# **Special Chapter**

# Introduction to Analysis of Variance (ANOVA)

Analysis of variance (ANOVA) is used to test the hypothesis that three or more population means are equal.

# **Hypothesis:**

$$H_0$$
:  $\mu_1 = \mu_2 = \mu_3 = \dots = \mu_k$ 

H<sub>a</sub>: Not all population means are equal

Dependent Variable (Response Variable) = Variable to be compared

Independent Variable (Factor or Treatment) = Names of categories or divisions

For example,

(Note)

#### **Assumptions of ANOVA**

- 1. For each population, the response variable is normally distributed.
- 2. The variance of the response variable is the same for all of the population.
- 3. The observations must be independent.

#### **A Conceptual Overview**

Suppose there are 3 populations. If the means for the three populations are equal, we would expect the three sample means to be close together. Hence, if the variability among the sample means is small, it supports  $H_0$ . If the variability among the sample means is large, it supports  $H_a$ .

We can then measure these variability by analyzing 2 types of variance estimates:

- Between-Treatments Estimate of Population Variance
- Within-Treatments Estimate of Population Variance

### Example:

We are trying to measure how employees at plants located in Atlanta, Dallas, and Seattle understand the concept of Total Quality Management. By doing so, 6 employees from each plant are selected to do the exam and the score for each employee was calculated. To see the difference in abilities of employees in different plants, we can employ ANOVA to test whether there is a difference in mean scores among employees from those plants.

Observation	Atlanta Plant	Dallas Plant	Seattle Plant		
1	85	71	59		
2	75	75	64 62 69		
3	82	73			
4	76	74			
5	71	69	75		
6	85	82	67		
Sample Mean	79	74	66		
Sample Variance	34	20	32		
Sample SD	5.83	4.47	5.66		

(Note: Demonstration via picture)

## Analysis of Variance (ANOVA)

**Test Statistic:** 

$$F = \frac{MSTR}{MSE}$$

Between-Treatments Estimate of Population Variance

MSTR = Mean Square due to Treatment

$$MSTR = \frac{\sum_{j=1}^{k} n_j (\bar{x}_j - \bar{\bar{x}})^2}{k-1}$$

Where, k - 1 = df between treatment (number of group -1)

Within-Treatments Estimate of Population Variance

MSE = Mean Square due to Error

$$MSE = \frac{\sum_{j=1}^{k} (n_j - 1)s_j^2}{n_T - k}$$

Where,  $n_T$  - k = df among errors (total number of observations – number of group)

Use the information from the previous table to determine whether there is a significant difference among the mean scores of employees from the three locations, given 0.05 level of significance.

 $\mathbf{Ex}\ \mathbf{1}$  Five observations were selected from each of three populations. The data obtained follow.

Observation	Sample 1	Sample 2	Sample 3	
1	32	44	33	
2	30	43	36	
3	30	44	35	
4	26	46	36	
5	32	48	40	
Sample mean	30	45	36	
Sample variance	6	4	6.5	

- a) Compute the between-treatments estimate of variance.
- b) Compute the within-treatments estimate of variance.
- c) At the 0.05 level of significance, can we reject the null hypothesis that the means of the three populations are equal?

Ex2

Four observations were selected from each of three populations (test scores of KKUIC students from CA, IA, and TM). The data obtained follow.

Observation	CA	IA	TM
1	165	174	169
2	149	164	154
3	156	180	161
4	142	158	148
Sample mean	153	169	158
Sample variance	96.67	97.33	82

- a) Compute the between-treatments estimate of variance.
- b) Compute the within-treatments estimate of variance.
- c) At the 0.05 level of significance, can we reject the null hypothesis that the means of the three populations are equal?

F - Distribution ( $\alpha$  = 0.05 in the Right Tail)

\ 4t	Numerator Degrees of Freedom								
$df_2 \setminus \frac{dr_1}{dr_2}$	1	2	3	4	5	6	7	8	9
df <sub>2</sub> 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29	1 161.45 18.513 10.128 7.7086 6.6079 5.9874 5.5914 5.3177 5.1174 4.9646 4.8443 4.7472 4.6672 4.6001 4.5431 4.4940 4.4513 4.4139 4.3807 4.3512 4.3248 4.3009 4.2793 4.2597 4.2417 4.2252 4.2100 4.1960 4.1830	2 199.50 19.000 9.5521 9.9443 5.7861 5.1433 4.7374 4.4590 4.2565 4.1028 3.9823 3.8853 3.8056 3.7389 3.6823 3.6337 3.5915 3.5546 3.5219 3.4928 3.4668 3.4434 3.4221 3.4028 3.3852 3.3690 3.3541 3.3404 3.3277		4 224.58 19.247 9.1172 6.3882 5.1922 4.5337 4.1203 3.8379 3.6331 3.4780 3.3567 3.2592 3.1791 3.1122 3.0556 3.0069 2.9647 2.9277 2.8951 2.8661 2.8401 2.8167 2.7955 2.7763 2.7587 2.7426 2.7278 2.7141 2.7014	_	6 233.99 19.330 8.9406 6.1631 4.9503 4.2839 3.8660 3.5806 3.3738 3.2172 3.0946 2.9961 2.9153 2.8477 2.7905 2.7413 2.6987 2.6613 2.6283 2.5990 2.5727 2.5491 2.5277 2.5082 2.4904 2.4741 2.4591 2.4453 2.4324	7 236.77 19.353 8.8867 6.0942 4.8759 4.2067 3.7870 3.5005 3.2927 3.1355 3.0123 2.9134 2.8321 2.7642 2.7066 2.6572 2.6143 2.5767 2.5435 2.5140 2.4876 2.4638 2.4422 2.4226 2.4047 2.3883 2.3732 2.3593 2.3463	238.88 19.371 8.8452 6.0410 4.8183 4.1468 3.7257 3.4381 3.2296 3.0717 2.9480 2.8486 2.7669 2.6987 2.6408 2.5911 2.5480 2.5102 2.4768 2.4471 2.4205 2.3965 2.3748 2.3551 2.3205 2.3053 2.2913 2.2783	240.54 19.385 8.8123 6.9988 4.7725 4.0990 3.6767 3.3881 3.1789 3.0204 2.8962 2.7964 2.7144 2.6458 2.5876 2.5377 2.4943 2.4563 2.4227 2.3928 2.3660 2.3419 2.3201 2.3002 2.2821 2.2655 2.2501 2.2360 2.2229
	4.1830 4.1709 4.0847 4.0012 3.9201	3.3277 3.3158 3.2317 3.1504 3.0718	2.9340 2.9223 2.8387 2.7581 2.6802		2.5454 2.5336 2.4495 2.3683 2.2899				
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19.164 3 10.128 9.5521 9.2766 4 7.7086 9.9443 6.5914 5 6.6079 5.7861 5.4095 6 5.9874 5.1433 4.7571 7 5.5914 4.7374 4.3468 8 5.3177 4.4590 4.0662 9 5.1174 4.2565 3.8625 10 4.9646 4.1028 3.7083 11 4.8443 3.9823 3.5874 12 4.7472 3.8853 3.4903 13 4.6672 3.8056 3.4105 14 4.6001 3.7389 3.3439 15 4.5431 3.6823 3.2874 16 4.4940 3.6337 3.2389 17 4.4513 3.5915 3.1968 18 4.4139 3.5546 3.1599 19 4.3807 3.5219 3.1274 20 4.3512 3.4928 3.0984 21 4.3248 3.4668 3.0725 22 4.3009 3.4434 3.0491 23 4.2793 3.4221 3.0280 24 4.2597 3.4028 3.0088 25 4.2417 3.3852 2.9912 26 4.252 3.3690 2.9752 27 4.2100 3.3541 2.9604 28 4.1960 3.3404 2.9467 29 4.1830 3.3277 2.9340 30 4.1709 3.3158 2.9223 40 4.0847 3.2317 2.8387 60 4.0012 3.1504 2.7581 120 3.9201 3.0718 2.6802	1 161.45 199.50 215.71 224.58 2 18.513 19.000 19.164 19.247 3 10.128 9.5521 9.2766 9.1172 4 7.7086 9.9443 6.5914 6.3882 5 6.6079 5.7861 5.4095 5.1922 6 5.9874 5.1433 4.7571 4.5337 7 5.5914 4.7374 4.3468 4.1203 8 5.3177 4.4590 4.0662 3.8379 9 5.1174 4.2565 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8.9406 4 7.7086 9.9443 6.5914 6.3882 6.2561 6.1631 5 6.6079 5.7861 5.4095 5.1922 5.0503 4.9503 6 5.9874 5.1433 4.7571 4.5337 4.3874 4.2839 7 5.5914 4.7374 4.3468 4.1203 3.9715 3.8660 8.53177 4.4590 4.0662 3.8379 3.6875 3.5806 9 5.1174 4.2565 3.8625 3.6331 3.4817 3.3738 10 4.9646 4.1028 3.7083 3.4780 3.3258 3.2172 11 4.8443 3.9823 3.5874 3.3567 3.2039 3.0946 12 4.7472 3.8853 3.4903 3.2592 3.1059 2.9961 13 4.6672 3.8056 3.4105 3.1791 3.0254 2.9153 14 4.6001 3.7389 3.3439 3.1122 2.9582 2.8477 15 4.5431 3.6823 3.2874 3.0556 2.9013 2.7905 16 4.4940 3.6337 3.2389 3.0069 2.8524 2.7413 19 4.3807 3.5219 3.1274 2.8951 2.7401 2.6283 19 4.3807 3.5219 3.1274 2.8951 2.7401 2.6283 20 4.3512 3.4928 3.0984 2.8661 2.7109 2.5990 21 4.3248 3.4668 3.0725 2.8401 2.6848 2.5727 24 4.2597 3.4028 3.0088 2.7763 2.6207 2.5082 24 4.2597 3.4028 3.0088 2.7763 2.6207 2.5082 29 4.1830 3.3277 2.9340 2.7014 2.5454 2.4324 3.009 3.4434 3.0491 2.8167 2.6613 2.5491 2.4 2.597 3.4028 3.0088 2.7763 2.6207 2.5082 24 4.2597 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2.7109 2.5990 2.5140 2.4471 4.2597 3.4028 3.0886 2.7955 2.8401 2.6888 2.5727 2.4876 2.4205 2.4208 2.4209 3.4443 3.0491 2.8167 2.6888 2.5727 2.4876 2.4205 2.4208 2.4217 3.3852 3.3098 2.9912 2.7569 2.3400 2.5277 2.4422 2.3748 2.4297 3.4221 3.0280 2.7955 2.6400 2.5277 2.4422 2.3748 2.4297 3.4221 3.0280 2.7955 2.6400 2.5277 2.4422 2.3748 2.4297 3.4221 3.0280 2.7955 2.6400 2.5277 2.4422 2.3748 2.4297 3.4221 3.0280 2.7955 2.6400 2.5277 2.4422 2.3748 2.4297 3.4217 3.3852 2.9912 2.7587 2.6613 2.5491 2.4383 2.3965 2.4206 2.3551 2.4206 2.3343 2.2960 2.7955 2.6400 2.5277 2.4422 2.3748 2.2417 3.3852 2.9912 2.7587 2.6613 2.5491 2.4383 2.3965 2.4206 2.3351 2.4260 2.3351 2.4260 2.3351 2.4260 2.3351 2.4260 2.3351 2.4260 2.3351 2.4260 2.3351 2.4260 2.3351 2.3408 2.3965 2.4266 2.3350 2.4260 2.3351 2.3060 2.7755 2.6400 2.5277 2.4422 2.3748 2.2411 2.3288 2.3965 2.4266 2.3363 2.3404 2.2588 2.4741 2.3883 2.3965 2.4266 2.3350 2.4205 2.3343 2.2662 2.3560 4.0012 3.1504 2.7581 2.5252 2.3683 2.2541 2.4533 2.2366 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