Chapter 5

Problems

5.1. Prove that it is impossible for two lines representing reversible, adiabatic processes on a PV diagram to intersect. (*Hint:* Assume that they do intersect, and complete the cycle with a line representing a reversible, isothermal process. Show that performance of this cycle violates the second law.)

5.2. A Carnot engine receives 250 $kJ\cdot s^{-1}$ of heat from a heat-source reservoir at 525°C and rejects heat to a heat-sink reservoir at 50°C. What are the power developed and the heat rejected?

5.3. The following heat engines produce power of 95,000 kW. Determine in each case the rates at which heat is absorbed from the hot reservoir and discarded to the cold reservoir.

(*a*) A Carnot engine operates between heat reservoirs at 750 K and 300 K.
(*b*) A practical engine operates between the same heat reservoirs but with a thermal efficiency $\eta = 0.35$.

5.4. A particular power plant operates with a heat-source reservoir at 350°C and a heat-sink reservoir at 30°C. It has a thermal efficiency equal to 55% of the Carnot-engine thermal efficiency for the same temperatures.

(*a*) What is the thermal efficiency of the plant?
(*b*) To what temperature must the heat-source reservoir be raised to increase the thermal efficiency of the plant to 35%? Again η is 55% of the Carnot-engine value.

5.5. An egg, initially at rest, is dropped onto a concrete surface; it breaks. Prove that the process is irreversible. In modeling this process treat the egg as the system, and assume the passage of sufficient time for the egg to return to its initial temperature.

5.6. Which is the more effective way to increase the thermal efficiency of a Carnot engine: to increase T_H with T_C constant, or to decrease T_C with T_H constant? For a real engine, which would be the more *practical* way?

5.7. Large quantities of liquefied natural gas (LNG) are shipped by ocean tanker. At the unloading port, provision is made for vaporization of the LNG so that it can be delivered to pipelines as gas. The LNG arrives in the tanker at atmospheric pressure and 113.7 K, and represents a possible heat sink for use as the cold reservoir of a heat engine. For unloading of LNG as a vapor at the rate of 9000 $m^3\cdot s^{-1}$, as measured at 25°C and 1.0133 bar, and assuming the availability of an adequate heat source at 30°C, what is the maximum possible power obtainable and what is the rate of heat transfer from the heat source? Assume that LNG at 25°C and 1.0133 bar is an ideal gas with the molar mass of 17. Also assume that the LNG vaporizes only, absorbing only its latent heat of 512 $kJ\cdot kg^{-1}$ at 113.7 K.

5.8. With respect to 1 kg of liquid water:

(*a*) Initially at 0°C, it is heated to 100°C by contact with a heat reservoir at 100°C. What is the entropy change of the water? Of the heat reservoir? What is ΔS_{total}?

(*b*) Initially at 0°C, it is first heated to 50°C by contact with a heat reservoir at 50°C and then to 100°C by contact with a reservoir at 100°C. What is ΔS_{total}?

(*c*) Explain how the water might be heated from 0°C to 100°C so that $\Delta S_{total} = 0$.

5.9. A rigid vessel of 0.06 m^3 volume contains an ideal gas, $C_V = (5/2)R$, at 500 K and 1 bar.

(*a*) If heat in the amount of 15,000 J is transferred to the gas, determine its entropy change.

(*b*) If the vessel is fitted with a stirrer that is rotated by a shaft so that work in the amount of 15,000 J is done on the gas, what is the entropy change of the gas if the process is adiabatic? What is ΔS_{total}? What is the irreversible feature of the process?

5.10. An ideal gas, $C_P = (7/2)R$, is heated in a steady-flow heat exchanger from 70°C to 190°C by another stream of the same ideal gas which enters at 320°C. The flow rates of the two streams are the same, and heat losses from the exchanger are negligible.

(*a*) Calculate the molar entropy changes of the two gas streams for both parallel cocurrent and countercurrent flow in the exchanger.

(*b*) What is ΔS_{total} in each case?

(*c*) Repeat parts (*a*) and (*b*) for countercurrent flow if the heating stream enters at 200°C.

5.11. For an ideal gas with constant heat capacities, show that:

(*a*) For a temperature change from T_1 to T_2, ΔS of the gas is greater when the change occurs at constant pressure than when it occurs at constant volume.

(*b*) For a pressure change from P_1 to P_2, the sign of ΔS for an isothermal change is opposite that for a constant-volume change.

5.12. For an ideal gas prove that:

$$\frac{\Delta S}{R} = \int_{T_0}^{T} \frac{C_V^{ig}}{R} \frac{dT}{T} + \ln \frac{V}{V_0}$$

5.13. A Carnot engine operates between two *finite* heat reservoirs of total heat capacity C_H^t and C_C^t.

(*a*) Develop an expression relating T_C to T_H at any time.

(*b*) Determine an expression for the work obtained as a function of C_H^t, C_C^t, T_H, and the initial temperatures T_{H_0} and T_{C_0}.

(*c*) What is the *maximum* work obtainable? This corresponds to infinite time, when the reservoirs attain the same temperature.

In approaching this problem, use the differential form of Carnot's equation,

$$\frac{dQ_H}{dQ_C} = -\frac{T_H}{T_C}$$

and a differential energy balance for the engine,

$$dW - dQ_C - dQ_H = 0$$

Here, Q_C and Q_H refer to the *reservoirs*.

5.14. A Carnot engine operates between an infinite hot reservoir and a *finite* cold reservoir of total heat capacity C_C^t.

(*a*) Determine an expression for the work obtained as a function of C_C^t, T_H (= constant), T_C, and the initial cold-reservoir temperature T_{C_0}.
(*b*) What is the *maximum* work obtainable? This corresponds to infinite time, when T_C becomes equal to T_H.

The approach to this problem is the same as for Prob. 5.13.

5.15. A heat engine operating in outer space can be assumed equivalent to a Carnot engine operating between reservoirs at temperatures T_H and T_C. The only way heat can be discarded from the engine is by radiation, the rate of which is given (approximately) by:

$$|\dot{Q}_C| = kAT_C^4$$

where k is a constant and A is the area of the radiator. Prove that, for fixed power output $|\dot{W}|$ and for fixed temperature T_H, the radiator area A is a minimum when the temperature ratio T_C/T_H is 0.75.

5.16. Imagine that a stream of fluid in steady-state flow serves as a heat source for an infinite set of Carnot engines, each of which absorbs a differential amount of heat from the fluid, causing its temperature to decrease by a differential amount, and each of which rejects a differential amount of heat to a heat reservoir at temperature T_σ. As a result of the operation of the Carnot engines, the temperature of the fluid decreases from T_1 to T_2. Equation (5.8) applies here in differential form, wherein η is defined as:

$$\eta \equiv dW/dQ$$

where Q is heat transfer with respect to the flowing fluid. Show that the total work of the Carnot engines is given by:

$$W = Q - T_\sigma \Delta S$$

where ΔS and Q both refer to the fluid. In a particular case, the fluid is an ideal gas, with $C_P = (7/2)R$, and the operating temperatures are $T_1 = 600$ K and $T_2 = 400$ K. If $T_\sigma = 300$ K, what is the value of W in J·mol^{-1}? How much heat is discarded to the heat reservoir at T_σ? What is the entropy change of the heat reservoir? What is ΔS_{total}?

5.17. A Carnot engine operates between temperature levels of 600 K and 300 K. It drives a Carnot refrigerator, which provides cooling at 250 K and discards heat at 300 K. Determine a numerical value for the ratio of heat extracted by the refrigerator ("cooling load") to the heat delivered to the engine ("heating load").

5.18. An ideal gas with constant heat capacity undergoes a change of state from conditions T_1, P_1 to conditions T_2, P_2. Determine ΔH (J·mol^{-1}) and ΔS (J·mol^{-1}·K^{-1}) for one of the following cases.

(*a*) $T_1 = 300$ K, $P_1 = 1.2$ bar, $T_2 = 450$ K, $P_2 = 6$ bar, $C_P/R = 7/2$
(*b*) $T_1 = 300$ K, $P_1 = 1.2$ bar, $T_2 = 500$ K, $P_2 = 6$ bar, $C_P/R = 7/2$
(*c*) $T_1 = 450$ K, $P_1 = 10$ bar, $T_2 = 300$ K, $P_2 = 2$ bar, $C_P/R = 5/2$
(*d*) $T_1 = 400$ K, $P_1 = 6$ bar, $T_2 = 300$ K, $P_2 = 1.2$ bar, $C_P/R = 9/2$
(*e*) $T_1 = 500$ K, $P_1 = 6$ bar, $T_2 = 300$ K, $P_2 = 1.2$ bar, $C_P/R = 4$

5.19. An ideal gas, $C_P = (7/2)R$ and $C_V = (5/2)R$, undergoes a cycle consisting of the following mechanically reversible steps:

- An adiabatic compression from P_1, V_1, T_1 to P_2, V_2, T_2
- An isobaric expansion from P_2, V_2, T_2 to $P_3 = P_2, V_3, T_3$
- An adiabatic expansion from P_3, V_3, T_3 to P_4, V_4, T_4
- A constant-volume process from P_4, V_4, T_4 to $P_1, V_1 = V_4, T_1$

Sketch this cycle on a *PV* diagram and determine its thermal efficiency if $T_1 = 200$°C, $T_2 = 1000$°C, and $T_3 = 1700$°C.

5.20. The infinite heat reservoir is an abstraction, often approximated in engineering applications by large bodies of air or water. Apply the closed-system form of the energy balance [Eq. (2.3)] to such a reservoir, treating it as a constant-volume system. How is it that heat transfer to or from the reservoir can be nonzero, yet the temperature of the reservoir remains constant?

5.21. One mole of an ideal gas, $C_P = (7/2)R$ and $C_V = (5/2)R$, is compressed adiabatically in a piston/cylinder device from 2 bar and 25°C to 7 bar. The process is irreversible and requires 35% more work than a reversible, adiabatic compression from the same initial state to the same final pressure. What is the entropy change of the gas?

5.22. A mass m of liquid water at temperature T_1 is mixed adiabatically and isobarically with an equal mass of liquid water at temperature T_2. Assuming constant C_P, show

$$\Delta S^t = \Delta S_{\text{total}} = S_G = 2mC_P \ln \frac{(T_1 + T_2)/2}{(T_1\, T_2)^{1/2}}$$

and prove that this is positive. What would be the result if the masses of the water were *different*, say, m_1 and m_2?

5.23. Reversible adiabatic processes are *isentropic*. Are isentropic processes necessarily reversible and adiabatic? If so, explain why; if not, give an illustrative example.

5.24. Prove that the mean heat capacities $\langle C_P \rangle_H$ and $\langle C_P \rangle_S$ are inherently *positive*, whether $T > T_0$ or $T < T_0$. Explain why they are well defined for $T = T_0$.

5.25. A reversible cycle executed by 1 mol of an ideal gas for which $C_P = (5/2)R$ and $C_V = (3/2)R$ consists of the following:

- Starting at $T_1 = 700$ K and $P_1 = 1.5$ bar, the gas is cooled at constant pressure to $T_2 = 350$ K.
- From 350 K and 1.5 bar, the gas is compressed isothermally to pressure P_2.
- The gas returns to its initial state along a path for which PT = constant.

What is the thermal efficiency of the cycle?

5.26. One mole of an ideal gas is compressed isothermally but irreversibly at 130°C from 2.5 bar to 6.5 bar in a piston/cylinder device. The work required is 30% greater than the work of reversible, isothermal compression. The heat transferred from the gas during compression flows to a heat reservoir at 25°C. Calculate the entropy changes of the gas, the heat reservoir, and ΔS_{total}.

5.27. For a steady-flow process at approximately atmospheric pressure, what is the entropy change of the gas:

(*a*) When 10 mol of SO_2 is heated from 200 to 1100°C?
(*b*) When 12 mol of propane is heated from 250 to 1200°C?
(*c*) When 20 kg of methane is heated from 100 to 800°C?
(*d*) When 10 mol of *n*-butane is heated from 150 to 1150°C?
(*e*) When 1000 kg of air is heated from 25 to 1000°C?
(*f*) When 20 mol of ammonia is heated from 100 to 800°C?
(*g*) When 10 mol of water is heated from 150 to 300°C?
(*h*) When 5 mol of chlorine is heated from 200 to 500°C?
(*i*) When 10 kg of ethylbenzene is heated from 300 to 700°C?

5.28. What is the entropy change of the gas, heated in a steady-flow process at approximately atmospheric pressure,

(*a*) When 800 kJ is added to 10 mol of ethylene initially at 200°C?
(*b*) When 2500 kJ is added to 15 mol of 1-butene initially at 260°C?
(*c*) When 10^6(Btu) is added to 40(lb mol) of ethylene initially at 500(°F)?

5.29. A device with no moving parts provides a steady stream of chilled air at −25°C and 1 bar. The feed to the device is compressed air at 25°C and 5 bar. In addition to the stream of chilled air, a second stream of warm air flows from the device at 75°C and 1 bar. Assuming adiabatic operation, what is the ratio of chilled air to warm air that the device produces? Assume that air is an ideal gas for which $C_P = (7/2)R$.

5.30. An inventor has devised a complicated nonflow process in which 1 mol of air is the working fluid. The net effects of the process are claimed to be:

- A change in state of the air from 250°C and 3 bar to 80°C and 1 bar.
- The production of 1800 J of work.
- The transfer of an undisclosed amount of heat to a heat reservoir at 30°C.

Determine whether the claimed performance of the process is consistent with the second law. Assume that air is an ideal gas for which $C_P = (7/2)R$.

5.31. Consider the heating of a house by a furnace, which serves as a heat-source reservoir at a high temperature T_F. The house acts as a heat-sink reservoir at temperature T, and heat $|Q|$ must be added to the house during a particular time interval to maintain this temperature. Heat $|Q|$ can of course be transferred directly from the furnace to the house, as is the usual practice. However, a third heat reservoir is readily available, namely, the surroundings at temperature T_σ, which can serve as another heat source, thus reducing the amount of heat required from the furnace. Given that $T_F = 810$ K, $T = 295$ K, $T_\sigma = 265$ K, and $|Q| = 1000$ kJ, determine the minimum amount of heat $|Q_F|$ that must be extracted from the heat-source reservoir (furnace) at T_F. No other sources of energy are available.

5.32. Consider the air conditioning of a house through use of solar energy. At a particular location, experiment has shown that solar radiation allows a large tank of pressurized water to be maintained at 175°C. During a particular time interval, heat in the amount of 1500 kJ must be extracted from the house to maintain its temperature at 24°C when the surroundings temperature is 33°C. Treating the tank of water, the house, and the surroundings as heat reservoirs, determine the minimum amount of heat that must be extracted from the tank of water by any device built to accomplish the required cooling of the house. No other sources of energy are available.

5.33. A refrigeration system cools a brine from 25°C to −15°C at a rate of 20 $\text{kg}\cdot\text{s}^{-1}$. Heat is discarded to the atmosphere at a temperature of 30°C. What is the power requirement if the thermodynamic efficiency of the system is 0.27? The specific heat of the brine is 3.5 $\text{kJ}\cdot\text{kg}^{-1}\cdot{}^\circ\text{C}^{-1}$.

5.34. An electric motor under steady load draws 9.7 amperes at 110 volts; it delivers 1.25(hp) of mechanical energy. The temperature of the surroundings is 300 K. What is the total rate of entropy generation in $\text{W}\cdot\text{K}^{-1}$?

5.35. A 25-ohm resistor at steady state draws a current of 10 amperes. Its temperature is 310 K; the temperature of the surroundings is 300 K. What is the total rate of entropy generation $\dot{S}_G$? What is its origin?

5.36. Show how the general rate form of the entropy balance, Eq. (5.16), reduces to Eq. (5.2) for the case of a closed system.

5.37. A list of common unit operations follows:

(*a*) Single-pipe heat exchanger
(*b*) Double-pipe heat exchanger
(*c*) Pump
(*d*) Gas compressor
(*e*) Gas turbine (expander)
(*f*) Throttle valve
(*g*) Nozzle

Develop a simplified form of the general steady-state entropy balance appropriate to each operation. State carefully, and justify, any assumptions you make.

5.38. Ten kmol per hour of air is throttled from upstream conditions of 25°C and 10 bar to a downstream pressure of 1.2 bar. Assume air to be an ideal gas with $C_P = (7/2)R$.

(*a*) What is the downstream temperature?
(*b*) What is the entropy change of the air in $\text{J·mol}^{-1}\text{·K}^{-1}$?
(*c*) What is the rate of entropy generation in W·K^{-1}?
(*d*) If the surroundings are at 20°C, what is the lost work?

5.39. A steady-flow adiabatic turbine (expander) accepts gas at conditions T_1, P_1, and discharges at conditions T_2, P_2. Assuming ideal gases, determine (per mole of gas) W, W_{ideal}, W_{lost}, and S_G for one of the following cases. Take $T_\sigma = 300$ K.

(*a*) $T_1 = 500$ K, $P_1 = 6$ bar, $T_2 = 371$ K, $P_2 = 1.2$ bar, $C_P/R = 7/2$
(*b*) $T_1 = 450$ K, $P_1 = 5$ bar, $T_2 = 376$ K, $P_2 = 2$ bar, $C_P/R = 4$
(*c*) $T_1 = 525$ K, $P_1 = 10$ bar, $T_2 = 458$ K, $P_2 = 3$ bar, $C_P/R = 11/2$
(*d*) $T_1 = 475$ K, $P_1 = 7$ bar, $T_2 = 372$ K, $P_2 = 1.5$ bar, $C_P/R = 9/2$
(*e*) $T_1 = 550$ K, $P_1 = 4$ bar, $T_2 = 403$ K, $P_2 = 1.2$ bar, $C_P/R = 5/2$

5.40. Consider the direct heat transfer from a heat reservoir at T_1 to another heat reservoir at temperature T_2, where $T_1 > T_2 > T_\sigma$. It is not obvious why the lost work of this process should depend on T_σ, the temperature of the surroundings, because the surroundings are not involved in the actual heat-transfer process. Through appropriate use of the Carnot-engine formula, show for the transfer of an amount of heat equal to $|Q|$ that

$$W_{\text{lost}} = T_\sigma |Q| \frac{T_1 - T_2}{T_1 T_2} = T_\sigma S_G$$

5.41. An ideal gas at 2500 kPa is throttled adiabatically to 150 kPa at the rate of 20 mol·s^{-1}. Determine $\dot{S}_G$ and $\dot{W}_{\text{lost}}$ if $T_\sigma = 300$ K.

5.42. An inventor claims to have devised a cyclic engine which exchanges heat with reservoirs at 25°C and 250°C, and which produces 0.45 kJ of work for each kJ of heat extracted from the hot reservoir. Is the claim believable?

5.43. Heat in the amount of 150 kJ is transferred directly from a hot reservoir at $T_H = 550$ K to two cooler reservoirs at $T_1 = 350$ K and $T_2 = 250$ K. The surroundings temperature is $T_\sigma = 300$ K. If the heat transferred to the reservoir at T_1 is half that transferred to the reservoir at T_2, calculate:

(*a*) The entropy generation in kJ·K^{-1}
(*b*) The lost work

How could the process be made reversible?

5.44. A nuclear power plant generates 750 MW; the reactor temperature is 315°C and a river with water temperature of 20°C is available.

(*a*) What is the maximum possible thermal efficiency of the plant, and what is the minimum rate at which heat must be discarded to the river?
(*b*) If the actual thermal efficiency of the plant is 60% of the maximum, at what rate must heat be discarded to the river, and what is the temperature rise of the river if it has a flow rate of 165 m^3·s^{-1}?

5.45. A single gas stream enters a process at conditions T_1, P_1, and leaves at pressure P_2. The process is adiabatic. Prove that the outlet temperature T_2 for the actual (irreversible) adiabatic process is greater than that for a *reversible* adiabatic process. Assume the gas is ideal with constant heat capacities.

5.46. A Hilsch vortex tube operates with no moving mechanical parts and splits a gas stream into two streams: one warmer and the other cooler than the entering stream. One such tube is reported to operate with air entering at 5 bar and 20°C, and air streams leaving at 27°C and –22°C, both at 1(atm). The mass flow rate of warm air leaving is six times that of the cool air. Are these results possible? Assume air to be an ideal gas at the conditions given.

5.47. (*a*) Air at 70(°F) and 1(atm) is cooled at the rate of 100,000(ft)^3(hr)$^{-1}$ to 20(°F) by refrigeration. For a surroundings temperature of 70(°F), what is the minimum power requirement in (hp)?
(*b*) Air at 25°C and 1(atm) is cooled at the rate of 3000 m^3·hr^{-1} to –8°C by refrigeration. For a surroundings temperature of 25°C, what is the minimum power requirement in kW?

5.48. A flue gas is cooled from 2000 to 300(°F), and the heat is used to generate saturated steam at 212(°F) in a boiler. The flue gas has a heat capacity given by:

$$\frac{C_P}{R} = 3.83 + 0.000306\, T/(R)$$

Water enters the boiler at 212(°F) and is vaporized at this temperature; its latent heat of vaporization is 970.3(Btu)(lb$_m$)$^{-1}$.

(*a*) With reference to a surroundings temperature of 70(°F), what is the lost work of this process in (Btu)(lb mole)$^{-1}$ of flue gas?

(*b*) With reference to a surroundings temperature of 70(°F), what is the maximum work, in (Btu)(lb mole)$^{-1}$ of flue gas, that can be accomplished by the saturated steam at 212(°F) if it condenses only, and does not subcool?

(*c*) How does the answer to part (*b*) compare with the maximum work theoretically obtainable from the flue gas itself as it is cooled from 2000 to 300(°F)?

5.49. A flue gas is cooled from 1100 to 150°C, and the heat is used to generate saturated steam at 100°C in a boiler. The flue gas has a heat capacity given by:

$$\frac{C_P}{R} = 3.83 + 0.000551\ T/K$$

Water enters the boiler at 100°C and is vaporized at this temperature; its latent heat of vaporization is 2256.9 kJ·kg^{-1}.

(*a*) With reference to a surroundings temperature of 25°C, what is the lost work of this process in kJ·mol^{-1} of flue gas?

(*b*) With reference to a surroundings temperature of 25°C, what is the maximum work, in kJ·mol^{-1} of flue gas, that can be accomplished by the saturated steam at 100°C if it condenses only, and does not subcool?

(*c*) How does the answer to part (*b*) compare with the maximum work theoretically obtainable from the flue gas itself as it is cooled from 1100 to 150°C?

5.50. Ethylene vapor is cooled at atmospheric pressure from 830 to 35°C by direct heat transfer to the surroundings at a temperature of 25°C. With respect to this surroundings temperature, what is the lost work of the process in kJ·mol^{-1}? Show that the same result is obtained as the work which can be derived from reversible heat engines operating with the ethylene vapor as heat source and the surroundings as sink. The heat capacity of ethylene is given in Table C.1 of App. C.