Chapter 1

Discrete Probability Distributions

Random Variable

A random variable is a numerical description of the outcome of an experiment. The particular numerical value of the random variable depends on the outcome of the experiment. A random variable can be classified as being either *discrete* or **continuous** depending on the numerical values it assumes.

1. Discrete Random Variables

= A random variable that may assume either a finite number of values or an infinite sequence of values such as 0, 1, 2, … is referred to as a discrete random variable. However, the outcomes of some experiments may not be described by numerical values (0, 1, 2, …). For example, gender (male and female). In this case, we can still describe these experimental outcomes numerically as follows:
(male = 0 and female = 1).

(Examples of Discrete Random Variables)

Experiment	**Random Variable**	**Possible Value**
1. Operate a restaurant for one day	Number of customers	0, 1, 2, 3, …
2. Sell an automobile	Gender of the customer	0 = male; 1 = female

Think about 4 more examples by yourself:

2. Continuous Random Variables

A random variable that may assume any numerical value in an interval or collection of intervals is called a **continuous random variable.** Experimental outcomes based on measurement scales such as time, weight, distance, and temperature can be described by continuous random variables.

(Examples of continuous random variables)

Experiment	**Random Variable**	**Possible Value**
1. Operate a bank	Time between customer arrivals (minute)	$x \geq 0$
2. Construct a new library	Percentage of project complete after 6 months	$0 \leq x \leq 100$

Think about 4 more examples by yourself:

Exercises

1. Consider the experiment of tossing a coin twice.

a. List the experimental outcomes.

b. Define a random variable that represents the number of heads occurring on the two tosses.

c. Show what value the random variable would assume for each of the experimental outcomes.

d. Is this random variable discrete or continuous?

2. Consider the experiment of a worker assembling a product.

a. Define a random variable that represents the time in minutes required to assemble the product.

b. What values may the random variable assume?

c. Is the random variable discrete or continuous?

3. Suppose we know home mortgage rates for 12 Florida lending institutions. Assume that the random variable is the number of lending institutions in this group that offers a 30-year fixed rate of 8.5% or less. What values may this random variable assume?

4. Listed below is a series of experiments and associated random variables. In each case, identify the values that the random variable can assume and state whether the random variable is discrete or continuous.

Experiment	Random Variable (x)
a. Take a 20-question examination	Number of questions answered correctly
b. Audit 50 tax returns	Number of returns containing errors
c. Observe an employee's work	Number of nonproductive hours in an eight-hour workday

Discrete Probability Distributions

The **probability distribution** for a random variable describes how probabilities are distributed over the values of the random variable. For a discrete random variable *x*, the probability distribution is defined by a **probability function**, denoted by *f(x)*. The probability function provides the probability for each value of the random variable.

Required conditions for a discrete probability function:

$f(x) \geq 0$

$\sum f(x) = 1$

Exercise

1. Consider the sales of automobiles at DiCarlo Motors in Saratoga, New York. Over the past 300 days of operation, Sales data show 54 days with no automobiles sold, 117 days with 1 automobile sold, 72 days with 2 automobiles sold, 42 days with 3 automobiles sold, 12 days with 4 automobiles sold, and 3 days with 5 automobiles sold. Suppose we consider the experiment of selecting a day of operation at DiCarlo Motors and define the random variable of interest as *x* = the number of automobiles sold during a day.

(Solution)

Probability Distribution for the Number of Automobiles Sold During a Day at Dicarlo Motors

x	**f(x)**
0	
1	
2	
3	
4	
5	
Total	

2. Drawing: Graphical Representation of the Probability Distribution for the Number of Automobiles Sold During a Day at Dicarlo Motors

Exercise

1. Suppose that for the experiment of rolling a die, we define that random variable x to be the number of dots on the upward face. For this experiment, n = 6 values are possible for the random variable; x = 1, 2, 3, 4, 5, 6. Fill in the following table.

x	**f(x)**
0	
1	
2	
3	
4	
5	
Total	

2. The probability distribution for the random variable x follows.

x	f(x)
20	0.20
25	0.15
30	0.25
35	0.40

   a. Is this probability distribution valid? Explain.

   b. What is the probability that x = 30?

   c. What is the probability that x is less than or equal to 25?

   d. What is the probability that x is greater than 30?

3. The director of admissions at Lakeville Community College subjectively assessed a probability distribution for x, the number of entering students, as follows.

x	**f(x)**
1000	0.15
1100	0.20
1200	0.30
1300	0.25
1400	0.10

   a. Is this probability distribution valid? Explain.

   b. What is the probability of 1200 or fewer entering students?

4. Table below shows the percent frequency distributions of job satisfaction scores for a sample of information systems (IS) senior executives and IS middle managers. The scores range from a low of 1 (very dissatisfied) to a high of 5 (very satisfied)

Percent Frequency Distribution of Job Satisfaction Scores for Information Systems Executives and Middle Managers

Job Satisfaction Score	***IS Senior Executives (%)***	***IS Middle Managers (%)***
1	*5*	*4*
2	*9*	*10*
3	*3*	*12*
4	*42*	*46*
5	*41*	*28*

a. Develop a probability distribution for the job satisfaction score of a senior executive.

b. Develop a probability distribution for the job satisfaction score of a middle manager.

c. What is the probability a senior executive will report a job satisfaction score of 4 or 5?

d. What is the probability a middle manager is very satisfied?

e. Compare the overall job satisfaction of senior executives and middle managers.

Expected Value and Variance

Expected Value

The expected value, or mean, of a random variable is a measure of the central location for the random variable.

Expected Value: $E(x) = \mu = \Sigma x f(x)$

Exercise

1. According to the case of DiCarlo Motors on page 4, how many automobiles are expected to sell per day? Also forecast the average monthly sales, in other words, how many automobiles are expected to sell in 30 days?

Variance

Variance helps to explain the variability (dispersion) in values of a random variable.

Variance of a Discrete Random Variable: $Var(x) = \sigma^2 = \Sigma(x - \mu)^2 f(x)$

Standard Deviation

$\sigma = \sqrt{\sigma^2}$

2. Using the information from the previous question, find variance and standard deviation of the discrete random variables.

3. The following table provides a probability distribution for the random variable x.

x	**f(x)**
3	0.25
6	0.50
9	0.25

a. Compute the expected value of x.

b. Compute the variance of x.

c. Compute the standard deviation of x.

4. The American Housing Survey reported the following data on the number of bedrooms in owner-occupied and renter-occupied houses in central cities:

	Number of Houses (1000s)	
Bedrooms	**Renter-Occupied**	**Owner-Occupied**
0	547	23
1	5012	541
2	6100	3832
3	2644	8690
4 or more	557	3783

a. Define a random variable x = number of bedrooms in renter-occupied houses and develop a probability distribution for the random variable. (Let x = 4 represent 4 or more bedrooms.)

b. Compute the expected value and variance for the number of bedrooms in renter-occupied houses.

c. Define a random variable y = number of bedrooms in owner-occupied houses and develop a probability distribution for the random variable. (Let y = 4 represent 4 or more bedrooms.)

d. Compute the expected value and variance for the number of bedrooms in owner occupied houses.

e. What observations can you make from a comparison of the number of bedrooms in renter-occupied versus owner-occupied homes?

Binomial Probability Distribution

The binomial probability distribution is associated with a multiple-step experiment that we call the binomial experiment.

A Binomial Experiment

Properties for a Binomial Experiment:

1. The experiment consists of a sequence of *n* identical trials.
2. Two outcomes are possible on each trial. We refer to one outcome as a *success* and the other outcome *failure.*
3. The probability of a success, denoted by *p*, does not change from trial to trial. Consequently, the probability of a failure, denoted by *1 – p*, does not change from trial to trial.
4. The trails are independent.

Examples

1. Consider the experiment of tossing a coin five times and on each toss observing whether the coin lands with a head or tail on its upward face. Suppose we want to count the number of heads appearing over the five tosses. Does this experiment show the properties of a binomial experiment? What is the random variable of interest?

2. Consider an insurance salesperson who visits 10 randomly selected families. The outcome associated with each visit is classified as a success if the family purchases an insurance policy and a failure if the family does not. From past experience, the sales person knows the probability that a randomly selected family will purchase an insurance policy is 10%. Does this experiment show the properties of a binomial experiment? What is the random variable of interest?