Chapter 5: Hypothesis Tests

The null hypothesis (H_0) = A tentative assumption

The alternative hypothesis (H_a) = The opposite of H_a

This chapter shows how hypothesis tests can be conducted about a population mean and a population proportion.

Developing Null and Alternative Hypotheses

Hypothesis tests about a population parameter may take one of three following forms:

1. H_0: $\mu \geq \mu_0$ and H_a: $\mu < \mu_0$
2. H_0: $\mu \leq \mu_0$ and H_a: $\mu > \mu_0$
3. H_0: $\mu = \mu_0$ and H_a: $\mu \neq \mu_0$

From above, 1. and 2. are called one-tailed tests. 3. is called a two-tailed test. In many situations, the choice of H_0 and H_a is not obvious and judgment is necessary to select the proper form.

If H_0 is no rejected (accepted) → No conclusion can be drawn.

If H_0 is rejected → We can accept H_a and make statistical inference accordingly.

Exercises

1. The manager of an automobile dealership is considering a new bonus plan designed to increase sales volume. Currently, the mean sales volume is 14 automobiles per month. The manager wants to conduct a research study to see whether the new bonus plan increases sales volume. To collect data on the plan, a sample of sales personnel will be allowed to sell under the new bonus plan for a one-month period.

a. Develop the null and alternative hypotheses most appropriate for this research situation.
b. Comment on the conclusion when H_0 cannot be rejected.
c. Comment on the conclusion when H_0 can be rejected.

2. Because of high production-changeover time and costs, a director of manufacturing must convince management that a proposed manufacturing method reduces costs before the new method can be implemented. The current production method operates with a mean cost of $220 per hour. A research study will measure the cost of the new method over a sample production period.
    a. Develop the null and alternative hypotheses most appropriate for this study.
    b. Comment on the conclusion when H0 cannot be rejected.
    c. Comment on the conclusion when H0 can be rejected.

Type I and Type II Errors

		Population Condition	
		H_0 True	**H_a True**
Conclusion	**Accept H_0**	Correct Conclusion	Type II Error
	Reject H_0	Type I Error	Correct Conclusion

Because hypothesis tests are based on sample information, the possibility of errors may occur.

Type I Error occurs when we reject H_0 while the population condition for H_0 is true.

Type II Error occurs when we accept H_0 while the population condition for H_a is true.

Level of significance (α) is the probability of making a Type I error when the null hypothesis is true as an equality.

In practice, to conduct the hypothesis test, we can specify the level of significance. By selecting α, we are controlling the probability of making a Type I error. Small values of α are preferred.

Significance tests are applications of hypothesis testing that only control for Type I error.

Exercises

1. Nielsen reported that young men in the United States watch 56.2 minutes of prime-time TV daily. A researcher believes that young men in Germany spend more time watching prime-time TV. A sample of German young men will be selected by the researcher and the time they spend watching TV in one day will be recorded. The sample results will be used to test the following null and alternative hypotheses.

   H_0: $\mu \leq 56.20$ and H_a: $\mu > 56.20$

   a. What is Type I error in this situation? What are the consequences of making this error?
   b. What is the Type II error in this situation? What are the consequences of making this error?

2. The label on a 3-quart container of orange juice claims that the orange juice contains an average of 1 gram of fat or less. Answer the following questions for a hypothesis test that could be used to test the claim on the label.
    a. Develop the appropriate null and alternative hypotheses.
    b. What is the Type I error in this situation? What are the consequences of making this error?
    c. What is the Type II error in this situation? What are the consequences of making this error?

3. Suppose a new production method will be implemented if a hypothesis test supports the conclusion that the new method reduces the mean operating cost per hour.
    a. State the appropriate null and alternative hypotheses if the mean cost for the current production method is $220 per hour.
    b. What is the Type I error in this situation? What are the consequences of making this error?
    c. What is the Type II error in this situation? What are the consequences of making this error?

Population Mean: σ Known

The methods presented in this section are exact if the sample is selected from a population that is normally distributed. In cases where it in not reasonable to assume the population is normally distributed, these methods are still applicable if the sample size is large enough. We provide some practical advice concerning the population distribution and the sample size at the end of this section.

One-Tailed Test

Lower Tail Test: $H_0: \mu \geq \mu_0$ and $H_a: \mu < \mu_0$

Upper Tail Test: $H_0: \mu \leq \mu_0$ and $H_a: \mu > \mu_0$

Test Statistic: Population Mean (σ known)

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$$

Rejection Rules

There are 2 approaches for rejection rule: p-value and critical value.

1. P-value approach: Reject H_0 if p-value $\leq \alpha$
2. Critical value approach: Reject H_0 if $z \leq z_\alpha$ or $z \geq z_\alpha$

(Notes)

Exercises

1. Consider the following hypothesis test:

$$H_0:\ \mu \ \leq \ 25$$
$$H_a:\ \mu \ > \ 25$$

A sample of 40 provided a sample mean of 26.4. The population standard deviation is 6.

 a. Compute the value of the test statistic.
 b. What is the p-value?
 c. At $\alpha = 0.01$, what is your conclusion?
 d. What is the rejection rule using the critical value? What is your conclusion?

2. Consider the following hypothesis test:

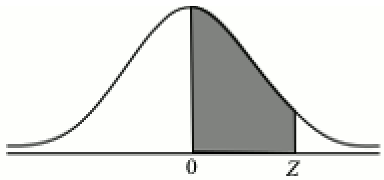
$$H_0:\ \mu \ \geq\ 80$$
$$H_a:\ \mu \ <\ 80$$

A sample of 100 is used and the population standard deviation is 12. Compute the p-value and state your conclusion for each of the following sample results. Use α = 0.01.

a. $\bar{x} = 78.50$
b. $\bar{x} = 77$
c. $\bar{x} = 75.50$
d. $\bar{x} = 81$

Standard Normal Probability Distribution

Standard Normal Density Function: $f(x) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$



Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998									
4.0	0.49997									
4.5	0.499997									
5.0	0.4999997									
6.0	0.499999999									