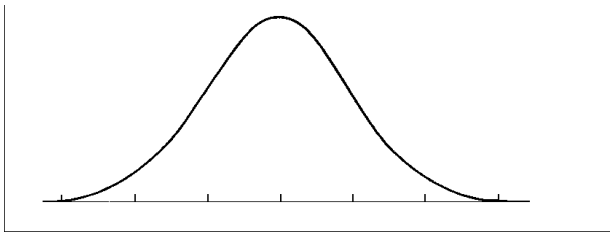
Chapter 2: Continuous Probability Distributions



For continuous random variables, the counterpart of the probability function is the probability density function, also denoted by *f(x).* The area under the graph of *f(x)* corresponding to a given interval does provide the probability that the continuous random variable *x* assumes a value in that interval.

1. Uniform Probability Distribution

Consider the random variable x representing the flight time of an airplane traveling from Chicago to New York. Suppose the flight time can be any value in the interval from 120 minutes to 140 minutes. Further assume that the probability of a flight time within any 1-minute interval is the same as the probability of a flight time within any other 1-minute interval contained in the larger interval from 120 to 140 minutes.

With 1-minute interval being equally likely, the random variable x is said to have a uniform probability distribution. The probability density function, which defines the uniform distribution for the flight-time random variable, is

$$f(x) = \begin{cases} 1/20 & for \ \ 120 \leq x \ \leq 140 \\ 0 & elsewhere \end{cases}$$

Draw a graph of the above probability density function:

The uniform probability function for a variable x is defined by the following formula:

$$f(x) = \begin{cases} \frac{1}{b-a} & for \ \ a \leq x \ \leq b \\ 0 & elsewhere \end{cases}$$

Exercise

1. According to the above question, what is the probability that the flight time must be between 120 and 130 minutes.

Area as a Measure of Probability

The Area Under the Graph f(x) = The Probability f(x)

2. What is the probability of a flight time between 128 and 136 minutes?

3. What is the probability of a flight time between 120 and 140 minutes?

Formulas:

Expected Value: $E(x) = \frac{a + b}{2}$

Variance $Var(x) = \frac{(b-a)^2}{12}$

Standard Deviation $SD(x) = \sqrt{Var(x)}$

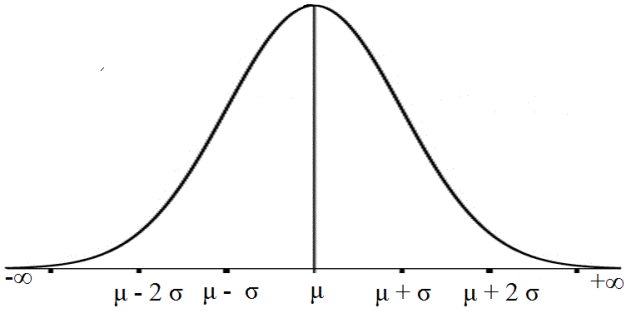
4. Using the above information, find E(x), Var(x), and SD(x).

5. The random variable x is known to be uniformly distributed between 10 and 20.
    a. Show the graph of the probability density function.
    b. Compute P(x = 15)
    c. Compute P(x < 15)
    d. Compute P(12 ≤ x ≤ 18)
    e. Compute E(x)
    f. Compute Var(x)

2. Normal Probability Distribution

Normal Curve

The bell-shape normal curve represents the normal distribution:



Normal Probability Density Function: $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$

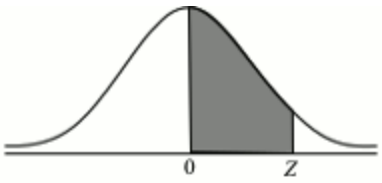
Where μ = mean, σ = SD, π 3.14159, e = 2.71828

The characteristics of the normal distribution:

a. The normal distributions are differentiated by two parameters: the mean and standard deviation.
b. The highest point on the normal curve is the mean, median, and mode of the distribution.
c. The mean of the distribution can be any numerical value (negative, zero, or positive)
d. The normal distribution is symmetric. The tails of the curve extend to infinity in both directions and never touch the horizontal axis.
e. The standard deviation determines how flat and wide the curve is. Larger values of the standard deviation result in wider and flatter curves.
f. Probabilities for the normal random variable are given by areas under the curve. The total area under the curve for the normal distribution is 1. The mean position divide the graph into 0.50 and 0.50.
g. The percentage of values in some commonly used intervals are:
    i. ± 1 σ → 68.3%
    ii. ± 2 σ → 95.4%
    iii. ± 3 σ → 99.7%

Standard Normal Probability Distribution

Standard Normal Density Function: $f(x) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$



Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998									
4.0	0.49997									
4.5	0.499997									
5.0	0.4999997									
6.0	0.499999999									

Exercises

1. Compute P(0.00 ≤ z ≤ 1.00)

2. Compute P(0.00 ≤ z ≤ 1.25)

3. Compute P(-1.00 ≤ z ≤ 1.00)

4. Find the probability of a z value between -2.00 and +2.00.

5. Compute the probability of obtaining a z value of at least 1.58.

6. Compute the probability of obtaining a z value of -0.50 or larger.

7. Compute the probability of obtaining a z value between 1.00 and 1.58.

8. Find a z value such that the probability of obtaining a larger z value is 0.10.

Computing Probabilities for Any Normal Distribution

The formula used to convert any normal random variable x with mean μ and standard deviation σ to the standard normal distribution follow:

$$z = \frac{x - \mu}{\sigma}$$

Examples: Grear Tire Company Problem

To launch a new product, Grear's managers believe that the mileage guarantee offered with the tire will be an important factor in the acceptance of the product. Before finalizing the tire mileage guarantee policy, Grear's managers want probability information about the number of miles the tires will last.

From actual road tests with the tires, Grear's engineering group estimates the mean tire mileage is 36,500 miles and that the standard deviation is 5,000. In addition, the data collected indicate a normal distribution is a reasonable assumption. What percentage of tires can be expected to last more than 40,000 miles?

Now assume that Grear is considering a guarantee that will provide a discount on replacement tires if the original tires do not exceed the mileage stated in the guarantee. What should the guaranteed mileage be if Grear wants no more than 10% of the tires to be eligible for the discount guarantee?

Exercises

1. Given that z is a standard normal random variable, find z for each situation
    a. The area between 0 and z is 0.4750
    b. The area between 0 and z is 0.2291
    c. The area to the right of z is 0.1314
    d. The area to the left of z is 0.6700

2. The average stock price for companies making up the S&P 500 is $30, and the standard deviation is $8.20 (*Business Week*, Special Annual Issue, Spring 2003). Assume the stock prices are normally distributed.
    a. What is the probability a company will have a stock price of at least $40?
    b. What is the probability a company will have a stock price no higher than $20?
    c. How high does a stock price have to be to put a company in the top 10%?

3. According to the Bureau of Labor Statistics, the average weekly pay for a U.S. production worker was $441.84 (*The World Almanac*, 2000). Assume that available data indicate that production worker wages were normally distributed with a standard deviation of $90.
    a. What is the probability that a worker earned between $400 and $500?
    b. How much did a production worker have to earn to be in the top 20% of wage earners?
    c. For a randomly selected production worker, what is the probability the worker earned less than $250 per week?