# Fluid Mechanics

## *Chapter 8*

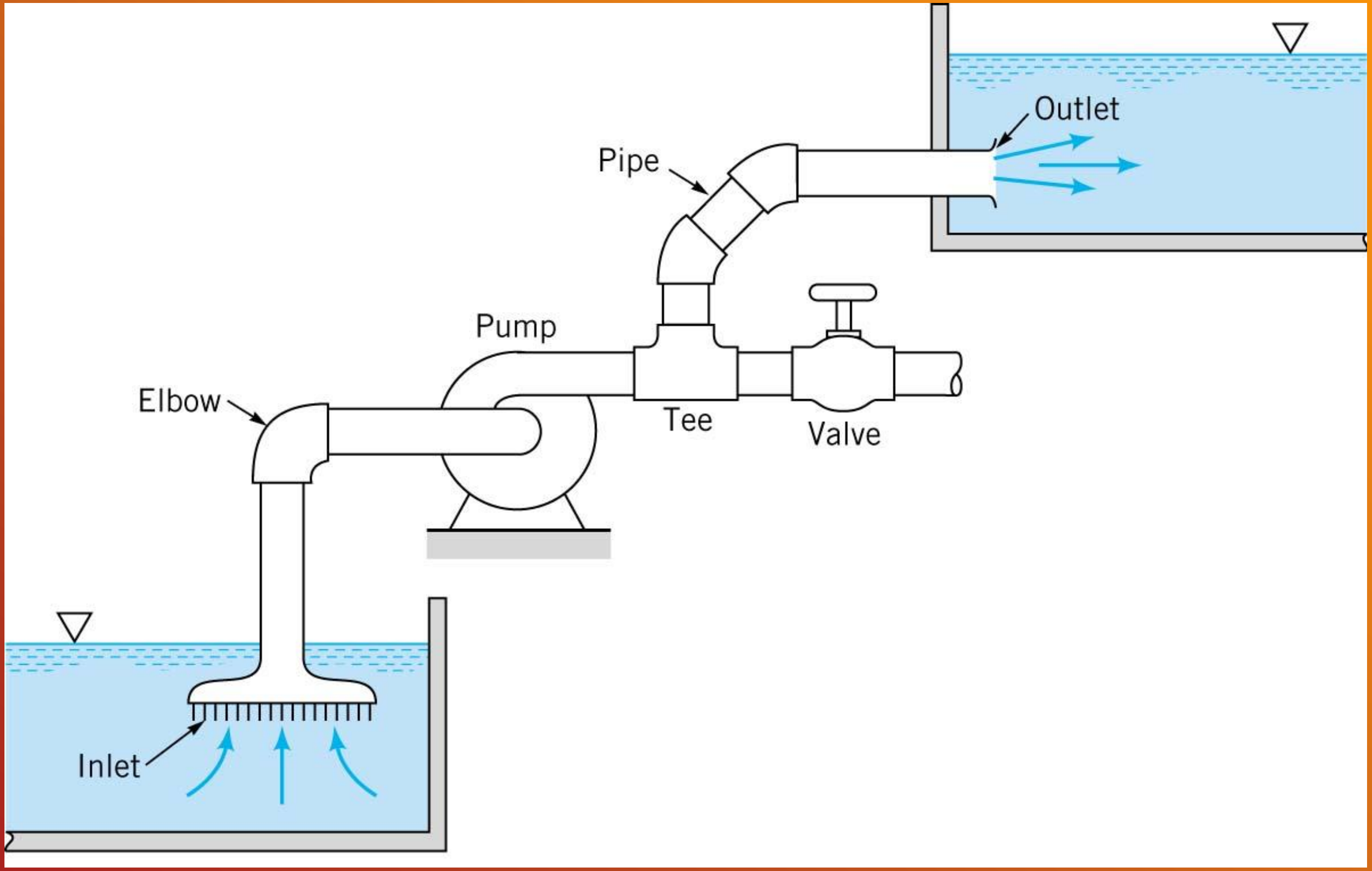
## *Viscous Flow in Pipes*

Asst.Prof.Dr.Denpong Soodphakdee

Department of Mechanical Engineering

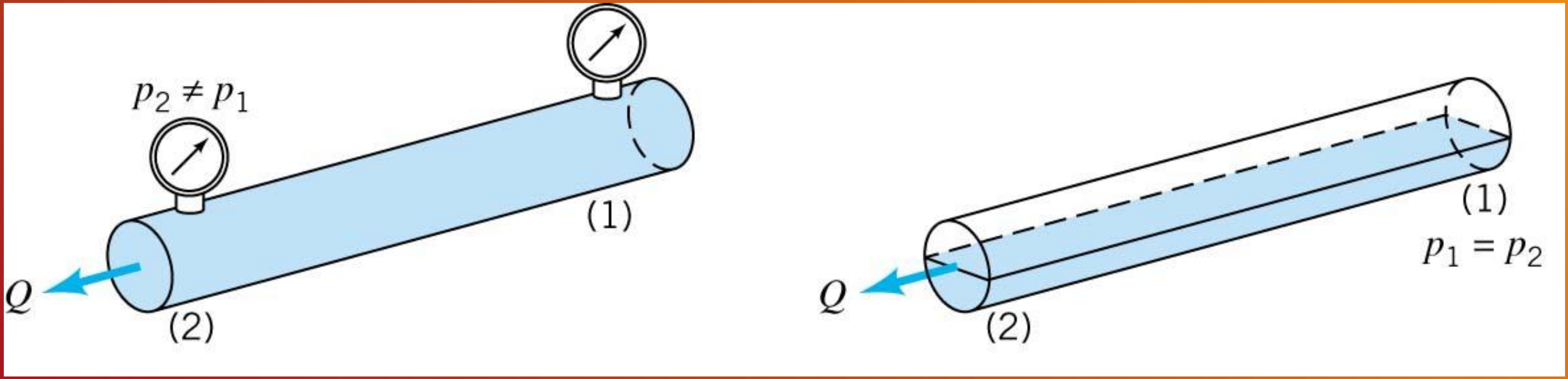
# Introduction

- Pipe flow is very important in our daily life.



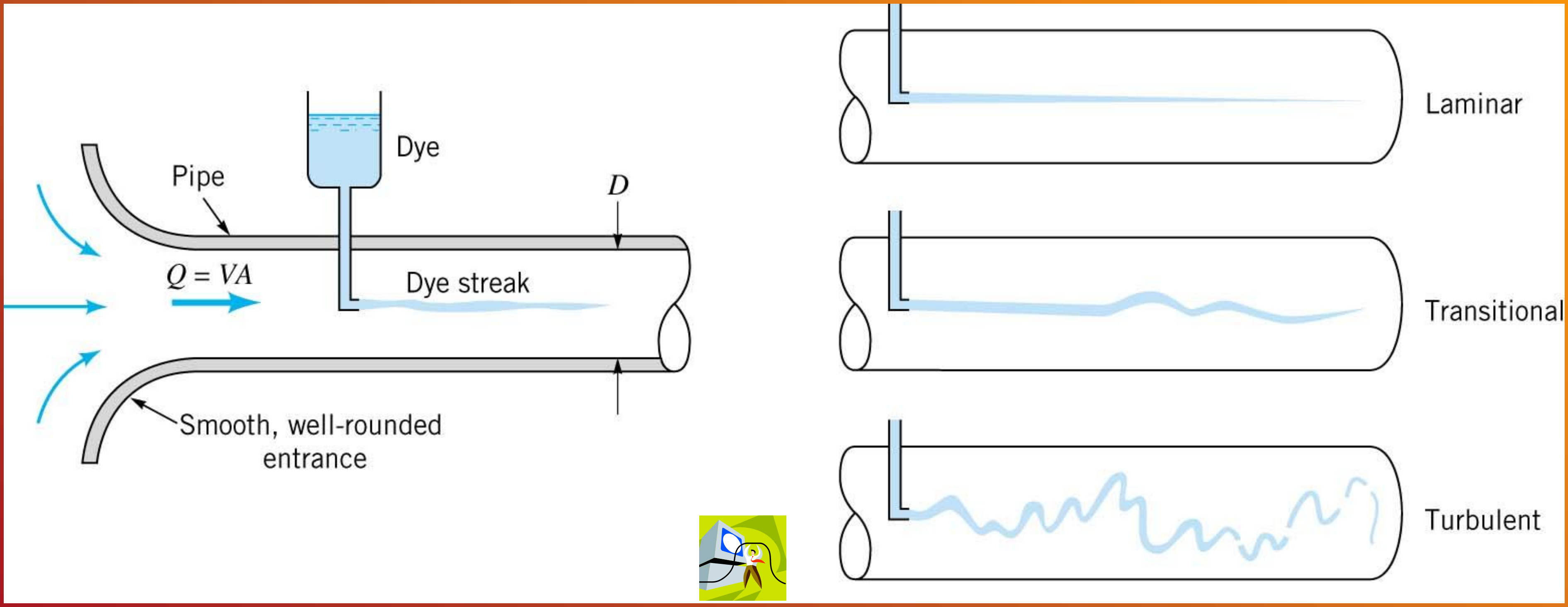
# General Characteristics of Pipe Flow

- There are assumptions on formulate and solve this important problem.
- The pipe is assumed to be completely full of the flowing fluid.
- The round shape fluid conveyer is assumed.



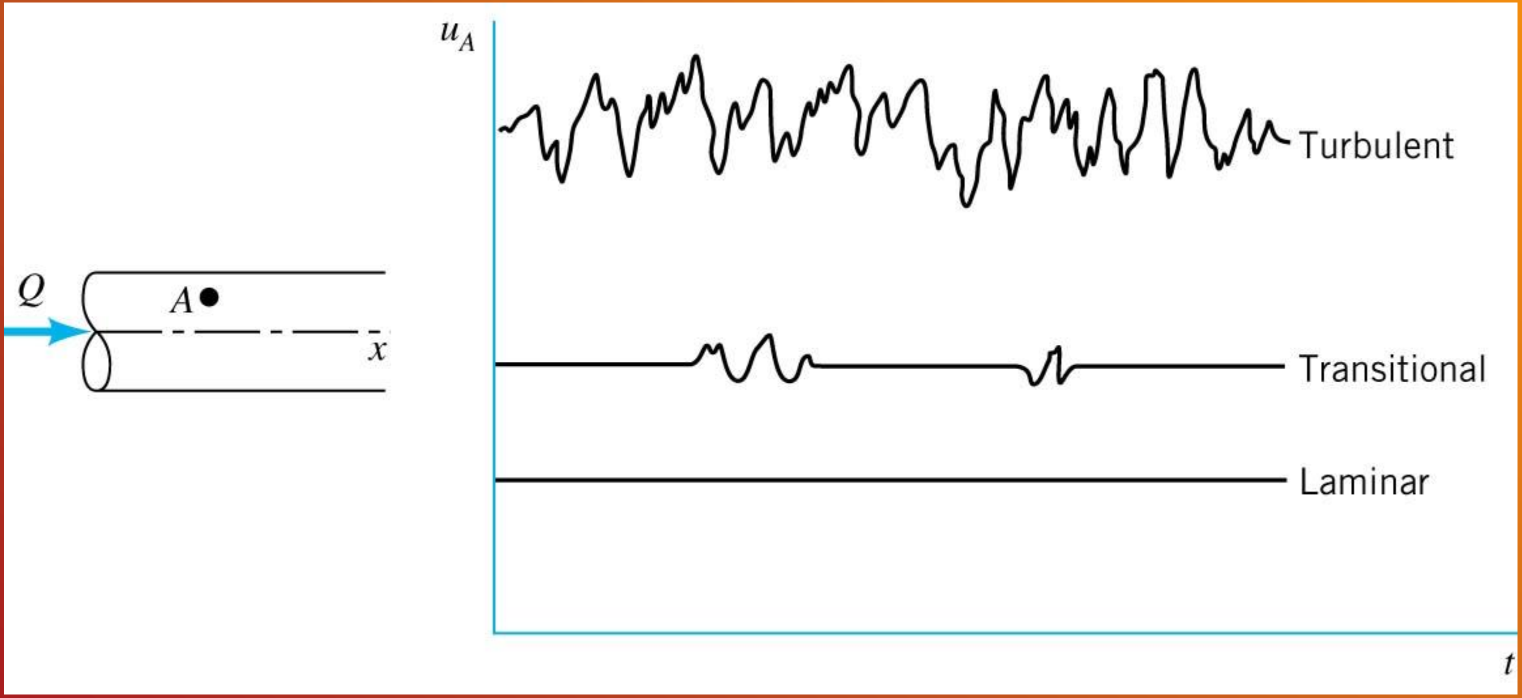
# Laminar or Turbulent Flow

- A flow may be laminar, transitional, or turbulent.



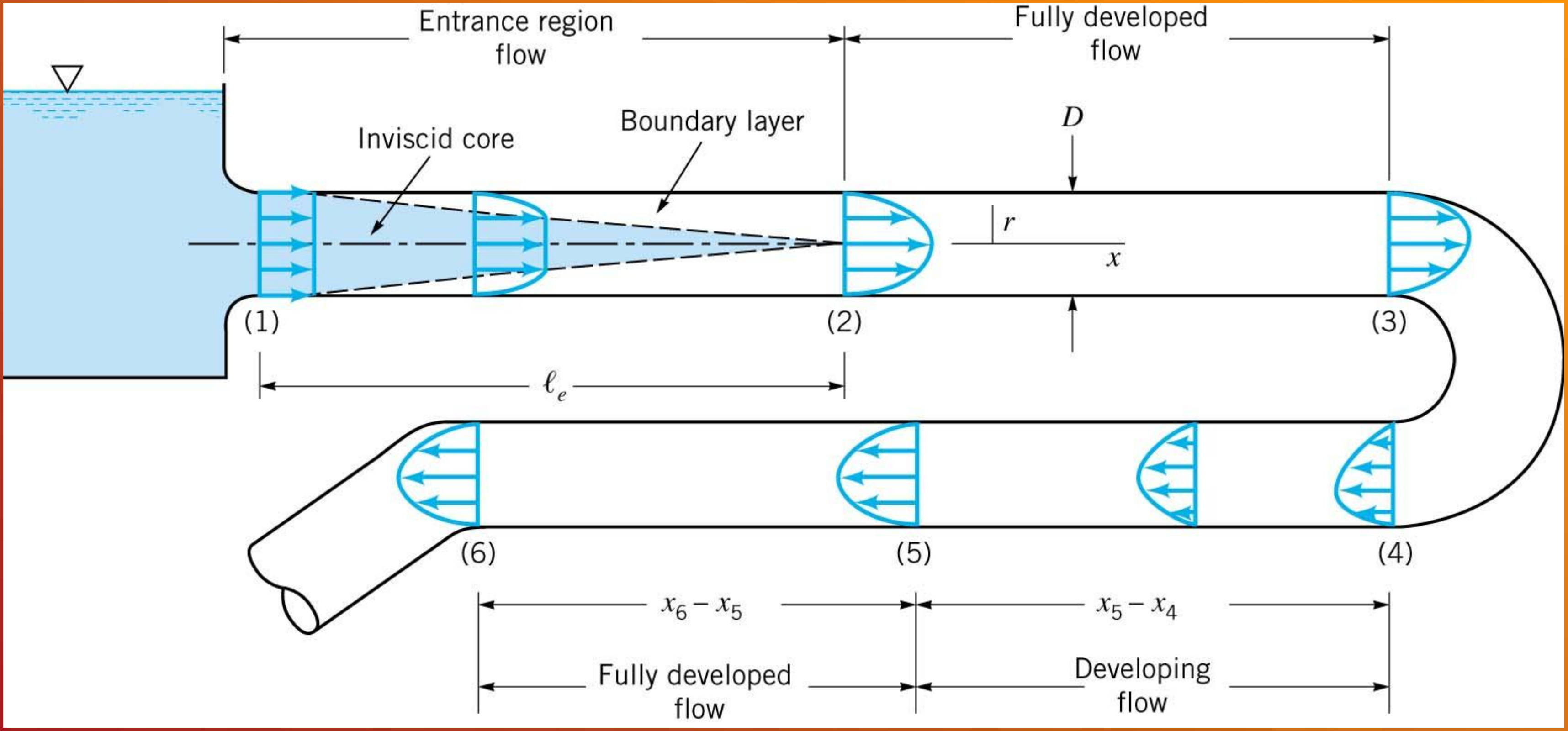
# Laminar or Turbulent Flow

- Time independence of fluid velocity at a point.
- Pipe flow characteristics are dependent on the value of Reynolds number, Re.



# Entrance Region and Fully Developed Flow

- Flow in the entrance region of a pipe is quite complex.



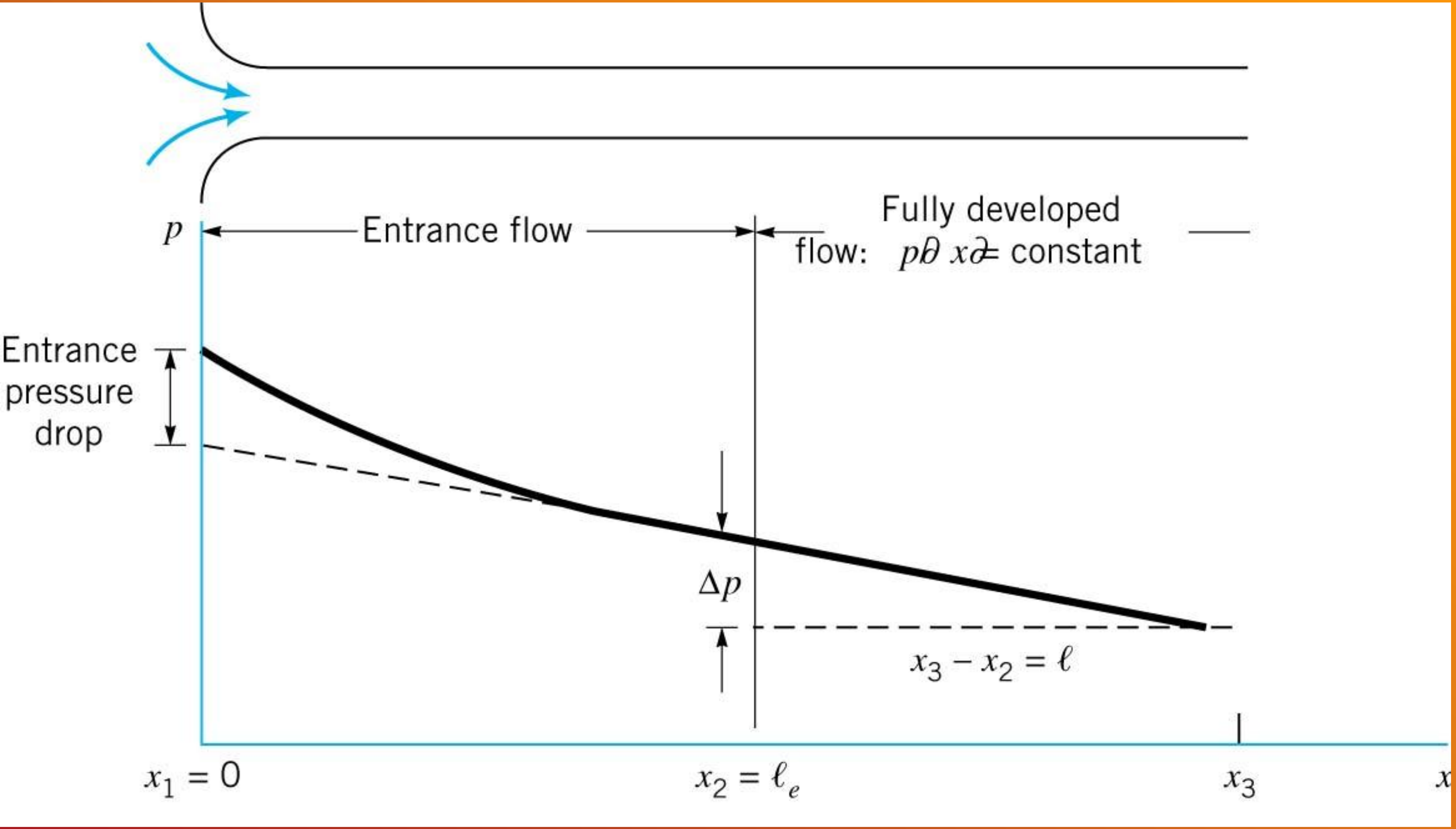
# Entrance Region and Fully Developed Flow

- The entrance length is a function of Reynolds number.

$$\frac{\ell_e}{D} = 0.06\,\mathrm{Re} \qquad \text{for laminar flow}$$

$$\frac{\ell_e}{D} = 4.4\left(\mathrm{Re}\right)^{1/6} \qquad \text{for turbulent flow}$$

# Pressure and Shear Stress



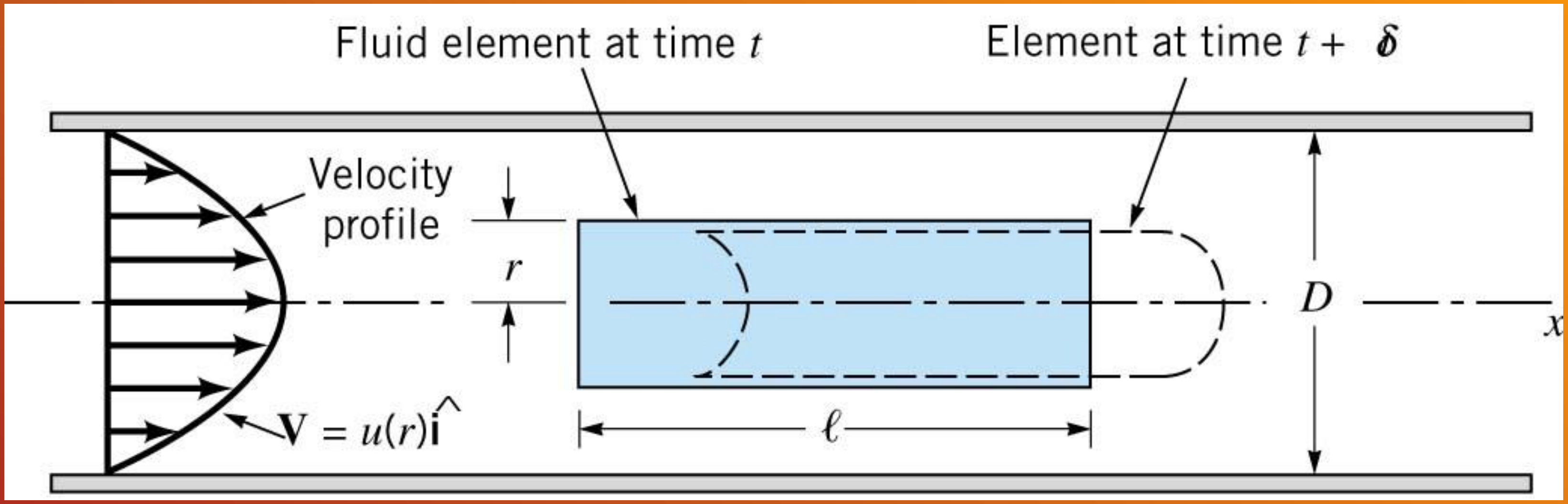
# Fully Developed Laminar Flow

- This is the case where the exact solution of Navier-Stokes equations can be obtained.
- Although most flows are turbulent rather than laminar, and many pipes are not long enough to allow the attainment of fully developed flow, a theoretical treatment and fully understanding of fully developed laminar flow is of considerable importance.

# Fully Developed Laminar Flow

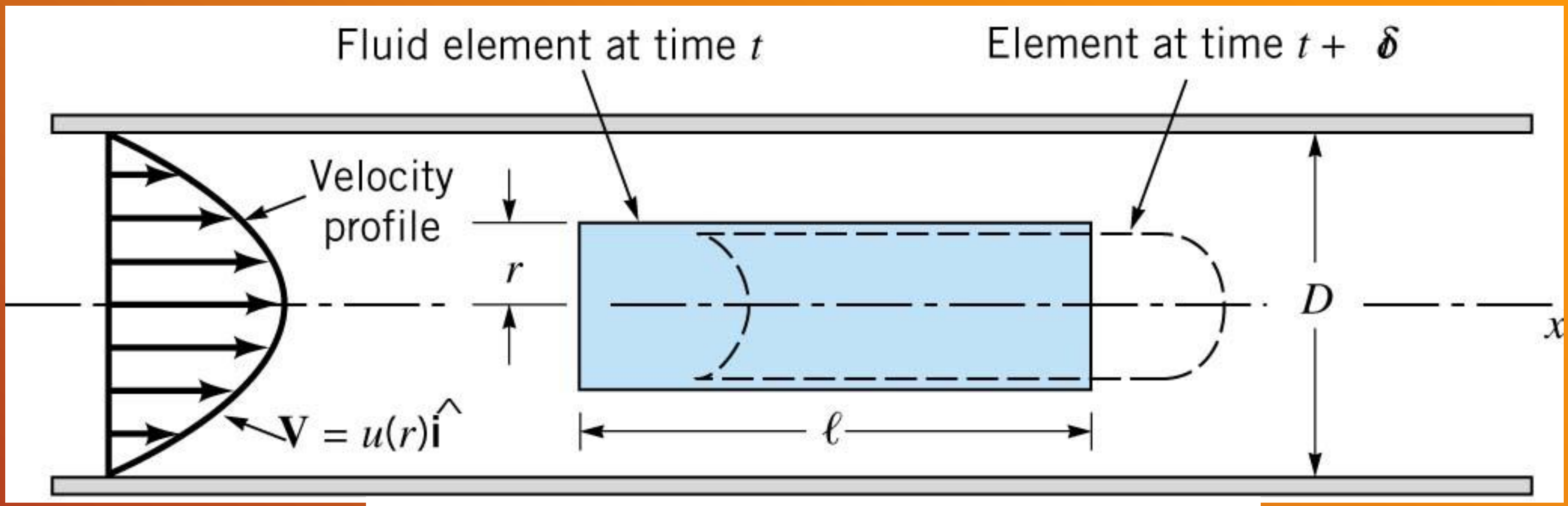
## From **F** = *ma* Applied Directly to a Fluid Element

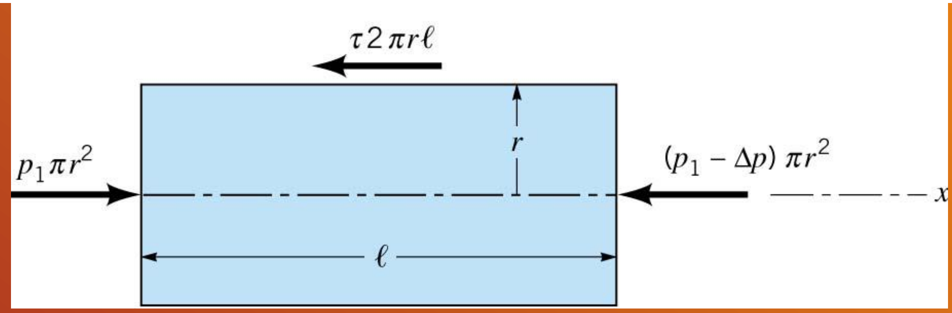
- Steady, fully developed pipe flow experiences no acceleration.



# Fully Developed Laminar Flow

## From **F** = *ma* Applied Directly to a Fluid Element

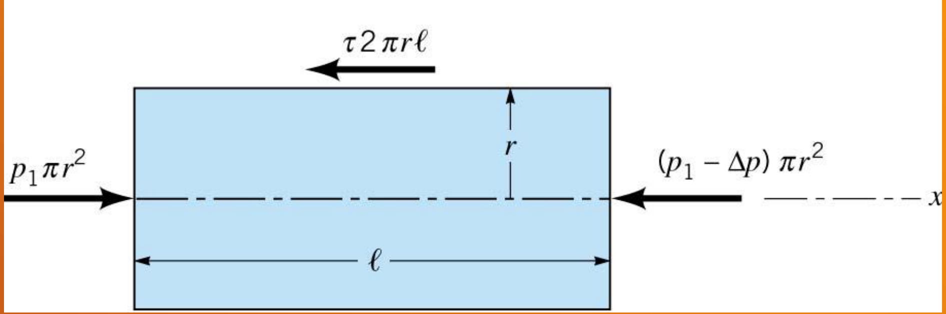



$$\left(p_1\right)\pi r^2 - \left(p_1 - \Delta p\right)\pi r^2 - \left(\tau\right)2\pi r\ell = 0$$

# Fully Developed Laminar Flow

## From **F** = *ma* Applied Directly to a Fluid Element




$$(p_1)\pi r^2 - (p_1 - \Delta p)\pi r^2 - (\tau)2\pi r\ell = 0$$

This can be simplified to

$$\frac{\Delta p}{\ell} = \frac{2\tau}{r}$$

# Fully Developed Laminar Flow

## From **F** = *ma* Applied Directly to a Fluid Element

$$\frac{\Delta p}{\ell} = \frac{2\tau}{r}$$

- Since neither $\Delta p$ or $\ell$ are functions of the radial coordinate, $r$, it is follow that $2\tau/r$ must also be independent of $r$. That is, $\tau = Cr$, where $C$ is a constant.
- Applying conditions for $r = 0$ and $r = D/2$, obtained

$$C = \frac{2\tau_w}{D} \longrightarrow \tau = \frac{2\tau_w r}{D} \longrightarrow \Delta p = \frac{4\ell\tau_w}{D} \longrightarrow \tau = \left(\frac{\Delta p}{2\ell}\right) r$$

# Fully Developed Laminar Flow

## From **F** = *ma* Applied Directly to a Fluid Element

- Referring shear stress for the pipe

$$\tau = -\mu \frac{du}{dr}$$

$$\frac{du}{dr} = -\frac{\tau}{\mu} = -\left(\frac{\Delta p}{2\mu\ell}\right) r$$

$$\int du = -\frac{\Delta p}{2\mu\ell}\int r\,dr$$

$$u = -\left(\frac{\Delta p}{4\mu\ell}\right) r^2 + C_1$$

since $u = 0$ at $r = D/2$

$$C_1 = \left(\frac{\Delta p}{16\mu\ell}\right) D^2$$

$$u(r) = \left(\frac{\Delta p D^2}{16\mu\ell}\right)\left[1 - \left(\frac{2r}{D}\right)^2\right]$$

$$= V_c\left[1 - \left(\frac{2r}{D}\right)^2\right]$$

# Fully Developed Laminar Flow

## From **F** = *ma* Applied Directly to a Fluid Element

$$V_c = \frac{\Delta p D^2}{16\mu\ell} \quad \text{maximum velocity}$$

$$Q = \int u dA = \int_{r=0}^{r=R} u(r) 2\pi r dr$$

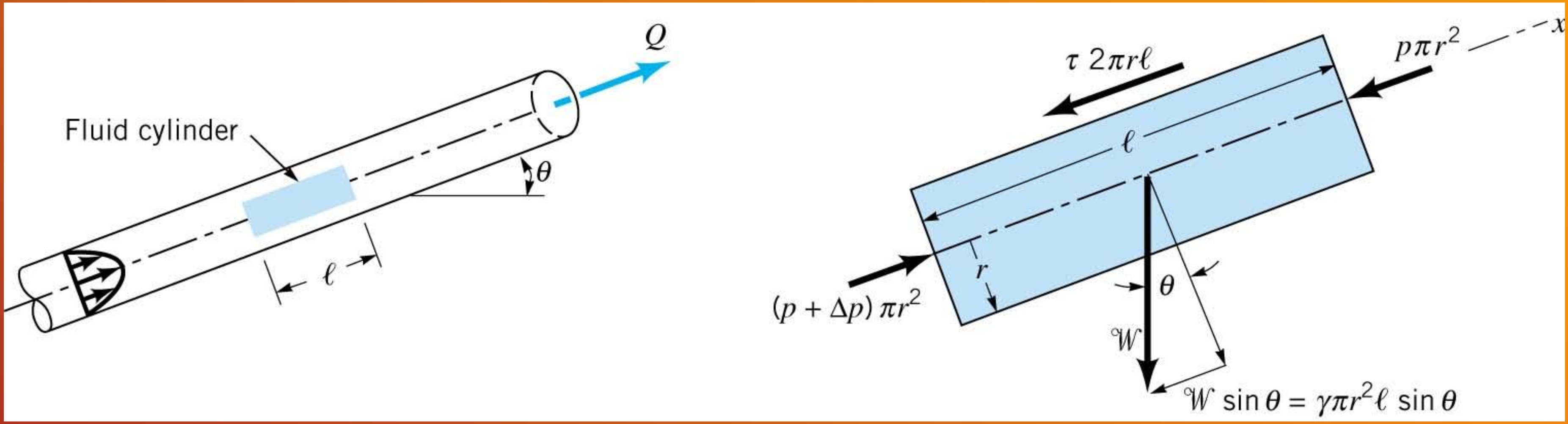
$$Q = \frac{\pi R^2 V_c}{2}$$

$$\boxed{Q = \frac{\pi D^4 \Delta p}{128\mu\ell}} \longleftarrow$$

**Poiseuille's law**

# Fully Developed Laminar Flow
## From **F** = *ma* Applied Directly to a Fluid Element

- For inclined pipe

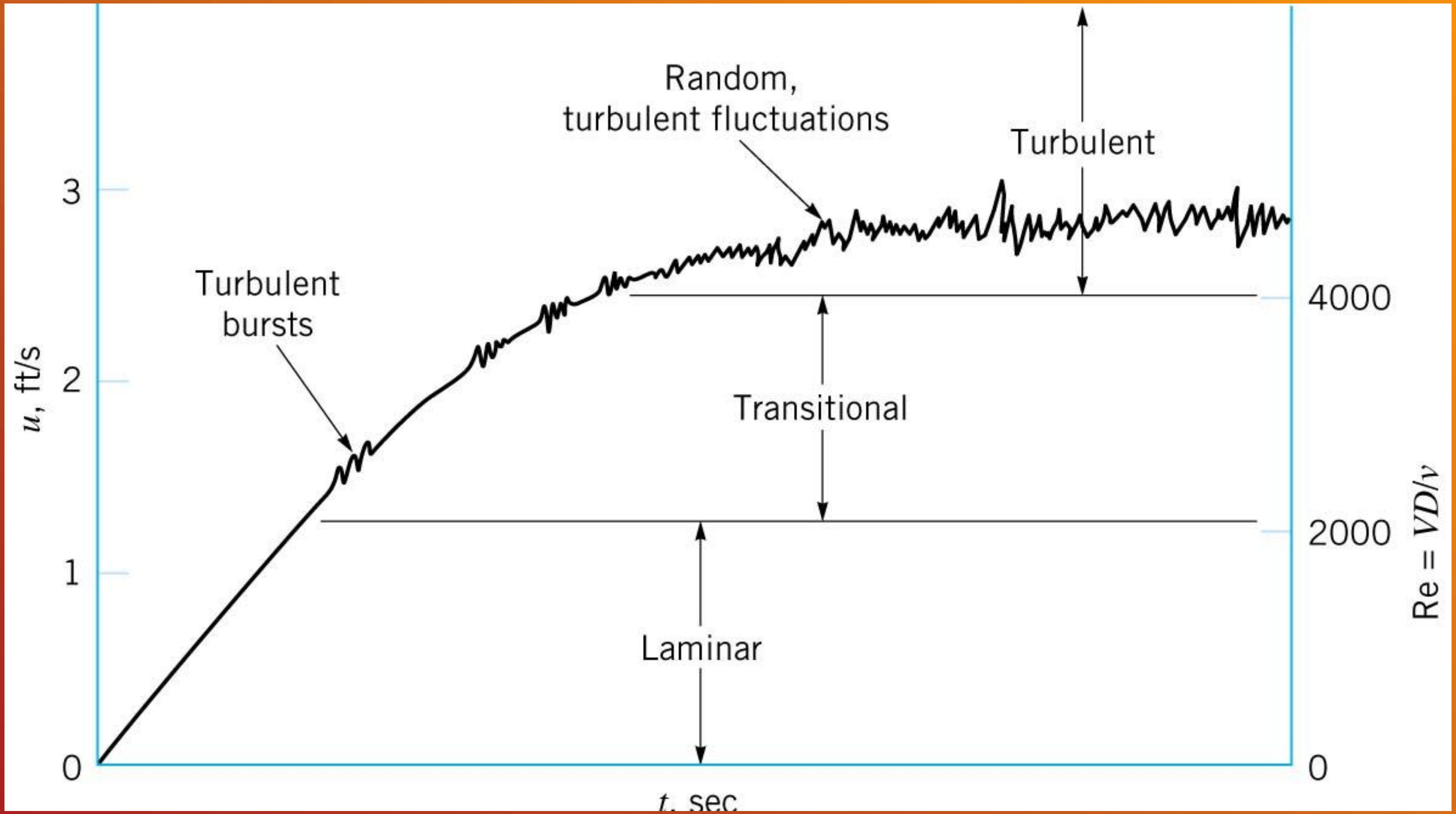



$$Q = \frac{\pi\left(\Delta p - \gamma \ell \sin\theta\right) D^4}{128\mu\ell}$$

# Fully Developed Turbulent Flow

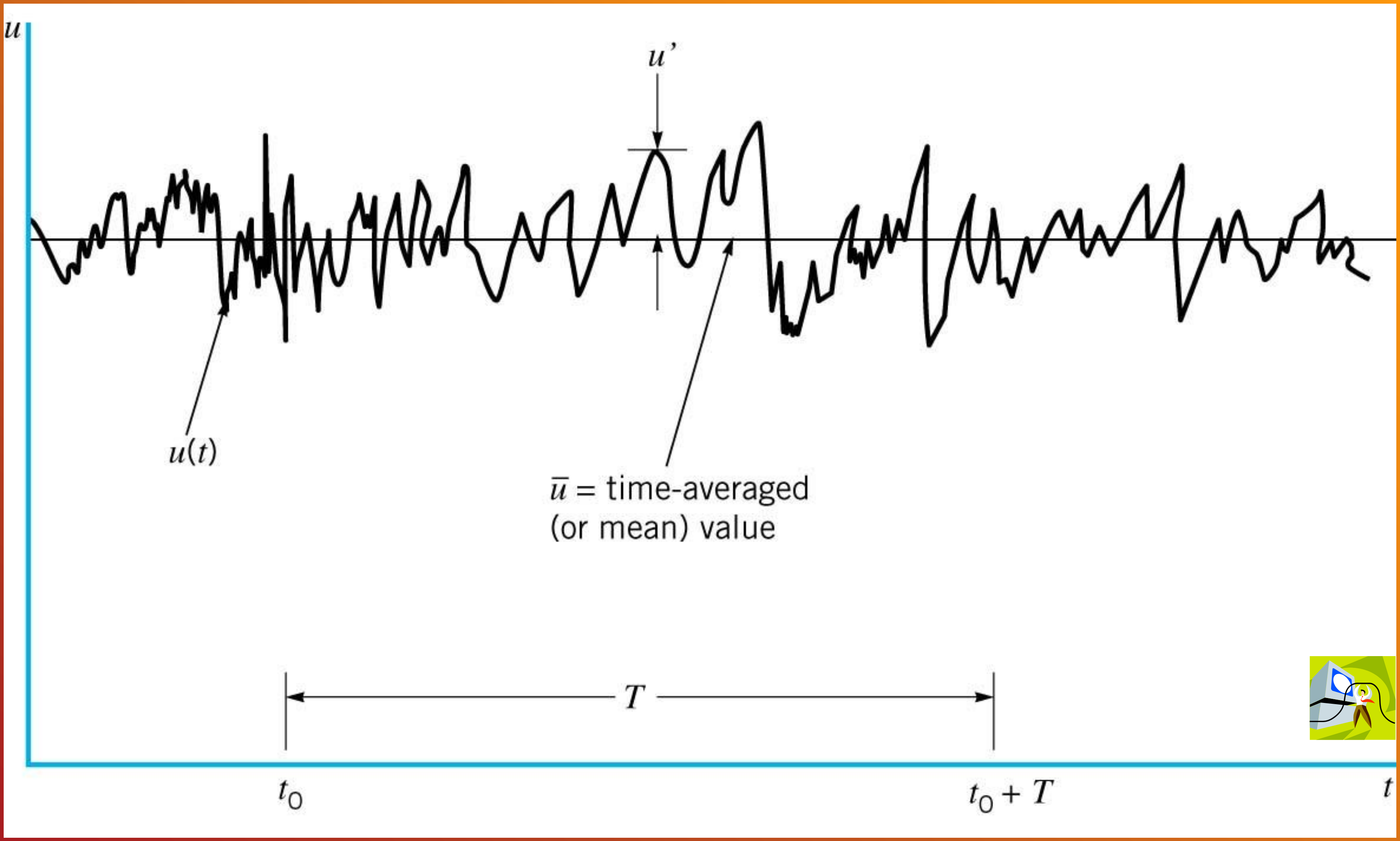
## Transition from Laminar to Turbulent Flow

- Turbulent flows involve randomly fluctuating parameters.



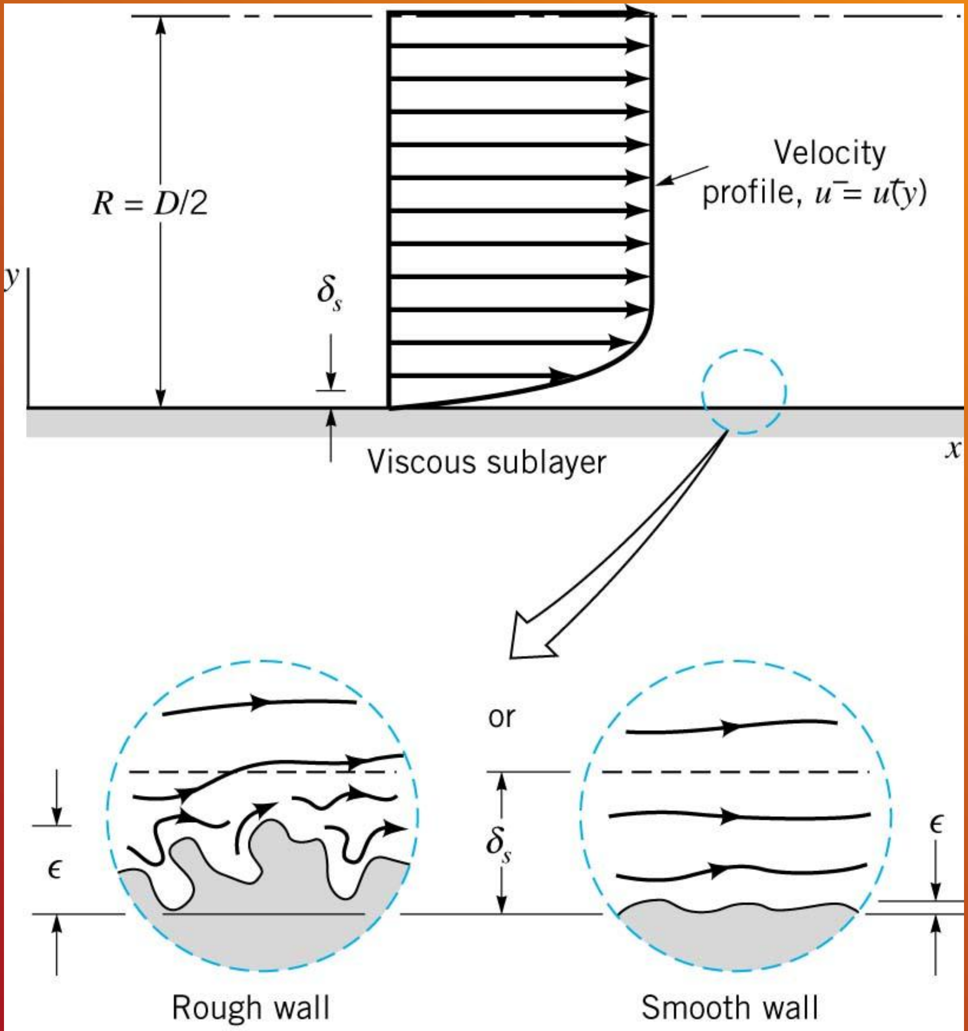
# Fully Developed Turbulent Flow

## Transition from Laminar to Turbulent Flow



# Dimensional Analysis of Pipe Flow

## The Moody Chart




$$\Delta p = F\left(V, D, \ell, \varepsilon, \mu, \rho\right)$$

$$\frac{\Delta p}{\frac{1}{2}\rho V^2} = \tilde{\phi}\left(\frac{\rho V D}{\mu}, \frac{\ell}{D}, \frac{\varepsilon}{D}\right)$$

$$\frac{\Delta p}{\frac{1}{2}\rho V^2} = \frac{\ell}{D}\phi\left(\frac{\rho V D}{\mu}, \frac{\varepsilon}{D}\right)$$

# Dimensional Analysis of Pipe Flow

## The Moody Chart

$$\Delta p = f\,\frac{\ell}{D}\,\frac{\rho V^2}{2}$$

where $f = \phi\left(\dfrac{\rho V D}{\mu}, \dfrac{\varepsilon}{D}\right)$

The energy equation for steady incompressible flow is given in chapter 5

$$\frac{p_1}{\gamma} + \alpha_1 \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_L$$

# Dimensional Analysis of Pipe Flow

## The Moody Chart

$$\frac{p_1}{\gamma}+\alpha_1\frac{V_1^2}{2g}+z_1=\frac{p_2}{\gamma}+\alpha_2\frac{V_2^2}{2g}+z_2+h_L$$

Where $h_L$ is the head loss between section (1) and (2). The $\alpha$ is the constant called *kinetic energy coefficient* obtained from the simplification assumption for non-uniform flow.

$$\int_1^2\frac{V^2}{2}\rho\mathbf{V}\cdot\hat{\mathbf{n}}\,dA=\dot{m}\left(\frac{\alpha_2\bar{V}_2^2}{2}-\frac{\alpha_1\bar{V}_1^2}{2}\right)$$

# Dimensional Analysis of Pipe Flow

## The Moody Chart

- The kinetic energy coefficient is defined by

$$\alpha = \frac{\int_A \left( V^2 / 2 \right) \rho \mathbf{V} \cdot \hat{\mathbf{n}} \, dA}{\dot{m} \bar{V}^2 / 2}$$

- For any velocity profile $\alpha \geq 1$, with $\alpha = 1$ only for uniform flow.
- This can be simplified

| | |
|---|---|
| **Laminar flow** | $\alpha = 2$ |
| **Turbulent flow** | $\alpha = 1$ |

# Dimensional Analysis of Pipe Flow
## The Moody Chart

$$\boxed{h_L = f\,\frac{\ell}{D}\,\frac{V^2}{2g}}$$ ← **Darcy-Weisbach equation**

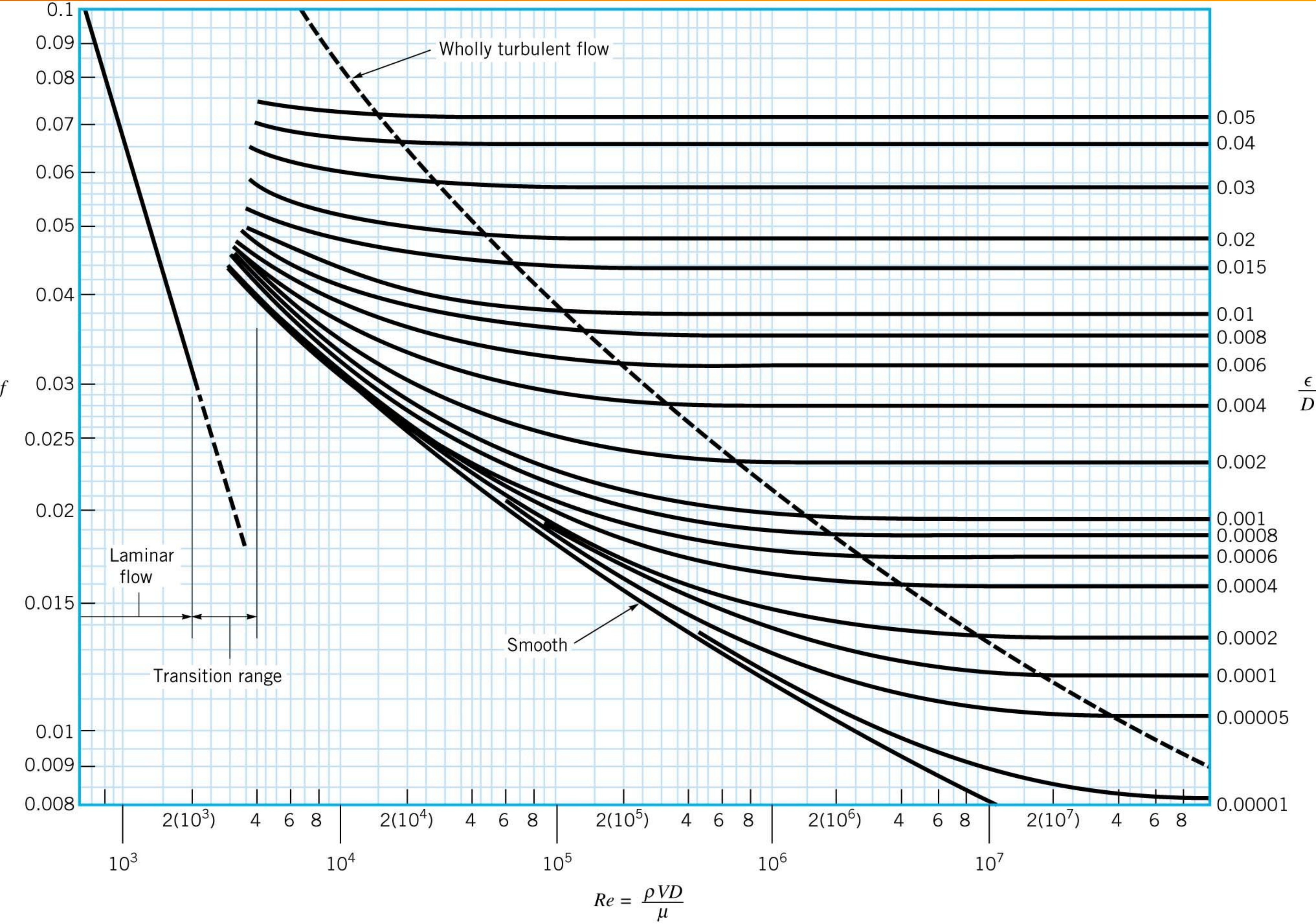
- The Darcy-Weisbach equation is valid for any fully developed, steady, incompressible flow.
- $f$ is the friction factor which is a function of Reynolds number and relative roughness.

$$f = \phi\left(\frac{\rho V D}{\mu}, \frac{\varepsilon}{D}\right)$$

# Dimensional Analysis of Pipe Flow

## The Moody Chart

| Pipe | Equivalent Roughness, $\varepsilon$ | |
|---|---|---|
| | Feet | Millimeters |
| Riveted steel | 0.003–0.03 | 0.9–9.0 |
| Concrete | 0.001–0.01 | 0.3–3.0 |
| Wood stave | 0.0006–0.003 | 0.18–0.9 |
| Cast iron | 0.00085 | 0.26 |
| Galvanized iron | 0.0005 | 0.15 |
| Commercial steel or wrought iron | 0.00015 | 0.045 |
| Drawn tubing | 0.000005 | 0.0015 |
| Plastic, glass | 0.0 (smooth) | 0.0 (smooth) |


Wholly turbulent flow
Laminar flow
Transition range
Smooth
$f$
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.025
0.02
0.015
0.01
0.009
0.008
$\frac{\epsilon}{D}$
0.05
0.04
0.03
0.02
0.015
0.01
0.008
0.006
0.004
0.002
0.001
0.0008
0.0006
0.0004
0.0002
0.0001
0.00005
0.00001
$10^3$ $2(10^3)$ 4 6 8 $10^4$ $2(10^4)$ 4 6 8 $10^5$ $2(10^5)$ 4 6 8 $10^6$ $2(10^6)$ 4 6 8 $10^7$ $2(10^7)$ 4 6 8
$Re = \frac{\rho VD}{\mu}$

# Dimensional Analysis of Pipe Flow

## The Moody Chart

- For **laminar** flow friction factor f is independent from pipe roughness which given friction factor:

$$\boxed{f = \frac{64}{\mathrm{Re}}}$$

- The **turbulent** portion of moody chart is represented by the Colebrook formula:

$$\boxed{\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right)}$$

*Students need to complete and assignment for this section.*

# Dimensional Analysis of Pipe Flow

## Minor Losses

- There are 2 types of losses occur in piping system those are
  - *Major losses* – the loss in straight pipe
  - *Minor losses* – the losses in pipe system components
- Losses due to pipe system components are given in term of ***loss coefficient, $K_L$*** or ***equivalent length, $\ell_{eq}$***.

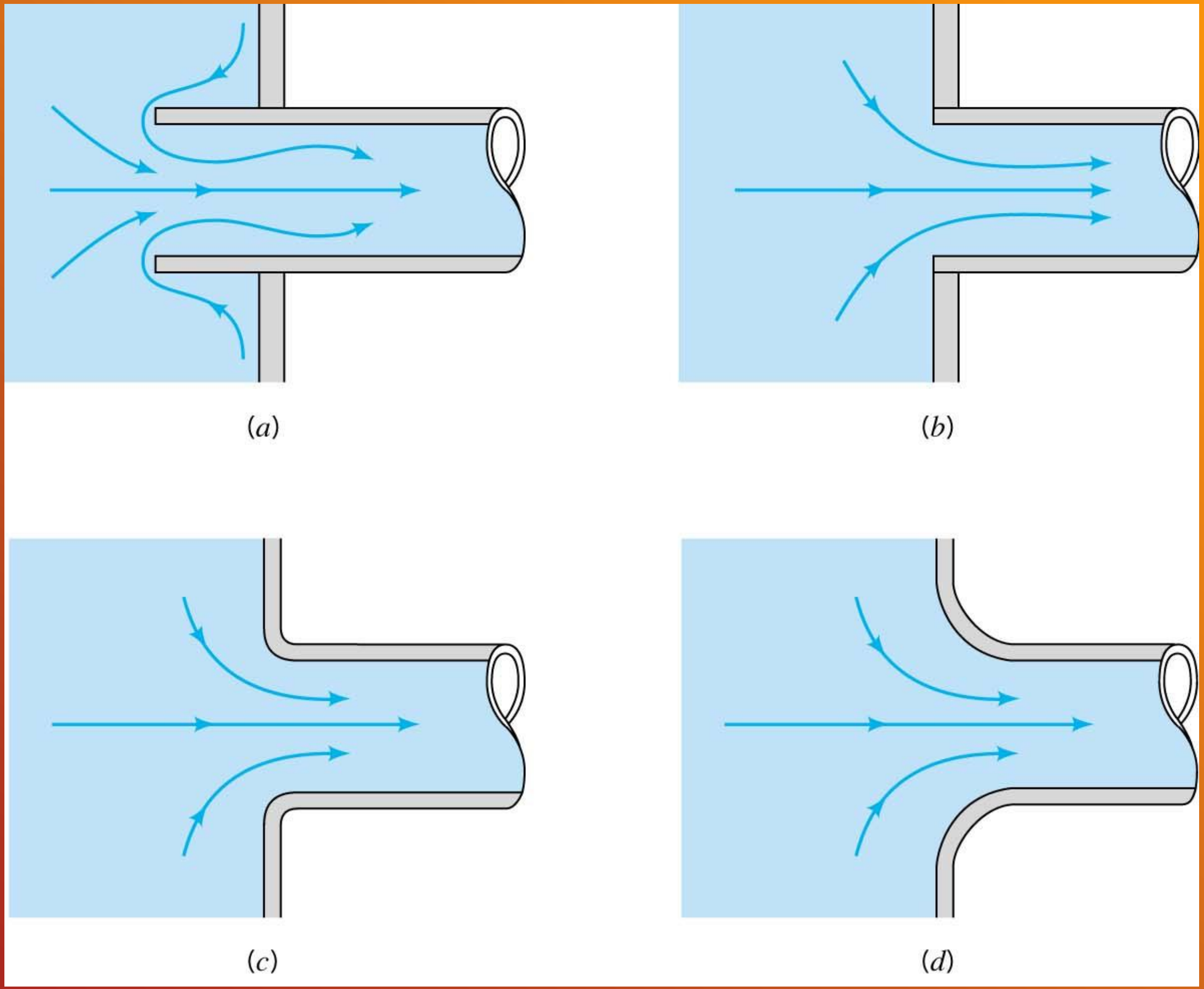
$$h_{Lm} = K_L \frac{V^2}{2g} = f \frac{\ell_{eq}}{D} \frac{V^2}{2g}$$

# Dimensional Analysis of Pipe Flow

## Minor Losses – Entrance flow conditions

$K_L = 0.8$

$K_L = 0.5$

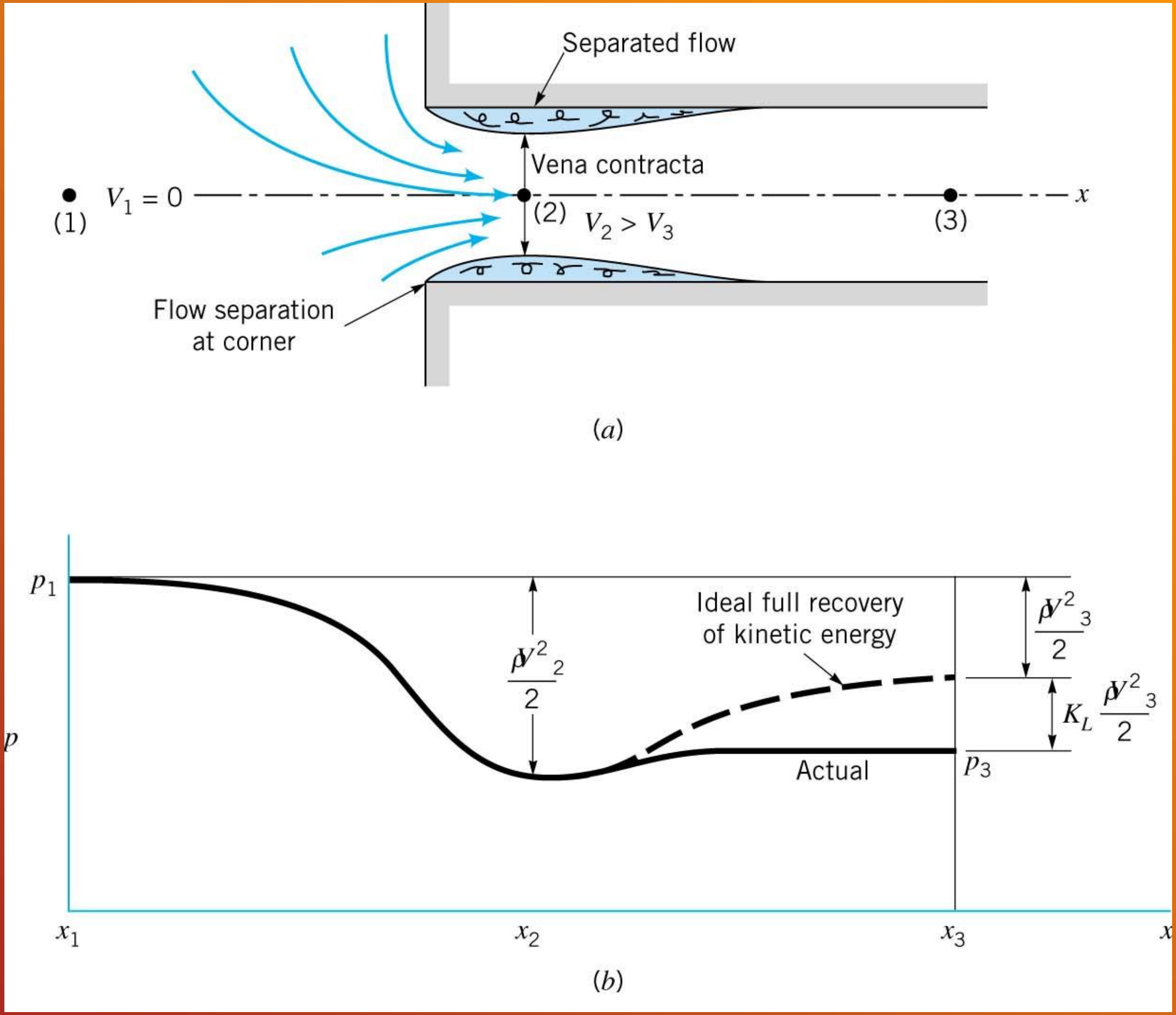



$K_L = 0.2$

$K_L = 0.04$

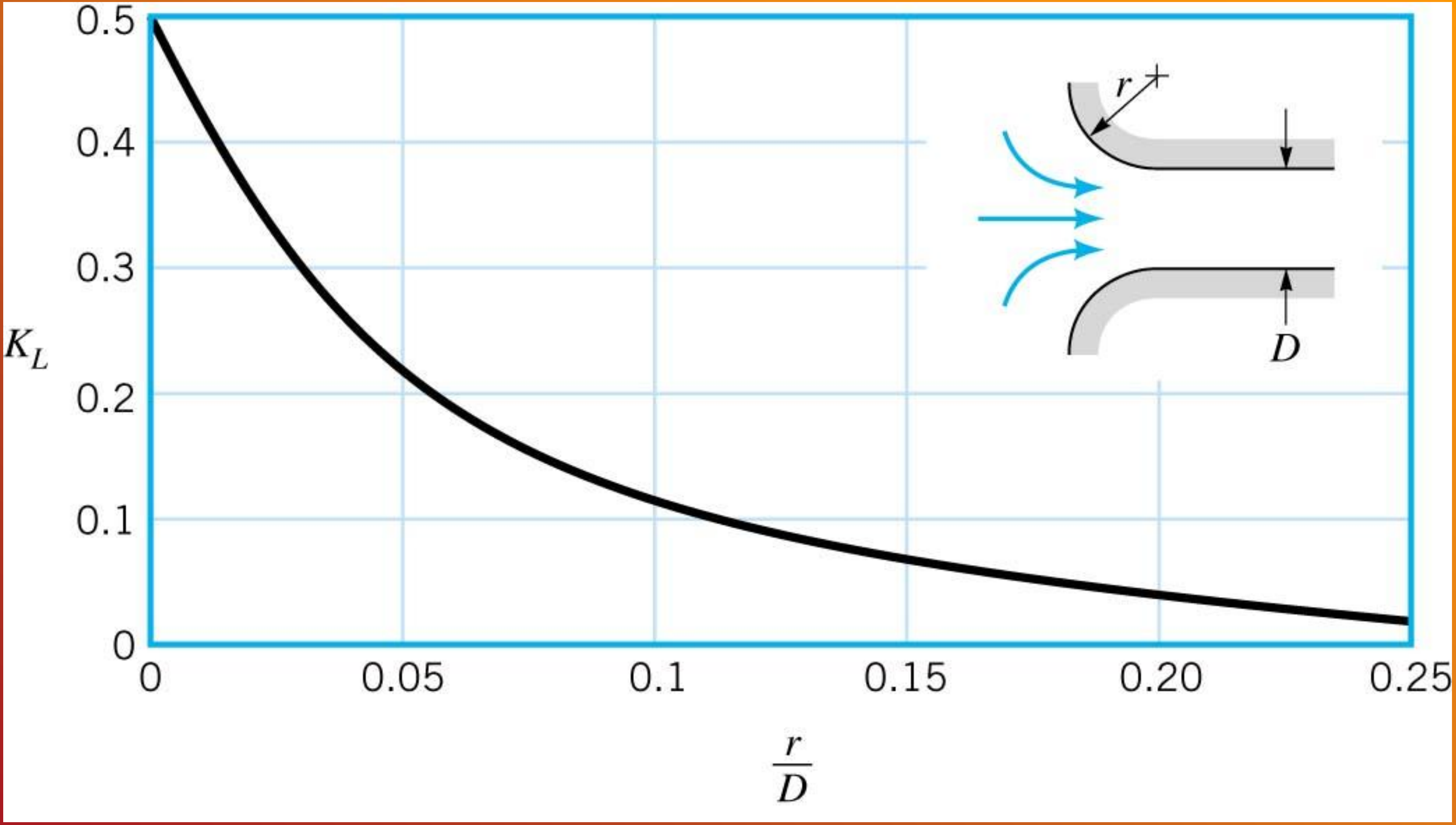
# Dimensional Analysis of Pipe Flow

## Minor Losses – Sharp-edged entrance



# Dimensional Analysis of Pipe Flow

## Minor Losses – Inlet edge



# Dimensional Analysis of Pipe Flow

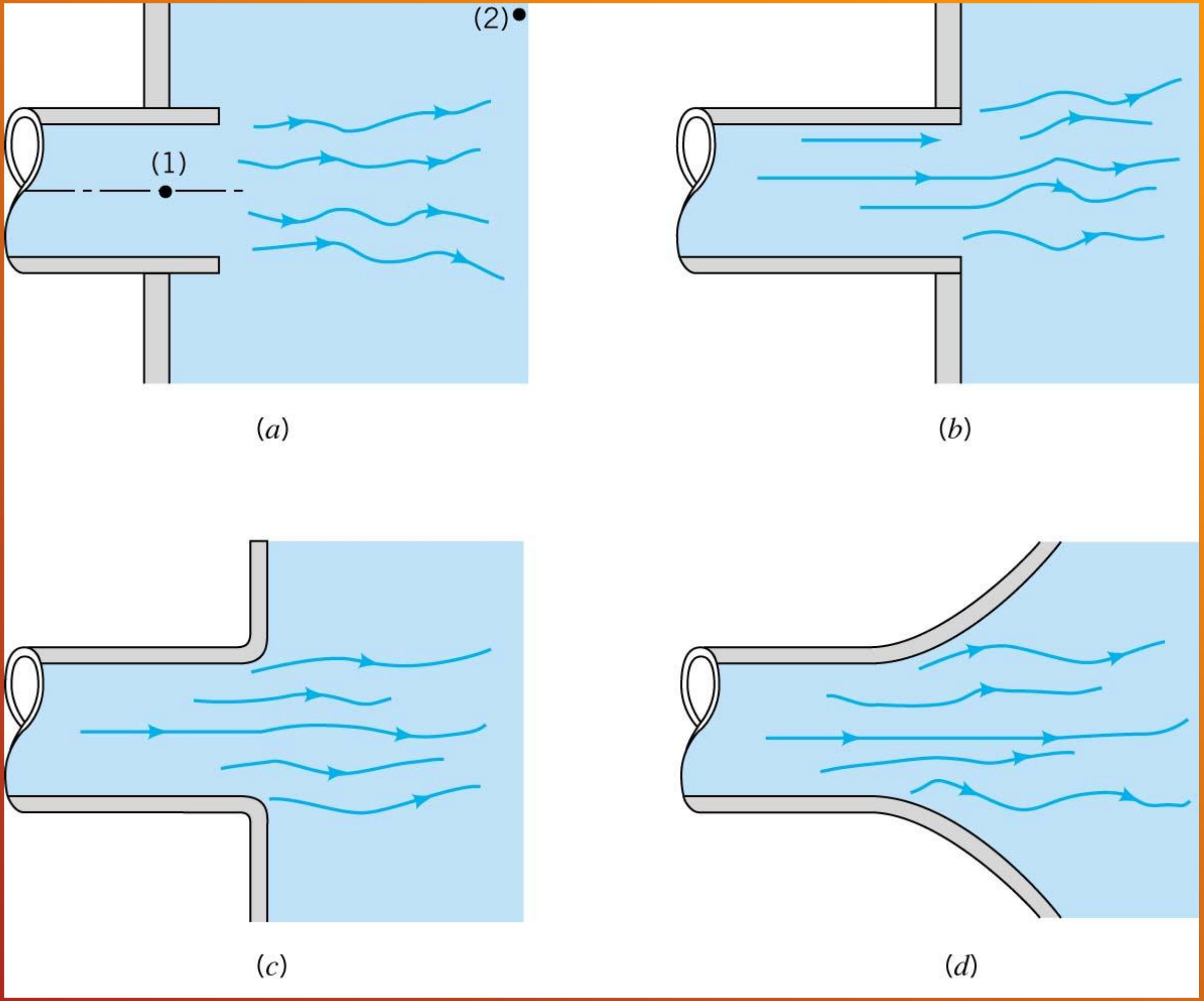
## Minor Losses – Exit flow

$K_L = 1.0$

$K_L = 1.0$

$K_L = 1.0$

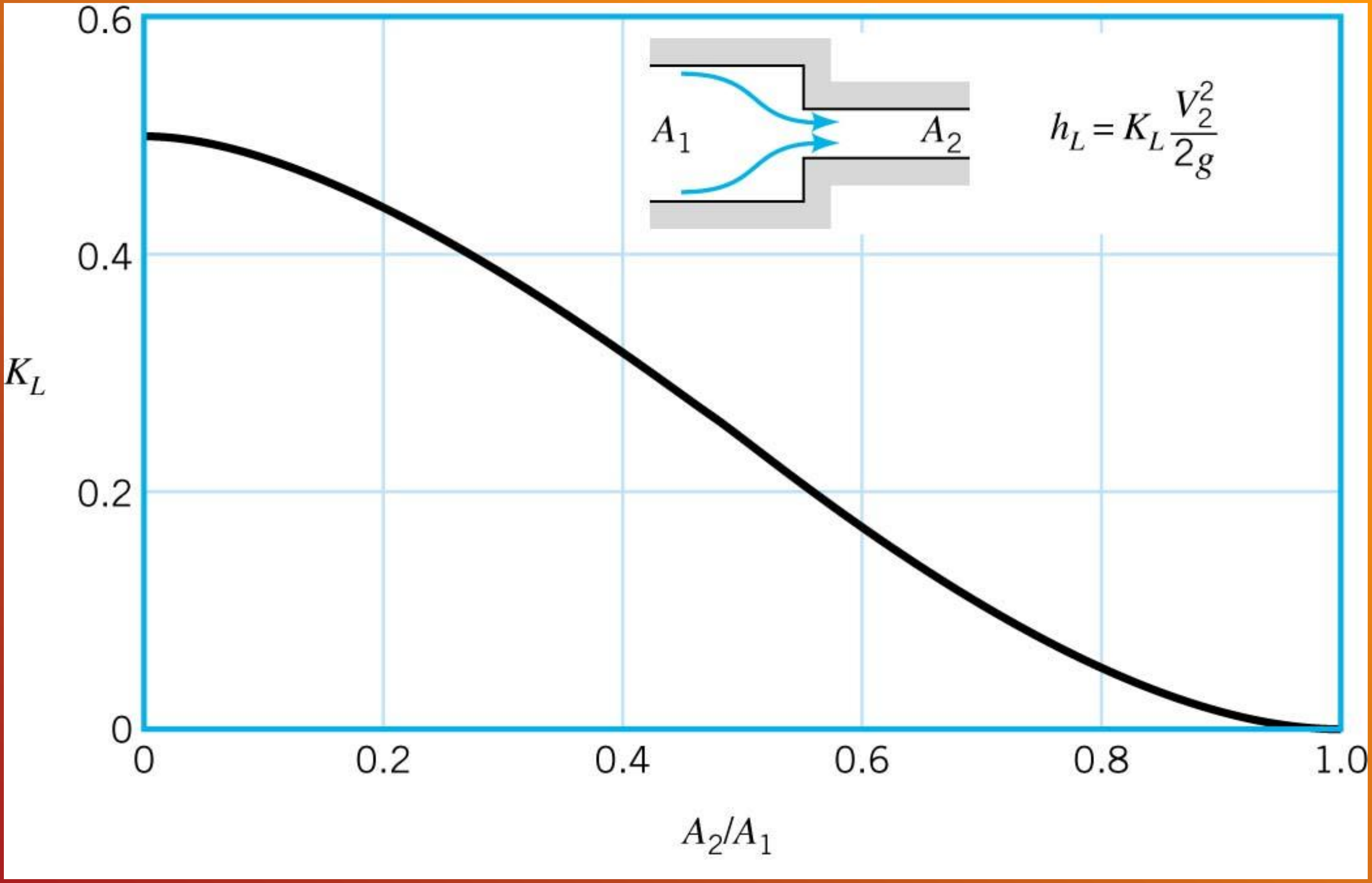
$K_L = 1.0$




(a) (b) (c) (d)

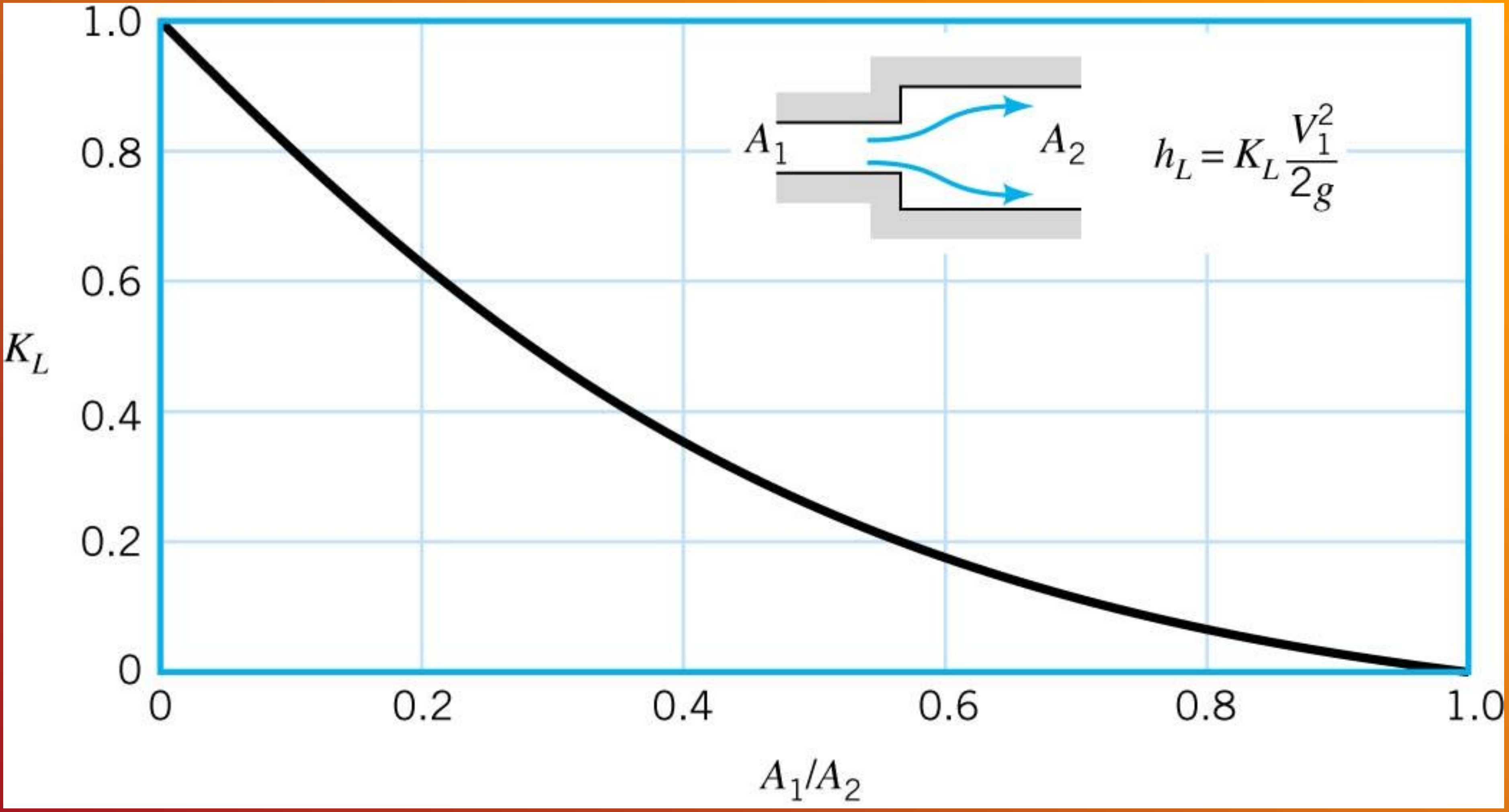
# Dimensional Analysis of Pipe Flow

## Minor Losses – Sudden contraction



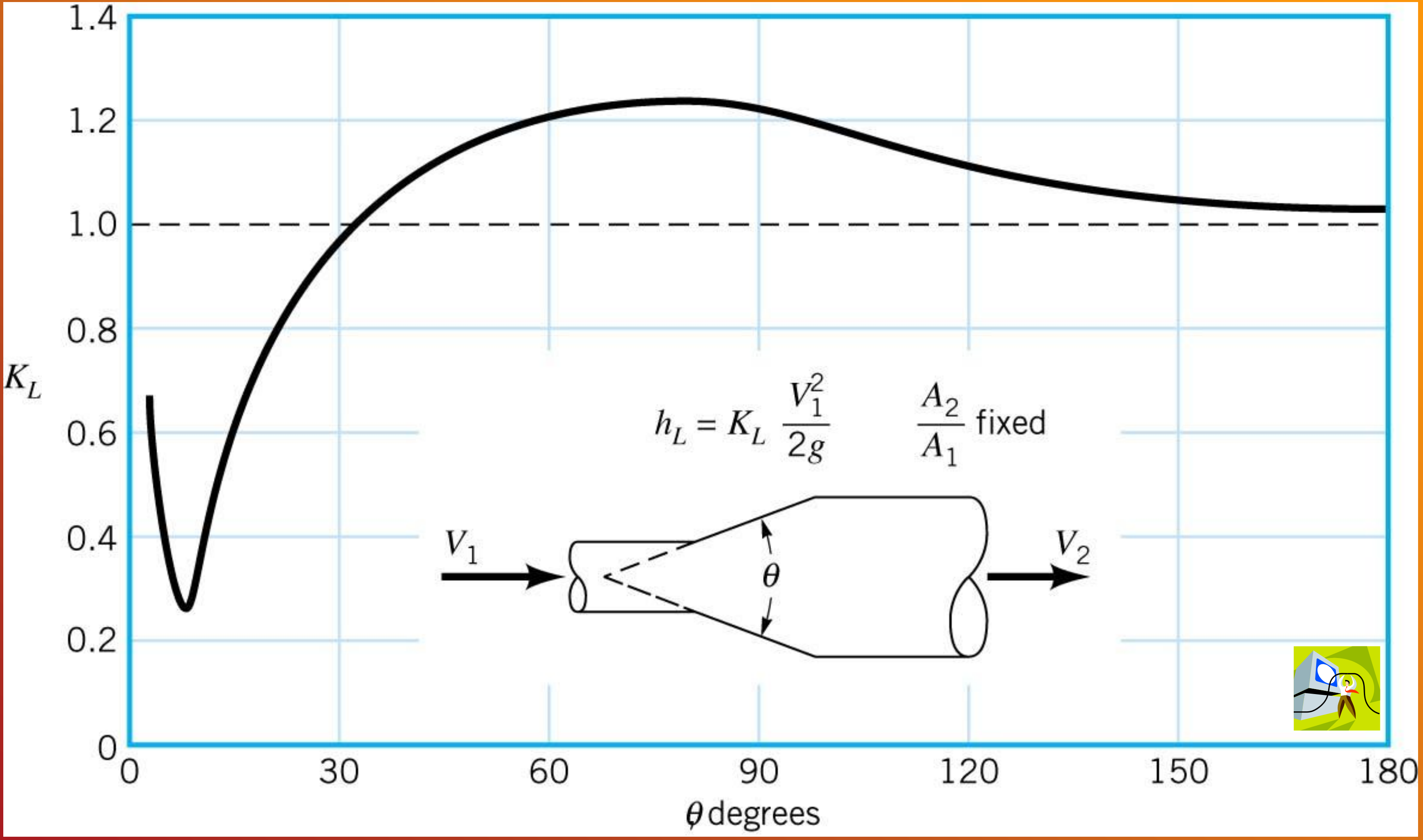
# Dimensional Analysis of Pipe Flow

## Minor Losses – Sudden expansion



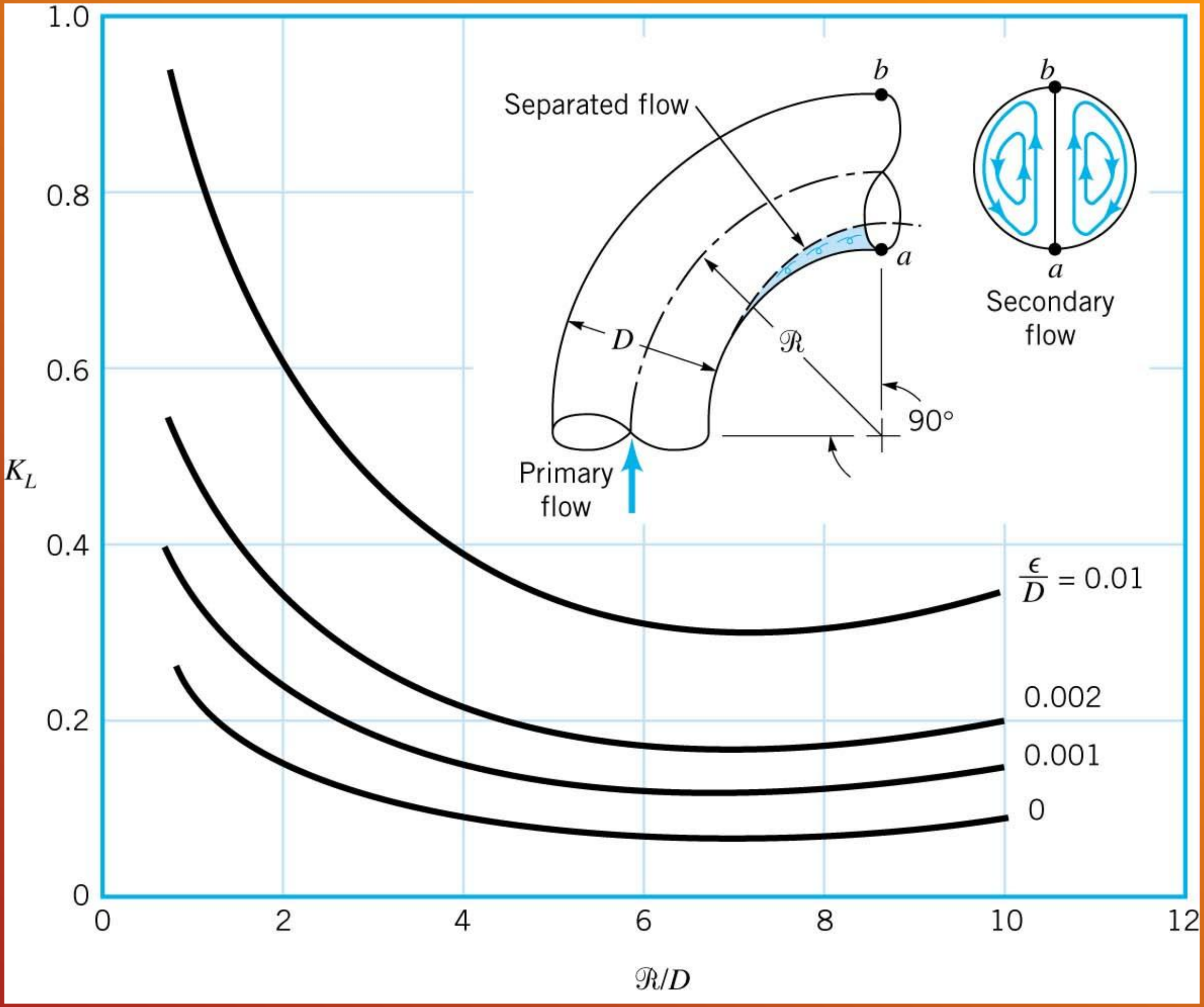
# Dimensional Analysis of Pipe Flow

## Minor Losses – Conical diffuser



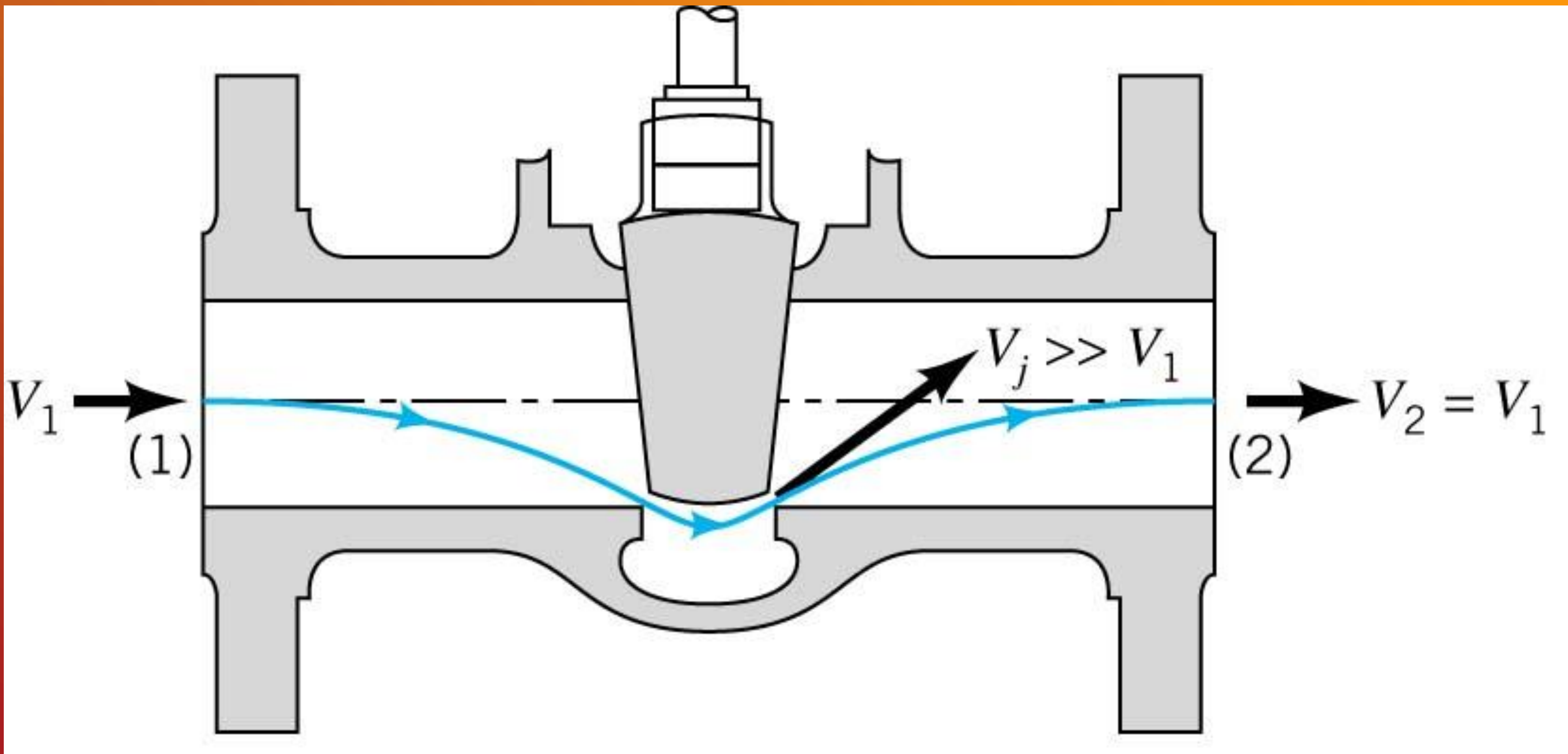
# Dimensional Analysis of Pipe Flow

## Minor Losses – 90° bend



# Dimensional Analysis of Pipe Flow

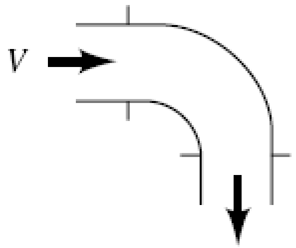
## Minor Losses – Valves

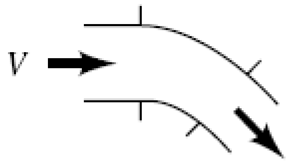


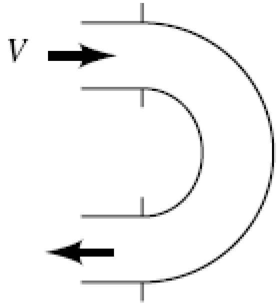
# Dimensional Analysis of Pipe Flow

## Minor Losses – Pipe system components

| Component | $K_L$ |
|---|---|
| **a. Elbows** | |
| Regular 90°, flanged | 0.3 |
| Regular 90°, threaded | 1.5 |
| Long radius 90°, flanged | 0.2 |
| Long radius 90°, threaded | 0.7 |
| Long radius 45°, flanged | 0.2 |
| Regular 45°, threaded | 0.4 |
| **b. 180° return bends** | |
| 180° return bend, flanged | 0.2 |
| 180° return bend, threaded | 1.5 |



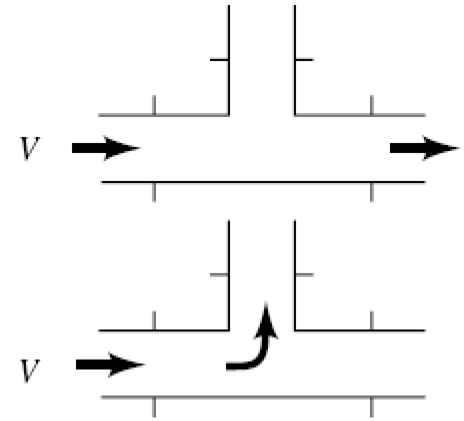



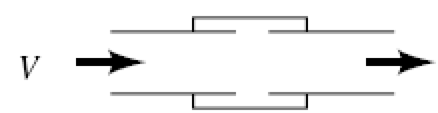



# Dimensional Analysis of Pipe Flow

## Minor Losses – Pipe system components

| Component | $K_L$ |
|---|---|
| **c. Tees** | |
| Line flow, flanged | 0.2 |
| Line flow, threaded | 0.9 |
| Branch flow, flanged | 1.0 |
| Branch flow, threaded | 2.0 |
| **d. Union, threaded** | 0.08 |



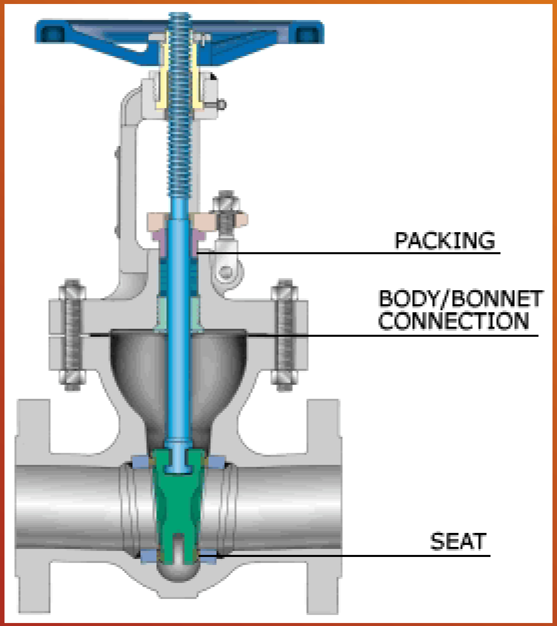



# Dimensional Analysis of Pipe Flow

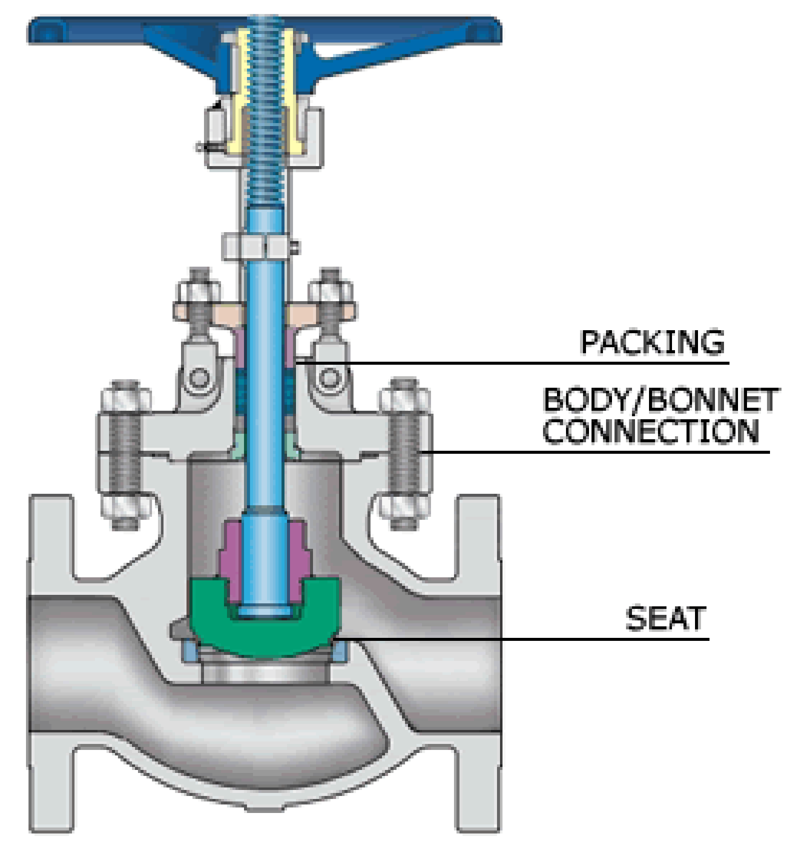
## Minor Losses – Pipe system components

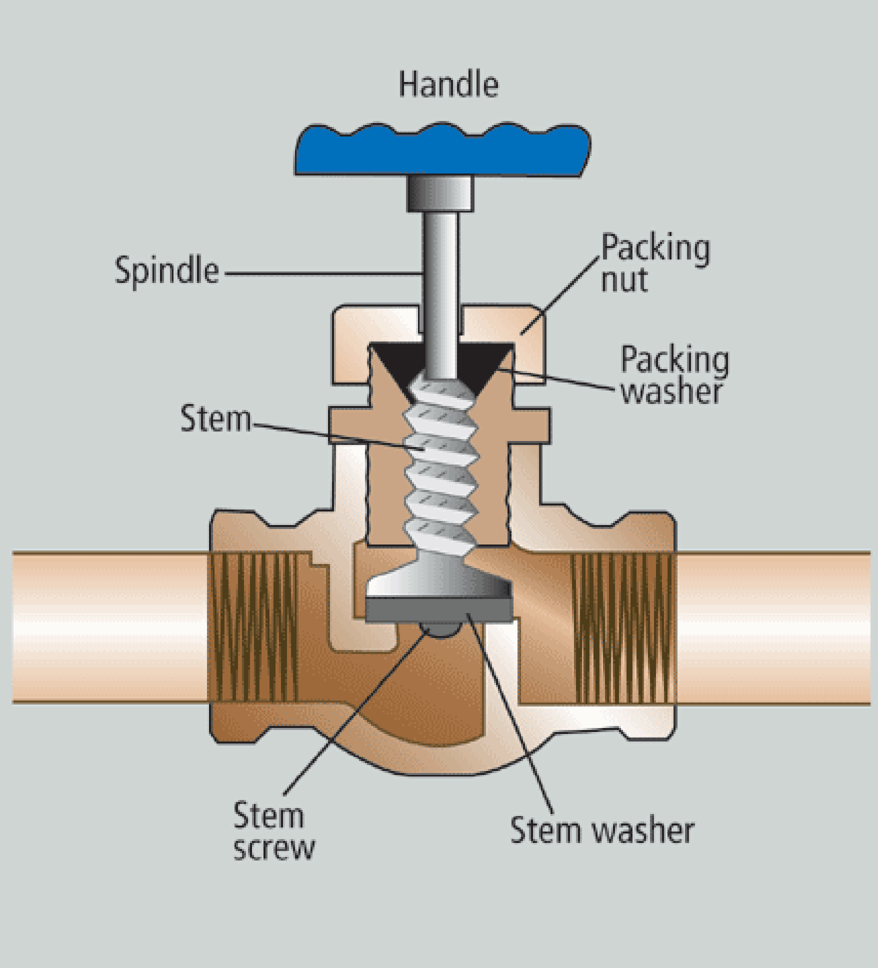
| Component | $K_L$ |
|---|---|
| **e. Valves** | |
| Globe, fully open | 10 |
| Angle, fully open | 2 |
| Gate, fully open | 0.15 |
| Gate, $\frac{1}{4}$ closed | 0.26 |
| Gate, $\frac{1}{2}$ closed | 2.1 |
| Gate, $\frac{3}{4}$ closed | 17 |
| Swing check, forward flow | 2 |
| Swing check, backward flow | $\infty$ |
| Ball valve, fully open | 0.05 |
| Ball valve, $\frac{1}{3}$ closed | 5.5 |
| Ball valve, $\frac{2}{3}$ closed | 210 |

# Gate Valves



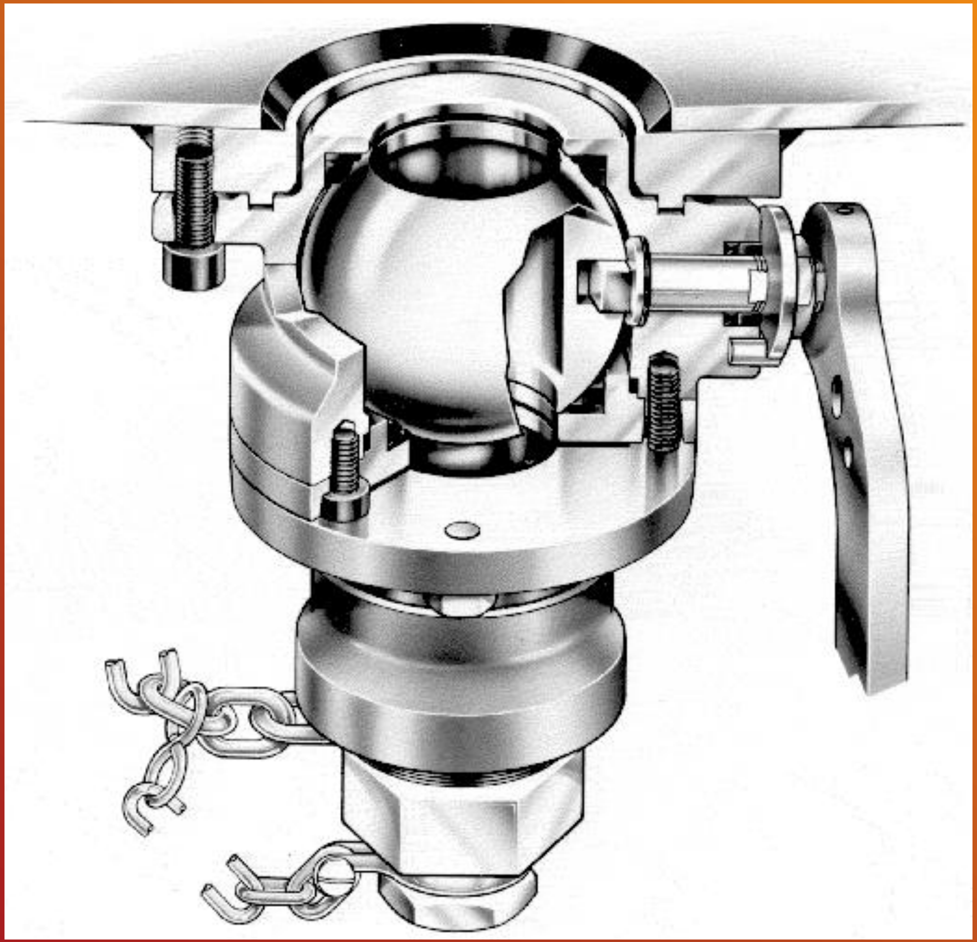

# Globe Valves



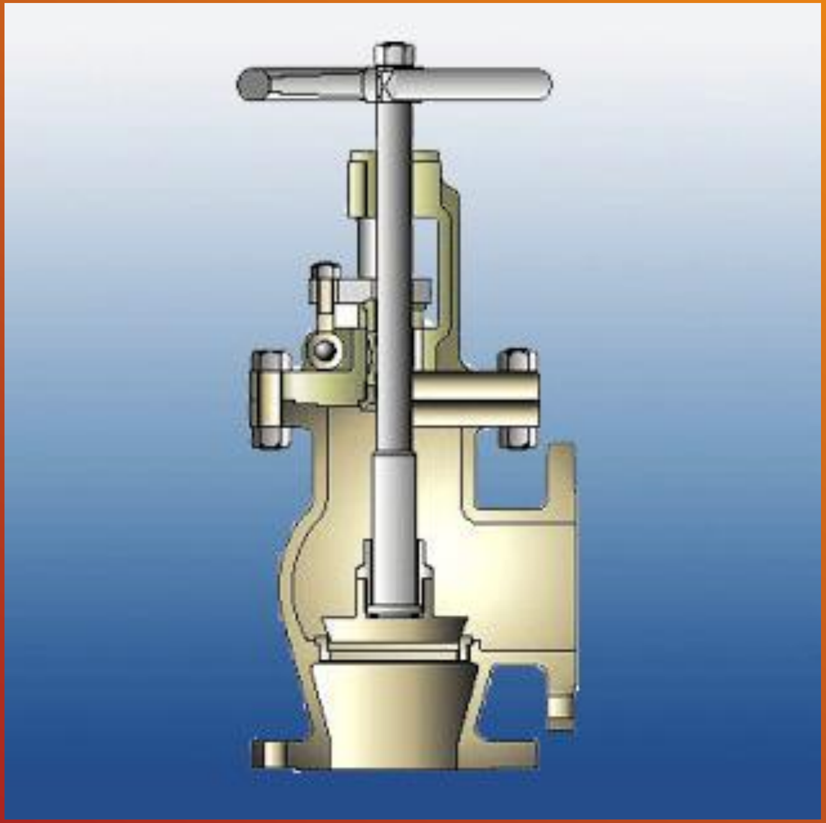



# Butterfly Valves

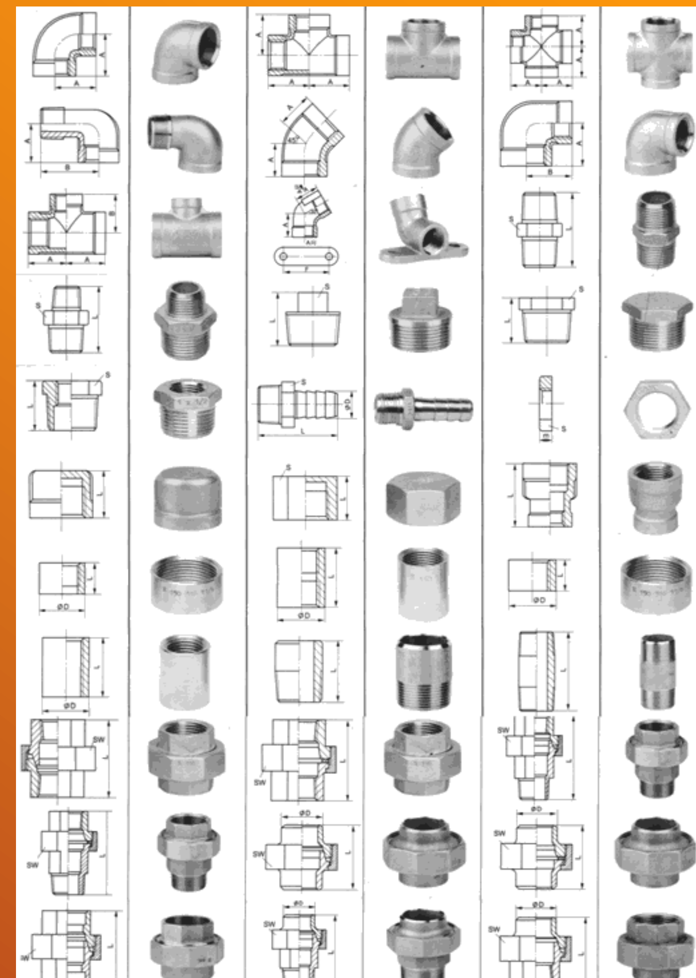
# Ball Valves



# Angle Valves



# Pipe Fittings



# Pipe Fittings

# Pipe Flow Example

## Single Pipe

- Pipe flow problems can be categorized by what parameters are given and what is to be calculated.

| Variable | Type I | Type II | Type III |
|---|---|---|---|
| **a. Fluid** | | | |
| Density | Given | Given | Given |
| Viscosity | Given | Given | Given |
| **b. Pipe** | | | |
| Diameter | Given | Given | Determine |
| Length | Given | Given | Given |
| Roughness | Given | Given | Given |
| **c. Flow** | | | |
| Flowrate or Average Velocity | Given | Determine | Given |
| **d. Pressure** | | | |
| Pressure Drop or Head Loss | Determine | Given | Given |