Chapter 4 EQUILIBRIUM OF RIGID BODIES

In the study of the ***equilibrium of rigid bodies***, i.e. the situation when the external forces acting on a rigid body ***form a system equivalent to zero***, we have

$$\Sigma \mathbf{F} = 0 \qquad \Sigma \mathbf{M_O} = \Sigma (\mathbf{r} \times \mathbf{F})$$

Resolving each force and each moment into its rectangular components, the necessary and sufficient conditions for the equilibrium of a rigid body are expressed by six scalar equations:

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma F_z = 0$$
$$\Sigma M_{\boldsymbol{x}} = 0 \qquad \Sigma M_{\boldsymbol{y}} = 0 \qquad \Sigma M_{\boldsymbol{z}} = 0$$

These equations can be used to determine unknown forces applied to the rigid body or unknown reactions exerted by its supports.

When solving a problem involving the equilibrium of a rigid body, it is essential to consider ***all*** of the forces acting on the body. Therefore, the first step in the solution of the problem should be to draw a ***free-body diagram*** showing the body under consideration and all of the unknown as well as known forces acting on it.

In the case of the ***equilibrium of two-dimensional structures***, each of the reactions exerted on the structure by its supports could involve one, two, or three unknowns, depending upon the type of support.

In the case of a two-dimensional structure, ***three equilibrium equations*** are used, namely

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma M_A = 0$$

where *A* is an arbitrary point in the plane of the structure.

$$\Sigma F_x = 0 \qquad \Sigma F_{\boldsymbol{y}} = 0 \qquad \Sigma M_{\boldsymbol{A}} = 0$$

These equations can be used to solve for three unknowns. While these three equilibrium equations cannot be ***augmented*** with additional equations, any one of them can be ***replaced*** by another equation. Therefore, we can write alternative sets of equilibrium equations, such as

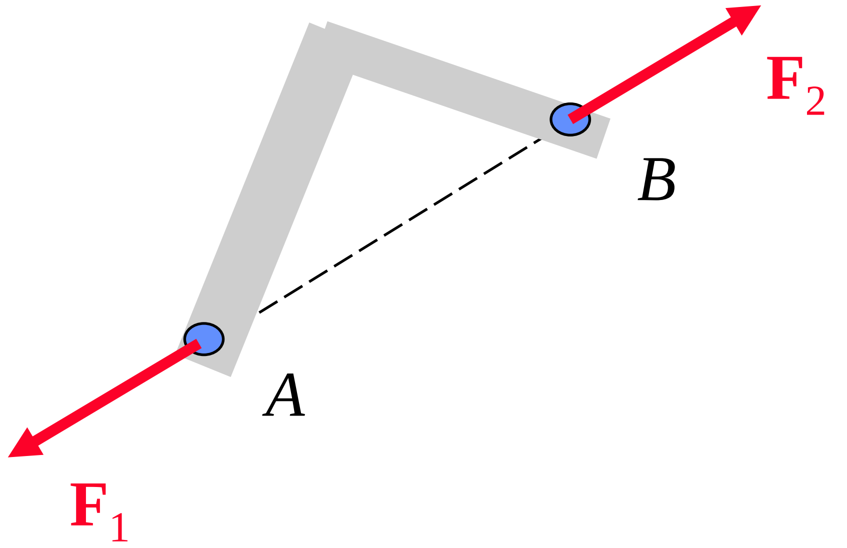
$$\Sigma F_{\boldsymbol{x}} = 0 \qquad \Sigma M_{\boldsymbol{A}} = 0 \qquad \Sigma M_{\boldsymbol{B}} = 0$$

where point B is chosen in such a way that the line AB is not parallel to the y axis, or

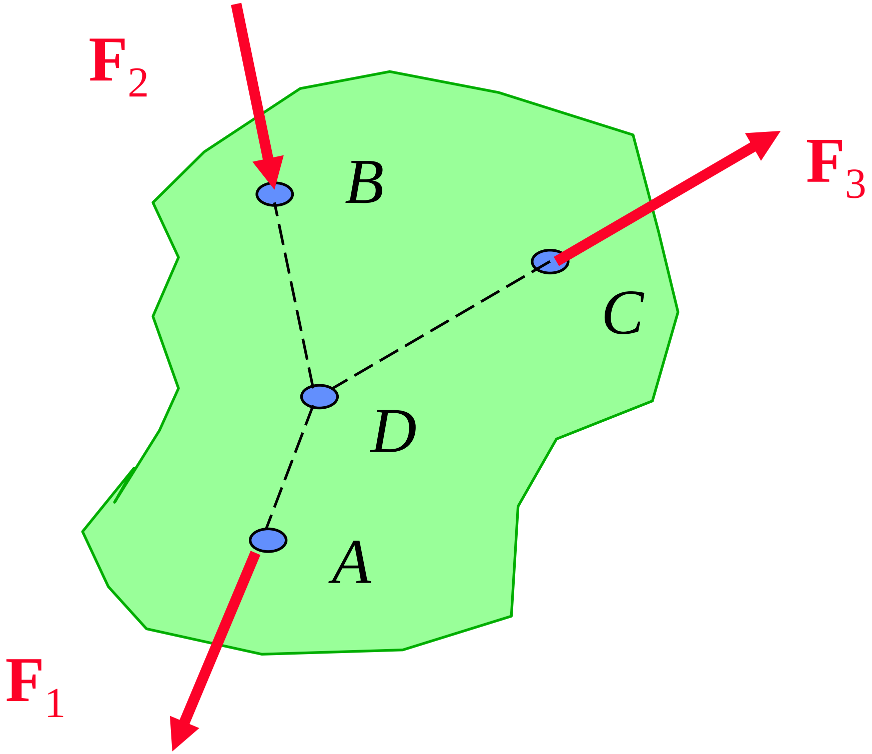
$$\Sigma M_{\boldsymbol{A}} = 0 \qquad \Sigma M_{\boldsymbol{B}} = 0 \qquad \Sigma M_{\boldsymbol{C}} = 0$$

where points A, B, *and* C do not lie in a straight line.

Since any set of equilibrium equations can be solved for only three unknowns, the reactions at the supports of a rigid two-dimensional structure cannot be completely determined if they involve ***more than three unknowns***; they are said to be ***statically indeterminate***. On the other hand, if the reactions involve ***fewer than three unknowns***, equilibrium will not be maintained under general loading conditions; the structure is said to be ***partially constrained***. The fact that the reactions involve exactly three unknowns is no guarantee that the equilibrium equations can be solved for all three unknowns. If the supports are arranged is such a way that the reactions are ***either concurrent or parallel***, the reactions are statically indeterminate, and the structure is said to be ***improperly constrained***.



Two particular cases of rigid body equilibrium are given special attention. A ***two-force body*** is a rigid body subjected to forces at only two points. The resultants $\mathbf{F_1}$ and $\mathbf{F_2}$ of these two forces must have the ***same magnitude, the same line of action, and opposite sense***.



A ***three-force body*** is a rigid body subjected to forces at only three points, and the resultants $\mathbf{F_1}$, $\mathbf{F_2}$,and $\mathbf{F_3}$ of these forces must be ***either concurrent or parallel***. This property provides an alternative approach to the solution of problems involving a three-force body.

When considering the ***equilibrium of a three-dimensional body***, each of the reactions exerted on the body by its supports can involve between one and six unknowns, depending upon the type of support.

In the general case of the equilibrium of a three-dimensional body, the six scalar equilibrium equations listed at the beginning of this review should be used and solved for ***six unknowns***. In most cases these equations are more conveniently obtained if we first write

$$\Sigma \mathbf{F} = 0 \qquad\qquad \Sigma \mathbf{M_O} = \Sigma (\mathbf{r} \times \mathbf{F})$$

and express the forces $\mathbf{F}$ and position vectors $\mathbf{r}$ in terms of scalar components and unit vectors. The vector product can then be computed either directly or by means of determinants, and the desired scalar equations obtained by equating to zero the coefficients of the unit vectors.

As many as three unknown reaction components can be eliminated from the computation of $\Sigma\, M_O$ through a judicious choice of point *O*. Also, the reactions at two points A and B can be eliminated from the solution of some problems by writing the equation $\Sigma\, M_{AB} = 0$, which involves the computation of the moments of the forces about an axis AB joining points A and B.

If the reactions involve more than six unknowns, some of the reactions are ***statically indeterminate***; if they involve fewer than six unknowns, the rigid body is only ***partially constrained***.

Even with six or more unknowns, the rigid body will be improperly constrained if the reactions associated with the given supports either are parallel or intersect the same line.