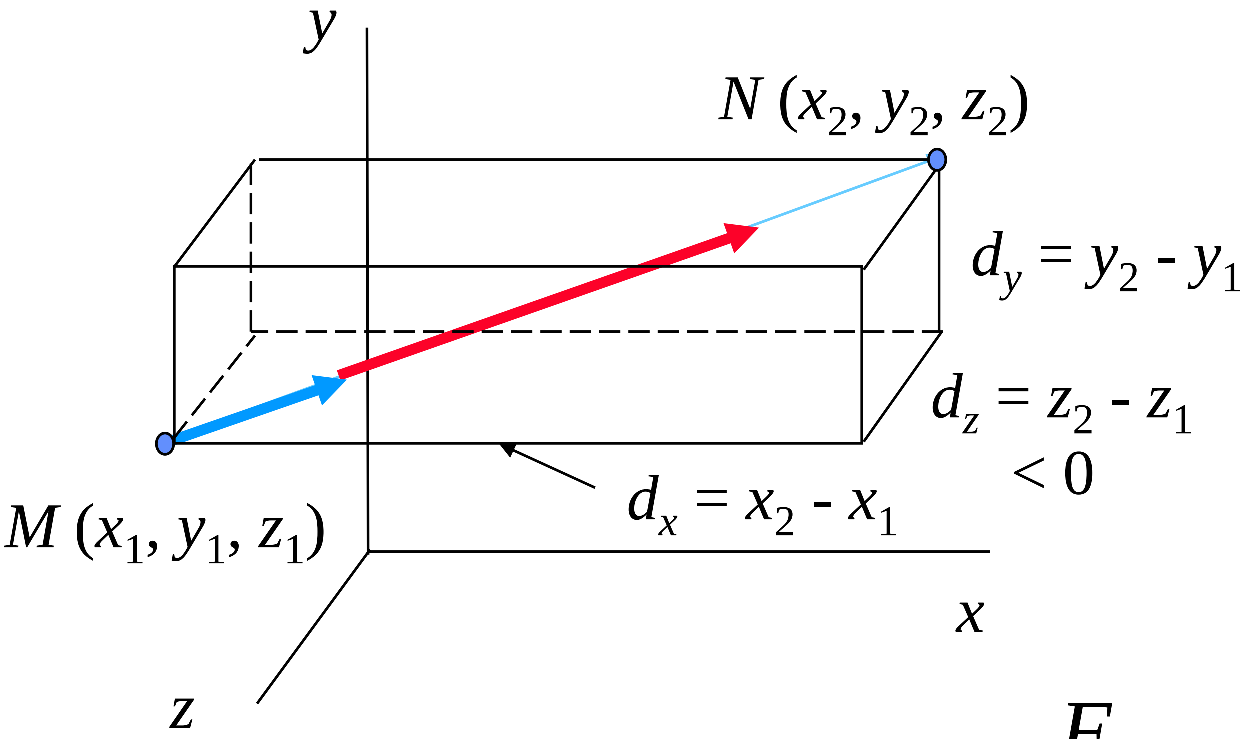


A force vector **F** in three-dimensions is defined by its magnitude F and two points M and N along its line of action. The vector $\overrightarrow{MN}$ joining points M and N is

$$\overrightarrow{MN} = d_x\,\mathbf{i} + d_y\,\mathbf{j} + d_z\,\mathbf{k}$$

The unit vector λ along the line of action of the force is

$$\lambda = \frac{\overrightarrow{MN}}{MN} = \frac{1}{d}\left(d_x\,\mathbf{i} + d_y\,\mathbf{j} + d_z\,\mathbf{k} \right)$$



$$d = \sqrt{d_x^{\,2} + d_y^{\,2} + d_z^{\,2}}$$

A force **F** is defined as the product of F and λ. Therefore,

$$\mathbf{F} = F\,\lambda = \frac{F}{d}\,(\,d_x\,\mathbf{i} + d_y\,\mathbf{j} + d_z\,\mathbf{k}\,)$$

From this it follows that

$$F_x = \frac{Fd_x}{d} \qquad F_y = \frac{Fd_y}{d} \qquad F_z = \frac{Fd_z}{d}$$

When ***two or more forces*** act on a particle in ***three-dimensions***, the rectangular components of their resultant **R** is obtained by adding the corresponding components of the given forces.

$$R_x = \Sigma F_x$$
$$R_y = \Sigma F_y$$
$$R_z = \Sigma F_z$$

The particle is in ***equilibrium*** when the resultant of all forces acting on it is zero.

To solve a problem involving a particle in equilibrium, draw a ***free-body diagram*** showing all the forces acting on the particle. The conditions which must be satisfied for particle equilibrium are

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma F_z = 0$$

In ***two-dimensions*** , only two of these equations are needed

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0$$