Chapter 7: Two-Sample Tests of Hypothesis

I. Two-Sample Tests of Hypothesis: Independent Samples

***Two-Sample Test of Means – Known σ

$$Z = \frac{\overline{X_1} - \overline{X_2}}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

Important Assumptions

1. The two samples must be unrelated, that is, independent.
2. The standard deviations for both populations must be known

Exercises

1. A sample of 40 observations is selected from one population with a population standard deviation of 5. The sample mean is 102. A sample of 50 observations is selected from a second population standard deviation of 6. The sample mean is 99. Conduct the following test hypothesis using the 0.04 significance level.
   H_0: $\mu_1 = \mu_2$ and H_a: $\mu_1 \neq \mu_2$
   a. Is this a one-tailed or a two-tailed test?
   b. Compute the value of the test statistic.
   c. What is the p-value?
   d. What is your decision regarding H_0?

2. Gibbs Baby Food Company wishes to compare the weight gain of infants using its brand versus its competitors. A sample of 40 babies using the Gibbs products revealed a mean weight gain of 7.6 pounds in the first three months after birth. For the Gibbs brand, the population standard deviation of the sample is 2.3 pounds. A sample of 55 babies suing the competitor's brand revealed a mean increase in weight of 8.1 pounds. The population standard deviation is 2.9 pounds. At the 0.05 significance level, can we conclude that babies using the Gibbs brand gained less weight? Compute the p-value and interpret it.

3. A financial analyst wants to compare the turnover rates, in percent, for shares of oil-related stocks versus other stocks, such as GE and IBM. She selected 32 oil-related stocks and 49 other stocks. The mean turnover rate of oil-related stocks is 31.4 percent and the population standard deviation 5.1 percent. For the other stocks, the mean rate was computed to be 34.9 percent and the population standard deviation 6.7 percent. Is there a significant difference in the turnover rates of the two types of stock? Use the 0.01 significance level.

***Two-Sample Test of Means – Unknown σ

$$t = \frac{\overline{X_1} - \overline{X_2}}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$$df = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{(s_1^2/n_1)^2}{n_1 - 1} + \frac{(s_2^2/n_2)^2}{n_2 - 1}}$$

Exercises

1. A recent article in The Wall Street Journal compared the cost of adopting children from China with that of Russia. For a sample of 16 adoptions from China, the mean cost was \$11,045 with a standard deviation of \$835. For a sample of 18 adoptions from Russia, the mean cost was \$12,840 with a standard deviation of \$1,545. Can we conclude the mean cost is larger for adopting children from Russia? Assume the two population standard deviations are not the same. Use the 0.05 significance level.

2. Given: H_0: $\mu_1 = \mu_2$ and H_a: $\mu_1 \neq \mu_2$

A random sample of 15 items from the first population showed a mean of 50 and a standard deviation of 5. A sample of 12 items for the second population showed a mean of 46 and a standard deviation of 15. Assume the sample populations do not have equal standard deviations and use the 0.05 significance level: a) determine the number of degrees of freedom, b) compute the value of the test statistic, and c) state your decision about the null hypothesis

II. Two-Sample Tests of Hypothesis: Independent Samples

***Paired t Test – Unknown σ

$$t = \frac{\bar{d}}{s_d/\sqrt{n}}$$

$\bar{d}$ is the mean of the difference between the paired or related observations.

s_d is the standard deviation of the differences between the paired or related observations

n is the number of paired observations.

df = n - 1

$$s_d = \sqrt{\frac{\sum(d - \bar{d})^2}{n-1}}$$

Exercises

1. Nickel Bank selected a sample of 10 residential properties and scheduled both firms for an appraisal. The results reported in $000 are:

Home	1	2	3	4	5	6	7	8	9	10
Schadek	235	210	231	242	205	230	231	210	225	249
Bowyer	228	205	219	240	198	223	227	215	222	245

At the 0.05 significant level, can we conclude that there is a difference in the mean appraised values of the homes?

2. The following sample information shows the number of defective units produced on the day shift and the afternoon shift for a sample of four days last month.

Day	1	2	3	4
Day Shift	10	12	15	19
Afternoon Shift	8	9	12	15

At the 0.05 significance level, can we conclude there are more defects produced on the afternoon shift?