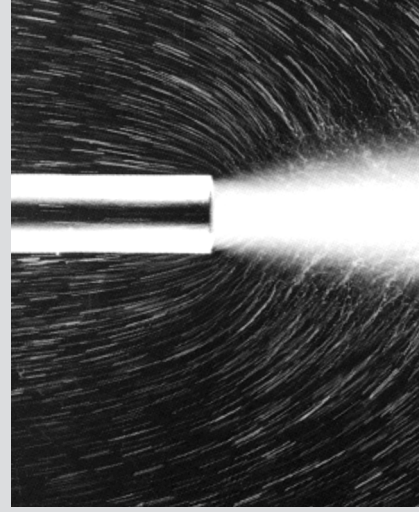


A jet of water injected into stationary water: Upon emerging from the slit at the left, the jet of fluid loses some of its momentum to the surrounding fluid. This causes the jet to slow down and its width to increase (air bubbles in water). (Photograph courtesy of ONERA, France.)

# 5

## *Finite Control Volume Analysis*



*Many fluid mechanics problems can be solved by using control volume analysis.*

Many practical problems in fluid mechanics require analysis of the behavior of the contents of a finite region in space (a control volume). For example, we may be asked to calculate the anchoring force required to hold a jet engine in place during a test. Or, we could be called on to determine the amount of time to allow for complete filling of a large storage tank. An estimate of how much power it would take to move water from one location to another at a higher elevation and several miles away may be sought. As you will learn by studying the material in this chapter, these and many other important questions can be readily answered with finite control volume analyses. The bases of this analysis method are some fundamental principles of physics, namely, conservation of mass, Newton's second law of motion, and the first and second<sup>1</sup> laws of thermodynamics. Thus, as one might expect, the resultant techniques are powerful and applicable to a wide variety of fluid mechanical circumstances that require engineering judgment. Furthermore, the finite control volume formulas are easy to interpret physically and thus are not difficult to use.

The control volume formulas are derived from the equations representing basic laws applied to a collection of mass (a system). The system statements are probably familiar to you presently. However, in fluid mechanics, the control volume or Eulerian view is generally less complicated and, therefore, more convenient to use than the system or Lagrangian view. The concept of a control volume and system occupying the same region of space at an instant (coincident condition) and use of the Reynolds transport theorem (Eqs. 4.19 and 4.23) are key elements in the derivation of the control volume equations.

Integrals are used throughout the chapter for generality. Volume integrals can accommodate spatial variations of the material properties of the contents of a control volume. Control surface area integrals allow for surface distributions of flow variables. However, in this chapter, for simplicity we often assume that flow variables are uniformly distributed over cross-sectional areas where fluid enters or leaves the control volume. This uniform flow is called one-dimensional flow. In Chapters 8 and 9, when we discuss velocity profiles and other flow variable distributions, the effects of nonuniformities will be covered in more detail.

<sup>1</sup>The section (Section 5.4) on the second law of thermodynamics may be omitted without loss of continuity in the text material.

Mostly steady flows are considered. However, some simple examples of unsteady flow analyses are introduced.

Although fixed, nondeforming control volumes are emphasized in this chapter, a few examples of moving, nondeforming control volumes and deforming control volumes are also included.

## 5.1 Conservation of Mass—The Continuity Equation

### 5.1.1 Derivation of the Continuity Equation

A system is defined as a collection of unchanging contents, so the conservation of mass principle for a system is simply stated as

$$\text{time rate of change of the system mass} = 0$$

or

$$\frac{DM_{\text{sys}}}{Dt} = 0 \quad (5.1)$$

*The amount of mass in a system is constant.*

where the system mass,  $M_{\text{sys}}$ , is more generally expressed as

$$M_{\text{sys}} = \int_{\text{sys}} \rho \, d\mathcal{V} \quad (5.2)$$

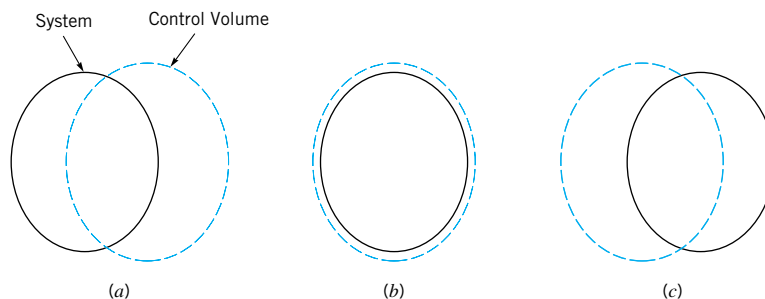
and the integration is over the volume of the system. In words, Eq. 5.2 states that the system mass is equal to the sum of all the density-volume element products for the contents of the system.

For a system and a fixed, nondeforming control volume that are coincident at an instant of time, as illustrated in Fig. 5.1, the Reynolds transport theorem (Eq. 4.19) with  $B = \text{mass}$  and  $b = 1$  allows us to state that

$$\frac{D}{Dt} \int_{\text{sys}} \rho \, d\mathcal{V} = \frac{\partial}{\partial t} \int_{\text{cv}} \rho \, d\mathcal{V} + \int_{\text{cs}} \rho \mathbf{V} \cdot \hat{\mathbf{n}} \, dA \quad (5.3)$$

or

$$\begin{array}{l} \text{time rate of change} \\ \text{of the mass of the} \\ \text{coincident system} \end{array} = \begin{array}{l} \text{time rate of change} \\ \text{of the mass of the} \\ \text{contents of the coin-} \\ \text{cident control volume} \end{array} + \begin{array}{l} \text{net rate of flow} \\ \text{of mass through} \\ \text{the control} \\ \text{surface} \end{array}$$



■ **FIGURE 5.1** System and control volume at three different instances of time. (a) System and control volume at time  $t - \delta t$ . (b) System and control volume at time  $t$ , coincident condition. (c) System and control volume at time  $t + \delta t$ .

In Eq. 5.3, we express the time rate of change of the system mass as the sum of two control volume quantities, the time rate of change of the mass of the contents of the control volume,

$$\frac{\partial}{\partial t} \int_{cv} \rho dV$$

and the net rate of mass flow through the control surface,

$$\int_{cs} \rho \mathbf{V} \cdot \hat{\mathbf{n}} dA$$

When a flow is steady, all field properties (i.e., properties at any specified point) including density remain constant with time and the time rate of change of the mass of the contents of the control volume is zero. That is,

$$\frac{\partial}{\partial t} \int_{cv} \rho dV = 0$$

The integrand,  $\mathbf{V} \cdot \hat{\mathbf{n}} dA$ , in the mass flowrate integral represents the product of the component of velocity,  $\mathbf{V}$ , perpendicular to the small portion of control surface and the differential area,  $dA$ . Thus,  $\mathbf{V} \cdot \hat{\mathbf{n}} dA$  is the volume flowrate through  $dA$  and  $\rho \mathbf{V} \cdot \hat{\mathbf{n}} dA$  is the mass flowrate through  $dA$ . Furthermore, the sign of the dot product  $\mathbf{V} \cdot \hat{\mathbf{n}}$  is “+” for flow *out* of the control volume and “−” for flow *into* the control volume since  $\hat{\mathbf{n}}$  is considered positive when it points out of the control volume. When all of the differential quantities,  $\rho \mathbf{V} \cdot \hat{\mathbf{n}} dA$ , are summed over the entire control surface, as indicated by the integral

$$\int_{cs} \rho \mathbf{V} \cdot \hat{\mathbf{n}} dA$$

the result is the net mass flowrate through the control surface, or

$$\int_{cs} \rho \mathbf{V} \cdot \hat{\mathbf{n}} dA = \sum \dot{m}_{out} - \sum \dot{m}_{in} \quad (5.4)$$

where  $\dot{m}$  is the mass flowrate (slug/s or kg/s). If the integral in Eq. 5.4 is positive, the net flow is out of the control volume; if the integral is negative, the net flow is into the control volume.

The control volume expression for *conservation of mass*, which is commonly called the *continuity equation*, for a fixed, nondeforming control volume is obtained by combining Eqs. 5.1, 5.2, and 5.3 to obtain

**The continuity equation is a statement that mass is conserved.**

$$\frac{\partial}{\partial t} \int_{cv} \rho dV + \int_{cs} \rho \mathbf{V} \cdot \hat{\mathbf{n}} dA = 0 \quad (5.5)$$

In words, Eq. 5.5 states that to conserve mass the time rate of change of the mass of the contents of the control volume plus the net rate of mass flow through the control surface must equal zero. Actually, the same result could have been obtained more directly by equating the rates of mass flow into and out of the control volume to the rates of accumulation and depletion of mass within the control volume (See [Section 3.6.2](#)). It is reassuring, however, to see that the Reynolds transport theorem works for this simple-to-understand case. This confidence will serve us well as we develop control volume expressions for other important principles.

**Mass flowrate equals the product of density and volume flowrate.**

An often-used expression for mass flowrate,  $\dot{m}$ , through a section of control surface having area  $A$  is

$$\dot{m} = \rho Q = \rho A V \quad (5.6)$$

where  $\rho$  is the fluid density,  $Q$  is the volume flowrate ( $\text{ft}^3/\text{s}$  or  $\text{m}^3/\text{s}$ ), and  $V$  is the component of fluid velocity perpendicular to area  $A$ . Since

$$\dot{m} = \int_A \rho \mathbf{V} \cdot \hat{\mathbf{n}} dA$$

application of Eq. 5.6 involves the use of *representative* or average values of fluid density,  $\rho$ , and fluid velocity,  $V$ . For incompressible flows,  $\rho$  is uniformly distributed over area  $A$ . For compressible flows, we will normally consider a uniformly distributed fluid density at each section of flow and allow density changes to occur only from section to section. The appropriate fluid velocity to use in Eq. 5.6 is the average value of the component of velocity normal to the section area involved. This average value,  $\bar{V}$ , is defined as

$$\bar{V} = \frac{\int_A \rho \mathbf{V} \cdot \hat{\mathbf{n}} dA}{\rho A} \quad (5.7)$$

If the velocity is considered uniformly distributed (one-dimensional flow) over the section area,  $A$ , then

$$\bar{V} = \frac{\int_A \rho \mathbf{V} \cdot \hat{\mathbf{n}} dA}{\rho A} = V \quad (5.8)$$

and the bar notation is not necessary (as in Example 5.1). When the flow is not uniformly distributed over the flow cross-sectional area, the bar notation reminds us that an average velocity is being used (as in Examples 5.2 and 5.4).



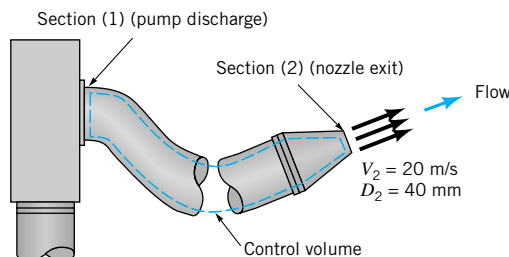
#### V5.1 Sink flow

### 5.1.2 Fixed, Nondeforming Control Volume

In many applications of fluid mechanics, an appropriate control volume to use is fixed and nondeforming. Several example problems that involve the continuity equation for fixed, nondeforming control volumes (Eq. 5.5) follow.

## EXAMPLE 5.1

Seawater flows steadily through a simple conical-shaped nozzle at the end of a fire hose as illustrated in Fig. E5.1. If the nozzle exit velocity must be at least 20 m/s, determine the minimum pumping capacity required in  $\text{m}^3/\text{s}$ .



■ FIGURE E5.1

## SOLUTION

The pumping capacity sought is the volume flowrate delivered by the fire pump to the hose and nozzle. Since we desire knowledge about the pump discharge flowrate and we have information about the nozzle exit flowrate, we link these two flowrates with the control volume designated with the dashed line in Fig. E5.1. This control volume contains, at any instant, seawater that is within the hose and nozzle from the pump discharge to the nozzle exit plane.

Equation 5.5 is applied to the contents of this control volume to give

$$\frac{\partial}{\partial t} \int_{cv} \rho \, dV + \int_{cs} \rho \mathbf{V} \cdot \hat{\mathbf{n}} \, dA = 0 \quad \text{0 (flow is steady)} \quad (1)$$

The time rate of change of the mass of the contents of this control volume is zero because the flow is steady. From Eq. 5.4, we see that the control surface integral in Eq. 1 involves mass flowrates at the pump discharge, section (1), and at the nozzle exit, section (2), or

$$\int_{cs} \rho \mathbf{V} \cdot \hat{\mathbf{n}} \, dA = \dot{m}_2 - \dot{m}_1 = 0$$

so that

$$\dot{m}_2 = \dot{m}_1 \quad (2)$$

Since the mass flowrate is equal to the product of fluid density,  $\rho$ , and volume flowrate,  $Q$ , (see Eq. 5.6), we obtain from Eq. 2

$$\rho_2 Q_2 = \rho_1 Q_1 \quad (3)$$

Liquid flow at low speeds, as in this example, may be considered incompressible. Therefore  $\rho_2 = \rho_1$  and from Eq. 3

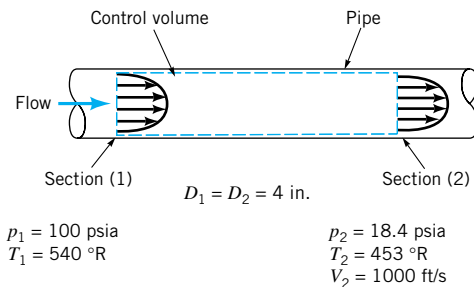
$$Q_2 = Q_1 \quad (4)$$

The pumping capacity is equal to the volume flowrate at the nozzle exit. If, for simplicity, the velocity distribution at the nozzle exit plane, section (2), is considered uniform (one-dimensional), then from Eqs. 4, 5.6, and 5.8

$$\begin{aligned} Q_1 &= Q_2 = V_2 A_2 \\ &= V_2 \frac{\pi}{4} D_2^2 = (20 \, \text{m/s}) \frac{\pi}{4} \left( \frac{40 \, \text{mm}}{1000 \, \text{mm/m}} \right)^2 = 0.0251 \, \text{m}^3/\text{s} \end{aligned} \quad (\text{Ans})$$

## EXAMPLE 5.2

Air flows steadily between two sections in a long, straight portion of 4-in. inside diameter pipe as indicated in Fig. E5.2. The uniformly distributed temperature and pressure at each section are given. If the average air velocity (nonuniform velocity distribution) at section (2) is 1000 ft/s, calculate the average air velocity at section (1).



■ FIGURE E5.2

## SOLUTION

The average fluid velocity at any section is that velocity which yields the section mass flowrate when multiplied by the section average fluid density and section area (Eq. 5.7). We relate the flows at sections (1) and (2) with the control volume designated with a dashed line in Fig. E5.2.

Equation 5.5 is applied to the contents of this control volume to obtain

$$\frac{\partial}{\partial t} \int_{cv} \rho dV + \int_{cs} \rho \mathbf{V} \cdot \hat{\mathbf{n}} dA = 0 \quad (\text{flow is steady})$$

The time rate of change of the mass of the contents of this control volume is zero because the flow is steady. The control surface integral involves mass flowrates at sections (1) and (2) so that from Eq. 5.4 we get

$$\int_{cs} \rho \mathbf{V} \cdot \hat{\mathbf{n}} dA = \dot{m}_2 - \dot{m}_1 = 0$$

or

$$\dot{m}_1 = \dot{m}_2 \quad (1)$$

and from Eqs. 1, 5.6, and 5.7 we obtain

$$\rho_1 A_1 \bar{V}_1 = \rho_2 A_2 \bar{V}_2 \quad (2)$$

or since  $A_1 = A_2$

$$\bar{V}_1 = \frac{\rho_2}{\rho_1} \bar{V}_2 \quad (3)$$

Air at the pressures and temperatures involved in this example problem behaves like an ideal gas. The ideal gas equation of state (Eq. 1.8) is

$$\rho = \frac{P}{RT} \quad (4)$$

Thus, combining Eqs. 3 and 4 we obtain

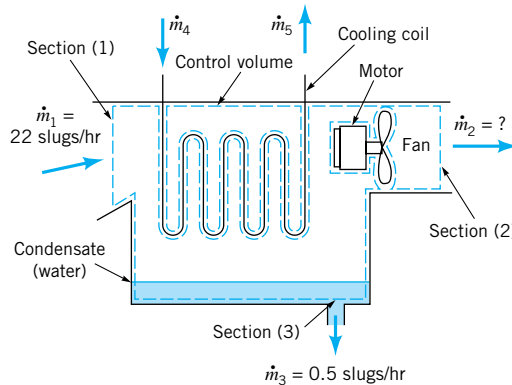
$$\begin{aligned} \bar{V}_1 &= \frac{p_2 T_1 \bar{V}_2}{p_1 T_2} \\ &= \frac{(18.4 \text{ psia})(540 \text{ °R})(1000 \text{ ft/s})}{(100 \text{ psia})(453 \text{ °R})} = 219 \text{ ft/s} \end{aligned}$$

(Ans)

We learn from this example that the continuity equation (Eq. 5.5) is valid for compressible as well as incompressible flows. Also, nonuniform velocity distributions can be handled with the average velocity concept.

## EXAMPLE 5.3

Moist air (a mixture of dry air and water vapor) enters a dehumidifier at the rate of 22 slugs/hr. Liquid water drains out of the dehumidifier at a rate of 0.5 slugs/hr. Determine the mass flowrate of the dry air and the water vapor leaving the dehumidifier. A simplified sketch of the process is provided in Fig. E5.3.



■ FIGURE E5.3

## SOLUTION

The unknown mass flowrate at section (2) is linked with the known flowrates at sections (1) and (3) with the control volume designated with a dashed line in Fig. E5.3. The contents of the control volume are the air and water vapor mixture and the condensate (liquid water) in the dehumidifier at any instant.

Not included in the control volume are the fan and its motor, and the condenser coils and refrigerant. Even though the flow in the vicinity of the fan blade is unsteady, it is unsteady in a cyclical way. Thus, the flowrates at sections (1), (2), and (3) appear steady and the time rate of change of the mass of the contents of the control volume may be considered equal to zero on a time-average basis. The application of Eqs. 5.4 and 5.5 to the control volume contents results in

$$\int_{cs} \rho \mathbf{V} \cdot \hat{\mathbf{n}} dA = -\dot{m}_1 + \dot{m}_2 + \dot{m}_3 = 0$$

or

$$\dot{m}_2 = \dot{m}_1 - \dot{m}_3 = 22 \text{ slugs/hr} - 0.5 \text{ slugs/hr} = 21.5 \text{ slugs/hr} \quad (\text{Ans})$$

Note that the continuity equation (Eq. 5.5) can be used when there is more than one stream of fluid flowing through the control volume.

The answer is the same regardless of which control volume is chosen. For example, if we select the same control volume as before except that we include the cooling coils to be within the control volume, the continuity equation becomes

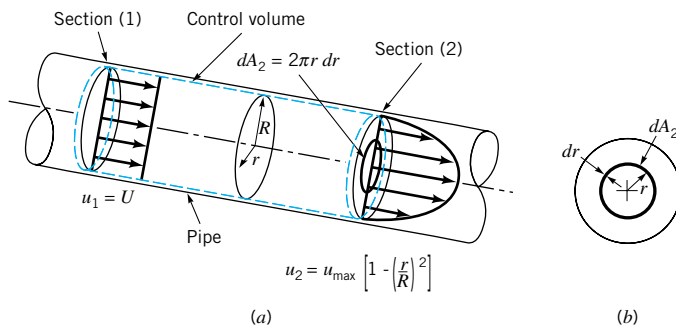
$$\dot{m}_2 = \dot{m}_1 - \dot{m}_3 + \dot{m}_4 - \dot{m}_5 \quad (1)$$

where  $\dot{m}_4$  is the mass flowrate of the cooling fluid flowing into the control volume, and  $\dot{m}_5$  is the flowrate out of the control volume through the cooling coil. Since the flow through the coils is steady, it follows that  $\dot{m}_4 = \dot{m}_5$ . Hence, Eq. 1 gives the same answer as obtained with the original control volume.



## EXAMPLE 5.4

Incompressible, laminar water flow develops in a straight pipe having radius  $R$  as indicated in Fig. E5.4. At section (1), the velocity profile is uniform; the velocity is equal to a constant value  $U$  and is parallel to the pipe axis everywhere. At section (2), the velocity profile is axisymmetric and parabolic, with zero velocity at the pipe wall and a maximum value of  $u_{\max}$  at the centerline. How are  $U$  and  $u_{\max}$  related? How are the average velocity at section (2),  $\bar{V}_2$ , and  $u_{\max}$  related?



■ FIGURE E5.4

## SOLUTION

An appropriate control volume is sketched (dashed lines) in Fig. E5.4a. The application of Eq. 5.5 to the contents of this control volume yields

$$\int_{cs} \rho \mathbf{V} \cdot \hat{\mathbf{n}} dA = 0$$

The surface integral is evaluated at sections (1) and (2) to give

$$-\rho_1 A_1 U + \int_{A_2} \rho \mathbf{V} \cdot \hat{\mathbf{n}} dA_2 = 0 \quad (1)$$

or, since the component of velocity,  $\mathbf{V}$ , perpendicular to the area at section (2) is  $u_2$ , and the element cross-sectional area,  $dA_2$ , is equal to  $2\pi r dr$  (see shaded area element in Fig. E5.4b), Eq. 1 becomes

$$-\rho_1 A_1 U + \rho_2 \int_0^R u_2 2\pi r dr = 0 \quad (2)$$

Since the flow is considered incompressible,  $\rho_1 = \rho_2$ . The parabolic velocity relationship for flow through section (2) is used in Eq. 2 to yield

$$-A_1 U + 2\pi u_{\max} \int_0^R \left[ 1 - \left( \frac{r}{R} \right)^2 \right] r dr = 0 \quad (3)$$

Integrating, we get from Eq. 3

$$-\pi R^2 U + 2\pi u_{\max} \left( \frac{r^2}{2} - \frac{r^4}{4R^2} \right)_0^R = 0$$

or

$$u_{\max} = 2U \quad (\text{Ans})$$

Since this flow is incompressible, we conclude from Eq. 5.8 that  $U$  is the average velocity at all sections of the control volume. Thus, the average velocity at section (2),  $\bar{V}_2$ , is one-half the maximum velocity,  $u_{\max}$ , there, or

$$\bar{V}_2 = \frac{u_{\max}}{2} \quad (\text{Ans})$$

## EXAMPLE 5.5

A bathtub is being filled with water from a faucet. The rate of flow from the faucet is steady at 9 gal/min. The tub volume is approximated by a rectangular space as indicated in Fig. E5.5. Estimate the time rate of change of the depth of water in the tub,  $\partial h / \partial t$ , in in./min at any instant.

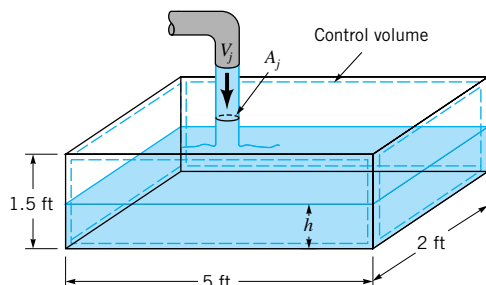


FIGURE E5.5

## SOLUTION

We will see later (Example 5.9) that this problem can also be solved with a deforming control volume that includes only the water in the tub at any instant. We presently use the fixed, nondeforming control volume outlined with a dashed line in Fig. E5.5. This control volume includes in it, at any instant, the water accumulated in the tub, some of the water flowing from the faucet into the tub, and some air. Application of Eqs. 5.4 and 5.5 to these contents of the control volume results in

$$\frac{\partial}{\partial t} \int_{\text{air volume}} \rho_{\text{air}} d\mathcal{V}_{\text{air}} + \frac{\partial}{\partial t} \int_{\text{water volume}} \rho_{\text{water}} d\mathcal{V}_{\text{water}} - \dot{m}_{\text{water}} + \dot{m}_{\text{air}} = 0$$

Note that the time rate of change of air mass and water mass are each not zero. Recognizing, however, that the air mass must be conserved, we know that the time rate of change of the mass of air in the control volume must be equal to the rate of air mass flow out of the control volume. For simplicity, we disregard any water evaporation that occurs. Thus, applying Eqs. 5.4 and 5.5 to the air only and to the water only, we obtain

$$\frac{\partial}{\partial t} \int_{\text{air volume}} \rho_{\text{air}} d\mathcal{V}_{\text{air}} + \dot{m}_{\text{air}} = 0$$

for air, and

$$\frac{\partial}{\partial t} \int_{\text{water volume}} \rho_{\text{water}} d\mathcal{V}_{\text{water}} = \dot{m}_{\text{water}} \quad (1)$$

for water. For the water,

$$\int_{\text{water volume}} \rho_{\text{water}} d\mathcal{V}_{\text{water}} = \rho_{\text{water}} [h(2 \text{ ft})(5 \text{ ft}) + (1.5 \text{ ft} - h)A_j] \quad (2)$$

where  $A_j$  is the cross-sectional area of the jet of water flowing from the faucet into the tub. Combining Eqs. 1 and 2, we obtain

$$\rho_{\text{water}} (10 \text{ ft}^2 - A_j) \frac{\partial h}{\partial t} = \dot{m}_{\text{water}}$$

and, thus

$$\frac{\partial h}{\partial t} = \frac{Q_{\text{water}}}{(10 \text{ ft}^2 - A_j)}$$

For  $A_j \ll 10 \text{ ft}^2$  we can conclude that

$$\frac{\partial h}{\partial t} = \frac{Q_{\text{water}}}{(10 \text{ ft}^2)} = \frac{(9 \text{ gal/min})(12 \text{ in./ft})}{(7.48 \text{ gal/ft}^3)(10 \text{ ft}^2)} = 1.44 \text{ in./min} \quad (\text{Ans})$$

The preceding example problems illustrate some important results of applying the conservation of mass principle to the contents of a fixed, nondeforming control volume. The dot product  $\mathbf{V} \cdot \hat{\mathbf{n}}$  is considered “+” for flow out of the control volume and “−” for flow into the control volume. Thus, mass flowrate out of the control volume is “+” and mass flowrate in is “−.” When the flow is steady, the time rate of change of the mass of the contents of the control volume

*The appropriate sign convention must be followed.*

$$\frac{\partial}{\partial t} \int_{\text{cv}} \rho dV$$

is zero and the net amount of mass flowrate,  $\dot{m}$ , through the control surface is therefore also zero

$$\sum \dot{m}_{\text{out}} - \sum \dot{m}_{\text{in}} = 0 \quad (5.9)$$

If the steady flow is also incompressible, the net amount of volume flowrate,  $Q$ , through the control surface is also zero:

$$\sum Q_{\text{out}} - \sum Q_{\text{in}} = 0 \quad (5.10)$$

An unsteady, but cyclical flow can be considered steady on a time-average basis. When the flow is unsteady, the instantaneous time rate of change of the mass of the contents of the control volume is not necessarily zero and can be an important variable. When the value of

$$\frac{\partial}{\partial t} \int_{\text{cv}} \rho dV$$

is “+,” the mass of the contents of the control volume is increasing. When it is “−,” the mass of the contents of the control volume is decreasing.

When the flow is uniformly distributed over the opening in the control surface (one-dimensional flow),

$$\dot{m} = \rho AV$$

where  $V$  is the uniform value of velocity component normal to the section area  $A$ . When the velocity is nonuniformly distributed over the opening in the control surface,

$$\dot{m} = \rho A \bar{V} \quad (5.11)$$

where  $\bar{V}$  is the average value of the component of velocity normal to the section area  $A$  as defined by Eq. 5.7.



#### V5.2 Shop vac filter

For steady flow involving only one stream of a specific fluid flowing through the control volume at sections (1) and (2),

$$\dot{m} = \rho_1 A_1 \bar{V}_1 = \rho_2 A_2 \bar{V}_2 \quad (5.12)$$

and for incompressible flow,

$$Q = A_1 \bar{V}_1 = A_2 \bar{V}_2 \quad (5.13)$$

For steady flow involving more than one stream of a specific fluid or more than one specific fluid flowing through the control volume,

$$\sum \dot{m}_{\text{in}} = \sum \dot{m}_{\text{out}}$$

The variety of example problems solved above should give the correct impression that the fixed, nondeforming control volume is versatile and useful.

### 5.1.3 Moving, Nondeforming Control Volume

It is sometimes necessary to use a nondeforming control volume attached to a moving reference frame. Examples include control volumes containing a gas turbine engine on an aircraft in flight, the exhaust stack of a ship at sea, and the gasoline tank of an automobile passing by.

*Some problems are most easily solved by using a moving control volume.*

As discussed in [Section 4.4.6](#), when a moving control volume is used, the fluid velocity relative to the moving control volume (relative velocity) is an important flow field variable. The relative velocity,  $\mathbf{W}$ , is the fluid velocity seen by an observer moving with the control volume. The control volume velocity,  $\mathbf{V}_{\text{cv}}$ , is the velocity of the control volume as seen from a fixed coordinate system. The absolute velocity,  $\mathbf{V}$ , is the fluid velocity seen by a stationary observer in a fixed coordinate system. These velocities are related to each other by the vector equation

$$\mathbf{V} = \mathbf{W} + \mathbf{V}_{\text{cv}} \quad (5.14)$$

which is the same as Eq. 4.22, introduced earlier.

For a system and a moving, nondeforming control volume that are coincident at an instant of time, the Reynolds transport theorem (Eq. 4.23) for a moving control volume leads to

$$\frac{DM_{\text{sys}}}{Dt} = \frac{\partial}{\partial t} \int_{\text{cv}} \rho \, d\mathcal{V} + \int_{\text{cs}} \rho \mathbf{W} \cdot \hat{\mathbf{n}} \, dA \quad (5.15)$$

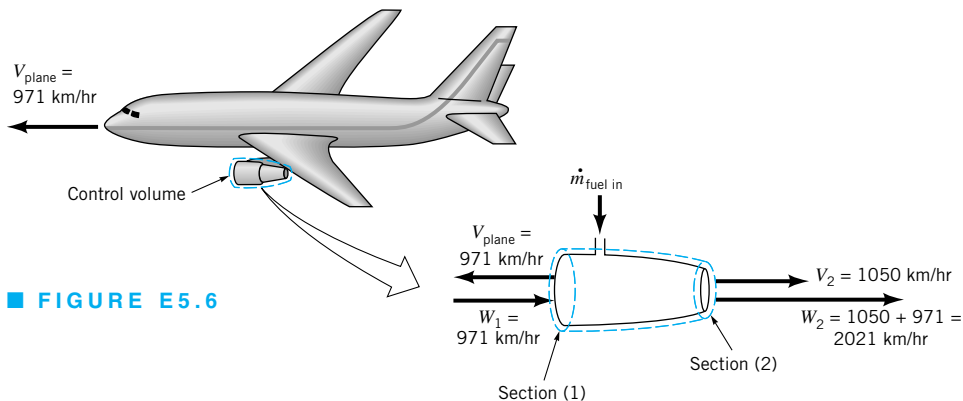
From Eqs. 5.1 and 5.15, we can get the control volume expression for conservation of mass (the continuity equation) for a moving, nondeforming control volume, namely,

$$\frac{\partial}{\partial t} \int_{\text{cv}} \rho \, d\mathcal{V} + \int_{\text{cs}} \rho \mathbf{W} \cdot \hat{\mathbf{n}} \, dA = 0 \quad (5.16)$$

Some examples of the application of Eq. 5.16 follow.

## EXAMPLE 5.6

An airplane moves forward at a speed of 971 km/hr as shown in Fig. E5.6. The frontal intake area of the jet engine is  $0.80 \, \text{m}^2$  and the entering air density is  $0.736 \, \text{kg/m}^3$ . A stationary observer determines that relative to the earth, the jet engine exhaust gases move away from the engine with a speed of 1050 km/hr. The engine exhaust area is  $0.558 \, \text{m}^2$ , and the exhaust gas density is  $0.515 \, \text{kg/m}^3$ . Estimate the mass flowrate of fuel into the engine in kg/hr.



■ FIGURE E5.6

## SOLUTION

The control volume, which moves with the airplane (see Fig. E5.6), surrounds the engine and its contents and includes all fluids involved at an instant. The application of Eq. 5.16 to these contents of the control volume yields

$$\frac{\partial}{\partial t} \int_{cv} \rho dV + \int_{cs} \rho \mathbf{W} \cdot \hat{\mathbf{n}} dA = 0 \quad (1)$$

0 (flow relative to moving control volume is considered steady on a time-average basis)

Assuming one-dimensional flow, we evaluate the surface integral in Eq. 1 and get

$$-\dot{m}_{\text{fuel in}} - \rho_1 A_1 W_1 + \rho_2 A_2 W_2 = 0$$

or

$$\dot{m}_{\text{fuel in}} = \rho_2 A_2 W_2 - \rho_1 A_1 W_1 \quad (2)$$

We consider the intake velocity,  $W_1$ , relative to the moving control volume, as being equal in magnitude to the speed of the airplane, 971 km/hr. The exhaust velocity,  $W_2$ , also needs to be measured relative to the moving control volume. Since a fixed observer noted that the exhaust gases were moving away from the engine at a speed of 1050 km/hr, the speed of the exhaust gases relative to the moving control volume,  $W_2$ , is determined as follows by using Eq. 5.14

$$V_2 = W_2 + V_{\text{plane}}$$

or

$$W_2 = V_2 - V_{\text{plane}} = 1050 \text{ km/hr} + 971 \text{ km/hr} = 2021 \text{ km/hr}$$

and is shown in Fig. E5.6b.

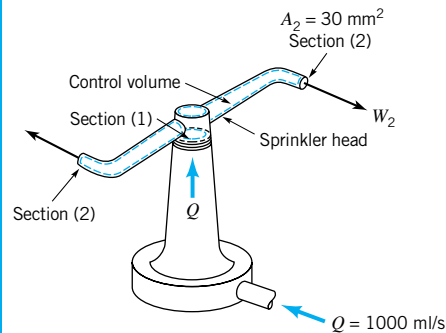
From Eq. 2,

$$\begin{aligned}
 \dot{m}_{\text{fuel in}} &= (0.515 \text{ kg/m}^3)(0.558 \text{ m}^2)(2021 \text{ km/hr})(1000 \text{ m/km}) \\
 &\quad - (0.736 \text{ kg/m}^3)(0.80 \text{ m}^2)(971 \text{ km/hr})(1000 \text{ m/km}) \\
 &= (580,800 - 571,700) \text{ kg/hr} \\
 \dot{m}_{\text{fuel in}} &= 9100 \text{ kg/hr} \quad (\text{Ans})
 \end{aligned}$$

Note that the fuel flowrate was obtained as the difference of two large, nearly equal numbers. Precise values of  $W_2$  and  $W_1$  are needed to obtain a modestly accurate value of  $\dot{m}_{\text{fuel in}}$ .

## EXAMPLE 5.7

Water enters a rotating lawn sprinkler through its base at the steady rate of 1000 ml/s as sketched in Fig. E5.7. If the exit area of each of the two nozzles is  $30 \text{ mm}^2$ , determine the average speed of the water leaving each nozzle, relative to the nozzle, if (a) the rotary sprinkler head is stationary, (b) the sprinkler head rotates at 600 rpm, and (c) the sprinkler head accelerates from 0 to 600 rpm.



■ FIGURE E5.7

## SOLUTION

We specify a control volume that contains the water in the rotary sprinkler head at any instant. This control volume is nondeforming, but it moves (rotates) with the sprinkler head. The application of Eq. 5.16 to the contents of this control volume for situation (a), (b), or (c) of the problem results in the same expression, namely,

$$\frac{\partial}{\partial t} \int_{\text{cv}} \rho \, dV + \int_{\text{cs}} \rho \mathbf{W} \cdot \hat{\mathbf{n}} \, dA = 0$$

0 flow is steady or the control volume is filled with an incompressible fluid

The flow is steady in the control volume reference frame when the control volume is stationary [part (a)] and when it moves [parts (b) and (c)]. Also, the control volume is filled with water. Thus, the time rate of change of the mass of the water in the control volume is zero. The control surface integral has a nonzero value only where water enters and leaves the control volume; thus,

$$\int_{cv} \rho \mathbf{W} \cdot \mathbf{n} dA = -\dot{m}_{in} + \dot{m}_{out} = 0$$

or

$$\dot{m}_{out} = \dot{m}_{in} \quad (1)$$

Since  $\dot{m}_{out} = 2\rho A_2 \bar{W}_2$  and  $\dot{m}_{in} = \rho Q$  it follows from Eq. 1 that

$$\bar{W}_2 = \frac{Q}{2A_2}$$

or

$$\bar{W}_2 = \frac{(1000 \text{ ml/s})(0.001 \text{ m}^3/\text{liter})(10^6 \text{ mm}^2/\text{m}^2)}{(1000 \text{ ml/liter})(2)(30 \text{ mm}^2)} = 16.7 \text{ m/s} = W_2 \quad (\text{Ans})$$

The value of  $W_2$  is independent of the speed of rotation of the sprinkler head and represents the average velocity of the water exiting from each nozzle with respect to the nozzle for cases (a), (b), (c). The velocity of water discharging from each nozzle, when viewed from a stationary reference (i.e.,  $V_2$ ), will vary as the rotation speed of the sprinkler head varies since from Eq. 5.14,

$$V_2 = W_2 - U$$

where  $U = \omega R$  is the speed of the nozzle and  $\omega$  and  $R$  are the angular velocity and radius of the sprinkler head, respectively.

When a moving, nondeforming control volume is used, the dot product sign convention used earlier for fixed, nondeforming control volume applications is still valid. Also, if the flow within the moving control volume is steady, or steady on a time-average basis, the time rate of change of the mass of the contents of the control volume is zero. Velocities seen from the control volume reference frame (relative velocities) must be used in the continuity equation. Relative and absolute velocities are related by a vector equation (Eq. 5.14), which also involves the control volume velocity.

### 5.1.4 Deforming Control Volume

Occasionally, a deforming control volume can simplify the solution of a problem. A deforming control volume involves changing volume size and control surface movement. Thus, the Reynolds transport theorem for a moving control volume can be used for this case, and Eqs. 4.23 and 5.1 lead to

$$\frac{DM_{sys}}{Dt} = \frac{\partial}{\partial t} \int_{cv} \rho dV + \int_{cs} \rho \mathbf{W} \cdot \hat{\mathbf{n}} dA = 0 \quad (5.17)$$

*Some problems are most easily solved by using a deforming control volume.*

The time rate of change term in Eq. 5.17,

$$\frac{\partial}{\partial t} \int_{cv} \rho dV$$

is usually nonzero and must be carefully evaluated because the extent of the control volume varies with time. The mass flowrate term in Eq. 5.17,

$$\int_{cs} \rho \mathbf{W} \cdot \hat{\mathbf{n}} dA$$

must be determined with the relative velocity,  $\mathbf{W}$ , the velocity referenced to the control surface. Since the control volume is deforming, the control surface velocity is not necessarily uniform and identical to the control volume velocity,  $\mathbf{V}_{cv}$ , as was true for moving, nondeforming control volumes. For the deforming control volume,

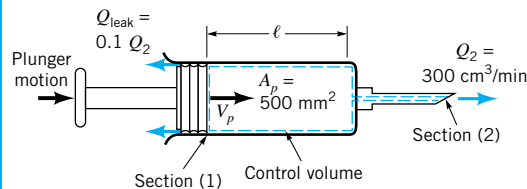
$$\mathbf{V} = \mathbf{W} + \mathbf{V}_{cs} \quad (5.18)$$

where  $\mathbf{V}_{cs}$  is the velocity of the control surface as seen by a fixed observer. The relative velocity,  $\mathbf{W}$ , must be ascertained with care wherever fluid crosses the control surface. Two example problems that illustrate the use of the continuity equation for a deforming control volume, Eq. 5.17, follow.

*The velocity of the surface of a deforming control volume is not the same at all points on the surface.*

## EXAMPLE 5.8

A syringe (Fig. E5.8) is used to inoculate a cow. The plunger has a face area of  $500 \text{ mm}^2$ . If the liquid in the syringe is to be injected steadily at a rate of  $300 \text{ cm}^3/\text{min}$ , at what speed should the plunger be advanced? The leakage rate past the plunger is 0.10 times the volume flowrate out of the needle.



■ FIGURE E5.8

## SOLUTION

The control volume selected for solving this problem is the deforming one illustrated in Fig. E5.8. Section (1) of the control surface moves with the plunger. The surface area of section (1),  $A_1$ , is considered equal to the circular area of the face of the plunger,  $A_p$ , although this is not strictly true, since leakage occurs. The difference is small, however. Thus,

$$A_1 = A_p \quad (1)$$

Liquid also leaves the needle through section (2), which involves fixed area  $A_2$ . The application of Eq. 5.17 to the contents of this control volume gives

$$\frac{\partial}{\partial t} \int_{cv} \rho dV + \dot{m}_2 + \rho Q_{\text{leak}} = 0 \quad (2)$$

Even though  $Q_{\text{leak}}$  and the flow through section area  $A_2$  are steady, the time rate of change



of the mass of liquid in the shrinking control volume is not zero because the control volume is getting smaller. To evaluate the first term of Eq. 2, we note that

$$\int_{cv} \rho dV = \rho(\ell A_1 + V_{\text{needle}}) \quad (3)$$

where  $\ell$  is the changing length of the control volume (see Fig. E5.8) and  $V_{\text{needle}}$  is the volume of the needle. From Eq. 3, we obtain

$$\frac{\partial}{\partial t} \int_{cv} \rho dV = \rho A_1 \frac{\partial \ell}{\partial t} \quad (4)$$

Note that

$$-\frac{\partial \ell}{\partial t} = V_p \quad (5)$$

where  $V_p$  is the speed of the plunger sought in the problem statement. Combining Eqs. 2, 4, and 5 we obtain

$$-\rho A_1 V_p + \dot{m}_2 + \rho Q_{\text{leak}} = 0 \quad (6)$$

However, from Eq. 5.6, we see that

$$\dot{m}_2 = \rho Q_2 \quad (7)$$

and Eq. 6 becomes

$$-\rho A_1 V_p + \rho Q_2 + \rho Q_{\text{leak}} = 0 \quad (8)$$

Solving Eq. 8 for  $V_p$  yields

$$V_p = \frac{Q_2 + Q_{\text{leak}}}{A_1} \quad (9)$$

Since  $Q_{\text{leak}} = 0.1Q_2$ , Eq. 9 becomes

$$V_p = \frac{Q_2 + 0.1Q_2}{A_1} = \frac{1.1Q_2}{A_1}$$

and

$$V_p = \frac{(1.1)(300 \text{ cm}^3/\text{min})}{(500 \text{ mm}^2)} \left( \frac{1000 \text{ mm}^3}{\text{cm}^3} \right) = 660 \text{ mm/min} \quad (\text{Ans})$$

## EXAMPLE 5.9

Solve the problem of **Example 5.5** using a deforming control volume that includes only the water accumulating in the bathtub.

## SOLUTION

For this deforming control volume, Eq. 5.17 leads to

$$\frac{\partial}{\partial t} \int_{\text{water volume}} \rho dV + \int_{cs} \rho \mathbf{W} \cdot \hat{\mathbf{n}} dA = 0 \quad (1)$$

The first term of Eq. 1 can be evaluated as

$$\begin{aligned}\frac{\partial}{\partial t} \int_{\text{water volume}} \rho dV &= \frac{\partial}{\partial t} [\rho h(2 \text{ ft})(5 \text{ ft})] \\ &= \rho (10 \text{ ft}^2) \frac{\partial h}{\partial t}\end{aligned}\quad (2)$$

The second term of Eq. 1 can be evaluated as

$$\int_{cs} \rho \mathbf{W} \cdot \hat{\mathbf{n}} dA = -\rho \left( V_j + \frac{\partial h}{\partial t} \right) A_j \quad (3)$$

where  $A_j$  and  $V_j$  are the cross-sectional area and velocity of the water flowing from the faucet into the tube. Thus, from Eqs. 1, 2, and 3 we obtain

$$\frac{\partial h}{\partial t} = \frac{V_j A_j}{(10 \text{ ft}^2 - A_j)} = \frac{Q_{\text{water}}}{(10 \text{ ft}^2 - A_j)}$$

or for  $A_j \ll 10 \text{ ft}^2$

$$\frac{\partial h}{\partial t} = \frac{9(\text{gal/min})(12 \text{ in./ft})}{(7.48 \text{ gal/ft}^3)(10 \text{ ft}^2)} = 1.44 \text{ in./min} \quad (\text{Ans})$$

Note that these results using a deforming control volume are the same as that obtained in Example 5.5 with a fixed control volume.

The conservation of mass principle is easily applied to the contents of a control volume. The appropriate selection of a specific kind of control volume (for example, fixed and non-deforming, moving and nondeforming, or deforming) can make the solution of a particular problem less complicated. In general, where fluid flows through the control surface, it is advisable to make the control surface perpendicular to the flow. In the sections ahead we learn that the conservation of mass principle is primarily used in combination with other important laws to solve problems.

## 5.2 Newton's Second Law—The Linear Momentum and Moment-of-Momentum Equations



### V5.3 Smokestack plume momentum

*Newton's second law deals with system momentum and forces.*

### 5.2.1 Derivation of the Linear Momentum Equation

Newton's second law of motion for a system is

$$\begin{array}{ll}\text{time rate of change of the} & = \text{sum of external forces} \\ \text{linear momentum of the system} & \text{acting on the system}\end{array}$$

Since momentum is mass times velocity, the momentum of a small particle of mass  $\rho dV$  is  $\mathbf{V}\rho dV$ . Thus, the momentum of the entire system is  $\int_{\text{sys}} \mathbf{V}\rho dV$  and Newton's law becomes

$$\frac{D}{Dt} \int_{\text{sys}} \mathbf{V}\rho dV = \sum \mathbf{F}_{\text{sys}} \quad (5.19)$$

Any reference or coordinate system for which this statement is true is called *inertial*. A fixed coordinate system is inertial. A coordinate system that moves in a straight line with constant velocity and is thus without acceleration is also inertial. We proceed to develop the control volume formula for this important law. When a control volume is coincident with a system at an instant of time, the forces acting on the system and the forces acting on the contents of the coincident control volume (see Fig. 5.2) are instantaneously identical, that is,

$$\sum \mathbf{F}_{\text{sys}} = \sum \mathbf{F}_{\text{contents of the coincident control volume}} \quad (5.20)$$

Furthermore, for a system and the contents of a coincident control volume that is fixed and nondeforming, the Reynolds transport theorem (Eq. 4.19 with  $b$  set equal to the velocity, and  $B_{\text{sys}}$  being the system momentum) allows us to conclude that

$$\frac{D}{Dt} \int_{\text{sys}} \mathbf{V} \rho \, d\mathcal{V} = \frac{\partial}{\partial t} \int_{\text{cv}} \mathbf{V} \rho \, d\mathcal{V} + \int_{\text{cs}} \mathbf{V} \rho \mathbf{V} \cdot \hat{\mathbf{n}} \, dA \quad (5.21)$$

or

time rate of change of the linear momentum of the system	=	time rate of change of the linear momentum of the contents of the control volume	+	net rate of flow of linear momentum through the control surface
---	---	--	---	--

Equation 5.21 states that the time rate of change of system linear momentum is expressed as the sum of the two control volume quantities: the time rate of change of the *linear momentum of the contents of the control volume*, and the net rate of *linear momentum flow through the control surface*. As particles of mass move into or out of a control volume through the control surface, they carry linear momentum in or out. Thus, linear momentum flow should seem no more unusual than mass flow.

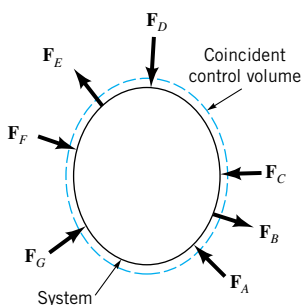
For a control volume that is fixed (and thus inertial) and nondeforming, Eqs. 5.19, 5.20, and 5.21 suggest that an appropriate mathematical statement of Newton's second law of motion is

*The linear momentum equation is a statement of Newton's second law.*

$$\frac{\partial}{\partial t} \int_{\text{cv}} \mathbf{V} \rho \, d\mathcal{V} + \int_{\text{cs}} \mathbf{V} \rho \mathbf{V} \cdot \hat{\mathbf{n}} \, dA = \sum \mathbf{F}_{\text{contents of the control volume}} \quad (5.22)$$

We call Eq. 5.22 the *linear momentum equation*.

In our application of the linear momentum equation, we initially confine ourselves to fixed, nondeforming control volumes for simplicity. Subsequently, we discuss the use of a moving but inertial, nondeforming control volume. We do not consider deforming control volumes and accelerating (noninertial) control volumes. If a control volume is noninertial,



■ **FIGURE 5.2** External forces acting on system and coincident control volume.

*Both surface and body forces act on the contents of the control volume.*

the acceleration components involved (for example, translation acceleration, Coriolis acceleration, and centrifugal acceleration) require consideration.

The forces involved in Eq. 5.22 are body and surface forces that act on what is contained in the control volume. The only body force we consider in this chapter is the one associated with the action of gravity. We experience this body force as weight. The surface forces are basically exerted on the contents of the control volume by material just outside the control volume in contact with material just inside the control volume. For example, a wall in contact with fluid can exert a reaction surface force on the fluid it bounds. Similarly, fluid just outside the control volume can push on fluid just inside the control volume at a common interface, usually an opening in the control surface through which fluid flow occurs. An immersed object can resist fluid motion with surface forces.

The linear momentum terms in the momentum equation deserve careful explanation. We clarify their physical significance in the following sections.

### 5.2.2 Application of the Linear Momentum Equation

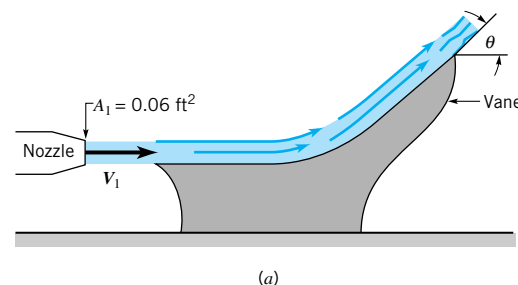
The linear momentum equation for an inertial control volume is a vector equation (Eq. 5.22). In engineering applications, components of this vector equation resolved along orthogonal coordinates, for example,  $x$ ,  $y$ , and  $z$  (rectangular coordinate system) or  $r$ ,  $\theta$ , and  $x$  (cylindrical coordinate system), will normally be used. A simple example involving steady, incompressible flow is considered first.

## EXAMPLE 5.10

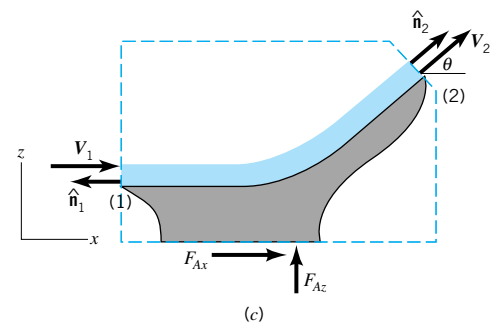
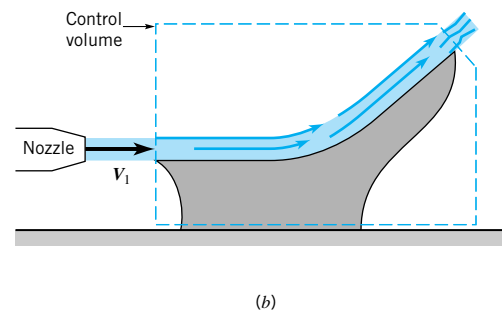


**V5.4 Force due to a water jet**

As shown in Fig. E5.10a, a horizontal jet of water exits a nozzle with a uniform speed of  $V_1 = 10$  ft/s, strikes a vane, and is turned through an angle  $\theta$ . Determine the anchoring force needed to hold the vane stationary. Neglect gravity and viscous effects.



■ FIGURE E5.10



## SOLUTION

We select a control volume that includes the vane and a portion of the water (see Figs. E5.10b,c) and apply the linear momentum equation to this fixed control volume. The  $x$  and  $z$  components of Eq. 5.22 become

$$\frac{\partial}{\partial t} \int_{cv} u \rho dV + \int_{cs} u \rho \mathbf{V} \cdot \hat{\mathbf{n}} dA = \sum F_x \quad (1)$$

0 (flow is steady)

and

$$\frac{\partial}{\partial t} \int_{cv} w \rho dV + \int_{cs} w \rho \mathbf{V} \cdot \hat{\mathbf{n}} dA = \sum F_z \quad (2)$$

0 (flow is steady)

where  $\mathbf{V} = u \hat{\mathbf{i}} + w \hat{\mathbf{k}}$ , and  $\sum F_x$  and  $\sum F_z$  are the net  $x$  and  $z$  components of force acting on the contents of the control volume.

The water enters and leaves the control volume as a free jet at atmospheric pressure. Hence, there is atmospheric pressure surrounding the entire control volume, and the net pressure force on the control volume surface is zero. If we neglect the weight of the water and vane, the only forces applied to the control volume contents are the horizontal and vertical components of the anchoring force,  $F_{Ax}$  and  $F_{Az}$ , respectively.

The only portions of the control surface across which fluid flows are section (1) (the entrance) where  $\mathbf{V} \cdot \hat{\mathbf{n}} = -V_1$  and section (2) (the exit) where  $\mathbf{V} \cdot \hat{\mathbf{n}} = +V_2$ . (Recall that the unit normal vector is directed out from the control surface.) Also, with negligible gravity and viscous effects, and since  $p_1 = p_2$ , the speed of the fluid remains constant, so that  $V_1 = V_2 = 10 \text{ ft/s}$  (see the Bernoulli equation, Eq. 3.6). Hence, at section (1),  $u = V_1$ ,  $w = 0$ , and at section (2),  $u = V_1 \cos \theta$ ,  $w = V_1 \sin \theta$ .

By using the above information, Eqs. 1 and 2 can be written as

$$V_1 \rho (-V_1) A_1 + V_1 \cos \theta \rho (V_1) A_2 = F_{Ax} \quad (3)$$

and

$$(0) \rho (-V_1) A_1 + V_1 \sin \theta \rho (V_1) A_2 = F_{Az} \quad (4)$$

Note that since the flow is uniform across the inlet and exit, the integrals simply reduce to multiplications. Equations 3 and 4 can be simplified by using conservation of mass which states that for this incompressible flow  $A_1 V_1 = A_2 V_2$ , or  $A_1 = A_2$  since  $V_1 = V_2$ . Thus

$$F_{Ax} = -\rho A_1 V_1^2 + \rho A_1 V_1^2 \cos \theta = -\rho A_1 V_1^2 (1 - \cos \theta) \quad (5)$$

and

$$F_{Az} = \rho A_1 V_1^2 \sin \theta \quad (6)$$

With the given data, we obtain

$$\begin{aligned} F_{Ax} &= -(1.94 \text{ slugs/ft}^3)(0.06 \text{ ft}^2)(10 \text{ ft/s})^2 (1 - \cos \theta) \\ &= -11.64(1 - \cos \theta) \text{ slugs} \cdot \text{ft/s}^2 = -11.64(1 - \cos \theta) \text{ lb} \end{aligned} \quad (\text{Ans})$$

and

$$\begin{aligned} F_{Az} &= (1.94 \text{ slugs/ft}^3)(0.06 \text{ ft}^2)(10 \text{ ft/s})^2 \sin \theta \\ &= 11.64 \sin \theta \text{ lb} \end{aligned} \quad (\text{Ans})$$

Note that if  $\theta = 0$  (i.e., the vane does not turn the water), the anchoring force is zero. The inviscid fluid merely slides along the vane without putting any force on it. If  $\theta = 90^\circ$ , then  $F_{Ax} = -11.64 \text{ lb}$  and  $F_{Az} = 11.64 \text{ lb}$ . It is necessary to push on the vane (and, hence, for the vane to push on the water) to the left ( $F_{Ax}$  is negative) and up in order to change the direction of flow of the water from horizontal to vertical. A momentum change requires a force. If  $\theta = 180^\circ$ , the water jet is turned back on itself. This requires no vertical force

( $F_{Az} = 0$ ), but the horizontal force ( $F_{Ax} = -23.3 \text{ lb}$ ) is two times that required if  $\theta = 90^\circ$ . This force must eliminate the incoming fluid momentum and create the outgoing momentum.

Note that the anchoring force (Eqs. 5, 6) can be written in terms of the mass flowrate,  $\dot{m} = \rho A_1 V_1$ , as

$$F_{Ax} = -\dot{m} V_1 (1 - \cos \theta)$$

and

$$F_{Az} = \dot{m} V_1 \sin \theta$$

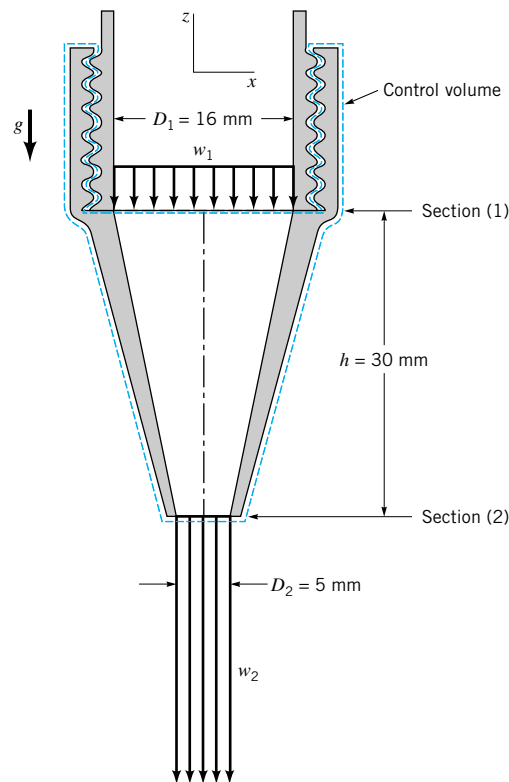
In this example the anchoring force is needed to produce the nonzero net momentum flowrate (mass flowrate times the change in  $x$  or  $z$  component of velocity) across the control surface.

## EXAMPLE 5.11

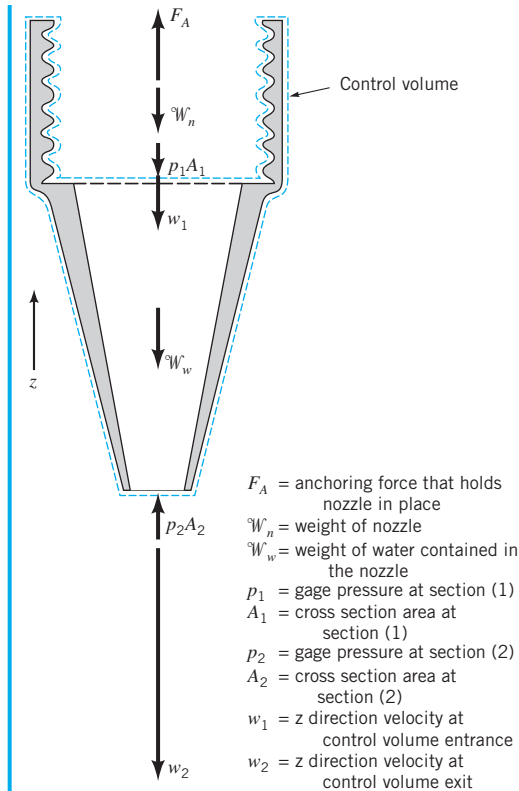
Determine the anchoring force required to hold in place a conical nozzle attached to the end of a laboratory sink faucet (see Fig. E5.11a) when the water flowrate is 0.6 liter/s. The nozzle mass is 0.1 kg. The nozzle inlet and exit diameters are 16 mm and 5 mm, respectively. The nozzle axis is vertical and the axial distance between sections (1) and (2) is 30 mm. The pressure at section (1) is 464 kPa.

## SOLUTION

The anchoring force sought is the reaction force between the faucet and nozzle threads. To evaluate this force we select a control volume that includes the entire nozzle and the water contained in the nozzle at an instant, as is indicated in Figs. E5.11a and E5.11b. All of the



■ FIGURE E5.11a



■ FIGURE E5.11b

vertical forces acting on the contents of this control volume are identified in Fig. E5.11b. The action of atmospheric pressure cancels out in every direction and is not shown. Gage pressure forces do not cancel out in the vertical direction and are shown. Application of the vertical or  $z$  direction component of Eq. 5.22 to the contents of this control volume leads to

$$\frac{\partial}{\partial t} \int_{cv} w \rho dV + \int_{cs} w \rho \mathbf{V} \cdot \hat{\mathbf{n}} dA = F_A - \mathcal{W}_n - p_1 A_1 - \mathcal{W}_w + p_2 A_2 \quad (1)$$

0 (flow is steady)

where  $w$  is the  $z$  direction component of fluid velocity, and the various parameters are identified in the figure.

Note that the positive direction is considered “up” for the forces. We will use this same sign convention for the fluid velocity,  $w$ , in Eq. 1. In Eq. 1, the dot product,  $\mathbf{V} \cdot \hat{\mathbf{n}}$ , is “+” for flow out of the control volume and “−” for flow into the control volume. For this particular example

$$\mathbf{V} \cdot \hat{\mathbf{n}} dA = \pm |w| dA \quad (2)$$

with the “+” used for flow out of the control volume and “−” used for flow in. To evaluate the control surface integral in Eq. 1, we need to assume a distribution for fluid velocity,  $w$ , and fluid density,  $\rho$ . For simplicity, we assume that  $w$  is uniformly distributed or constant, with magnitudes of  $w_1$  and  $w_2$  over cross-sectional areas  $A_1$  and  $A_2$ . Also, this flow is incompressible so the fluid density,  $\rho$ , is constant throughout. Proceeding further we obtain for Eq. 1

$$\begin{aligned}
 & (-\dot{m}_1)(-w_1) + \dot{m}_2(-w_2) \\
 & = F_A - \mathcal{W}_n - p_1A_1 - \mathcal{W}_w + p_2A_2
 \end{aligned} \quad (3)$$

where  $\dot{m} = \rho AV$  is the mass flowrate.

Note that  $-w_1$  and  $-w_2$  are used because both of these velocities are “down”. Also,  $-\dot{m}_1$  is used because it is associated with flow the control volume. Similarly,  $+\dot{m}_2$  is used because it is associated with flow out of the control volume. Solving Eq. 3 for the anchoring force,  $F_A$ , we obtain

$$F_A = \dot{m}_1w_1 - \dot{m}_2w_2 + \mathcal{W}_n + p_1A_1 + \mathcal{W}_w - p_2A_2 \quad (4)$$

From the conservation of mass equation, Eq. 5.12, we obtain

$$\dot{m}_1 = \dot{m}_2 = \dot{m} \quad (5)$$

which when combined with Eq. 4 gives

$$F_A = \dot{m}(w_1 - w_2) + \mathcal{W}_n + p_1A_1 + \mathcal{W}_w - p_2A_2 \quad (6)$$

It is instructive to note how the anchoring force is affected by the different actions involved. As expected, the nozzle weight,  $\mathcal{W}_n$ , the water weight,  $\mathcal{W}_w$ , and gage pressure force at section (1),  $p_1A_1$ , all increase the anchoring force, while the gage pressure force at section (2),  $p_2A_2$ , acts to decrease the anchoring force. The change in the vertical momentum flowrate,  $\dot{m}(w_1 - w_2)$ , will, in this instance, decrease the anchoring force because this change is negative ( $w_2 > w_1$ ).

To complete this example we use quantities given in the problem statement to quantify the terms on the right-hand side of Eq. 6.

From Eq. 5.6,

$$\dot{m} = \rho w_1A_1 = \rho Q = (999 \text{ kg/m}^3)(0.6 \text{ liter/s})(10^{-3} \text{ m}^3/\text{liter}) = 0.599 \text{ kg/s} \quad (7)$$

and

$$w_1 = \frac{Q}{A_1} = \frac{Q}{\pi(D_1^2/4)} = \frac{(0.6 \text{ liter/s})(10^{-3} \text{ m}^3/\text{liter})}{\pi(16 \text{ mm})^2/4(1000^2 \text{ mm}^2/\text{m}^2)} = 2.98 \text{ m/s} \quad (8)$$

Also from Eq. 5.6,

$$w_2 = \frac{Q}{A_2} = \frac{Q}{\pi(D_2^2/4)} = \frac{(0.6 \text{ liter/s})(10^{-3} \text{ m}^3/\text{liter})}{\pi(5 \text{ mm})^2/4(1000^2 \text{ mm}^2/\text{m}^2)} = 30.6 \text{ m/s} \quad (9)$$

The weight of the nozzle,  $\mathcal{W}_n$ , can be obtained from the nozzle mass,  $m_n$ , with

$$\mathcal{W}_n = m_ng = (0.1 \text{ kg})(9.81 \text{ m/s}^2) = 0.981 \text{ N} \quad (10)$$

The weight of the water in the control volume,  $\mathcal{W}_w$ , can be obtained from the water density,  $\rho$ , and the volume of water,  $\mathcal{V}_w$ , in the truncated cone of height  $h$ . That is,

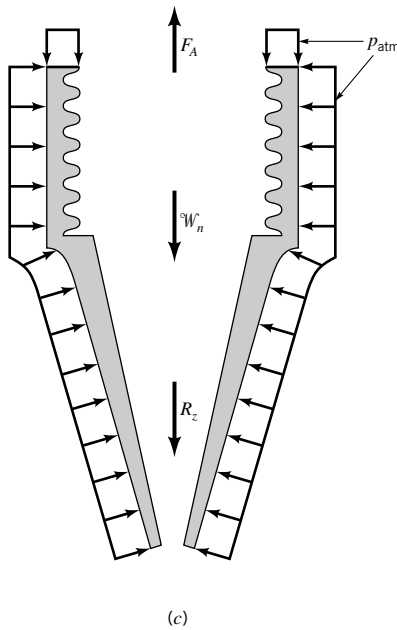
$$\mathcal{W}_w = \rho \mathcal{V}_w g = \rho \frac{1}{12} \pi h (D_1^2 + D_2^2 + D_1 D_2) g$$

Thus,

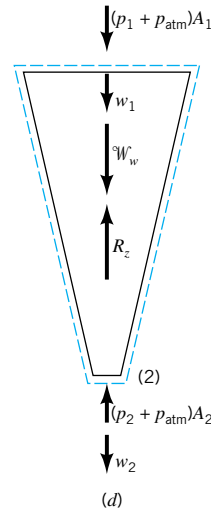
$$\begin{aligned}
 \mathcal{W}_w &= (999 \text{ kg/m}^3) \frac{1}{12} \pi \frac{(30 \text{ mm})}{(1000 \text{ mm/m})} \\
 &\times \left[ \frac{(16 \text{ mm})^2 + (5 \text{ mm})^2 + (16 \text{ mm})(5 \text{ mm})}{(1000^2 \text{ mm}^2/\text{m}^2)} \right] (9.81 \text{ m/s}^2) = 0.0278 \text{ N}
 \end{aligned} \quad (11)$$

The gage pressure at section (2),  $p_2$ , is zero since, as discussed in Section 3.6.1, when a subsonic flow discharges to the atmosphere as in the present situation, the discharge pressure is





■ FIGURE E5.11c



■ FIGURE E5.11d

essentially atmospheric. The anchoring force,  $F_A$ , can now be determined from Eqs. 6 through 11 with

$$\begin{aligned} F_A &= (0.599 \text{ kg/s})(2.98 \text{ m/s} - 30.6 \text{ m/s}) + 0.981 \text{ N} \\ &\quad + (464 \text{ kPa})(1000 \text{ Pa/kPa}) \frac{\pi(16 \text{ mm})^2}{4(1000^2 \text{ mm}^2/\text{m}^2)} \\ &\quad + 0.0278 \text{ N} - 0 \end{aligned}$$

or

$$F_A = -16.5 \text{ N} + 0.981 \text{ N} + 93.3 \text{ N} + 0.0278 \text{ N} = 77.8 \text{ N} \quad (\text{Ans})$$

Since the anchoring force,  $F_A$ , is positive, it acts upward in the  $z$  direction. The nozzle would be pushed off the pipe if it were not fastened securely.

The control volume selected above to solve problems such as these is not unique. The following is an alternate solution that involves two other control volumes—one containing only the nozzle and the other containing only the water in the nozzle. These control volumes are shown in Figs. E5.11c and E5.11d along with the vertical forces acting on the contents of each control volume. The new force involved,  $R_z$ , represents the interaction between the water and the conical inside surface of the nozzle. It includes the net pressure and viscous forces at this interface.

Application of Eq. 5.22 to the contents of the control volume of Fig. E5.11c leads to

$$F_A = \dot{W}_n + R_z - p_{\text{atm}}(A_1 - A_2) \quad (12)$$

The term  $p_{\text{atm}}(A_1 - A_2)$  is the resultant force from the atmospheric pressure acting upon the exterior surface of the nozzle (i.e., that portion of the surface of the nozzle that is not in contact with the water). Recall that the pressure force on a curved surface (such as the exterior surface of the nozzle) is equal to the pressure times the projection of the surface area on a plane perpendicular to the axis of the nozzle. The projection of this area on a plane perpendicular to the  $z$  direction is  $A_1 - A_2$ . The effect of the atmospheric pressure on the

internal area (between the nozzle and the water) is already included in  $R_z$  which represents the net force on this area.

Similarly, for the control volume of Fig. E5.11d we obtain

$$R_z = \dot{m}(w_1 - w_2) + \mathcal{W}_w + (p_1 + p_{\text{atm}})A_1 - (p_2 - p_{\text{atm}})A_2 \quad (13)$$

where  $p_1$  and  $p_2$  are gage pressures. From Eq. 13 it is clear that the value of  $R_z$  depends on the value of the atmospheric pressure,  $p_{\text{atm}}$ , since  $A_1 \neq A_2$ . That is, we must use absolute pressure, not gage pressure, to obtain the correct value of  $R_z$ .

By combining Eqs. 12 and 13 we obtain the same result for  $F_A$  as before (Eq. 6):

$$F_A = \dot{m}(w_1 - w_2) + \mathcal{W}_n + p_1A_1 + W_w - p_2A_2$$

Note that although the force between the fluid and the nozzle wall,  $R_z$ , is a function of  $p_{\text{atm}}$ , the anchoring force,  $F_A$ , is not. That is, we were correct in using gage pressure when solving for  $F_A$  by means of the original control volume shown in Fig. E5.11b.

Several important generalities about the application of the linear momentum equation (Eq. 5.22) are apparent in the example just considered.

1. When the flow is uniformly distributed over a section of the control surface where flow into or out of the control volume occurs, the integral operations are simplified. Thus, one-dimensional flows are easier to work with than flows involving nonuniform velocity distributions.
2. Linear momentum is directional; it can have components in as many as three orthogonal coordinate directions. Furthermore, along any one coordinate, the linear momentum of a fluid particle can be in the positive or negative direction and thus be considered as a positive or a negative quantity. In Example 5.11, only the linear momentum in the  $z$  direction was considered (all of it was in the negative  $z$  direction and was hence treated as being negative).
3. The flow of positive or negative linear momentum *into* a control volume involves a negative  $\mathbf{V} \cdot \hat{\mathbf{n}}$  product. Momentum flow *out* of the control volume involves a positive  $\mathbf{V} \cdot \hat{\mathbf{n}}$  product. The correct algebraic sign (+ or -) to assign to momentum flow ( $\mathbf{V} \rho \mathbf{V} \cdot \hat{\mathbf{n}} dA$ ) will depend on the sense of the velocity (+ in positive coordinate direction, - in negative coordinate direction) and the  $\mathbf{V} \cdot \hat{\mathbf{n}}$  product (+ for flow out of the control volume, - for flow into the control volume). In Example 5.11, the momentum flow into the control volume past section (1) was a positive (+) quantity while the momentum flow out of the control volume at section (2) was a negative (-) quantity.
4. The time rate of change of the linear momentum of the contents of a nondeforming control volume (i.e.,  $\partial/\partial t \int_{\text{cv}} \mathbf{V} \rho d\mathcal{V}$ ) is zero for steady flow. The momentum problems considered in this text all involve steady flow.
5. If the control surface is selected so that it is perpendicular to the flow where fluid enters or leaves the control volume, the surface force exerted at these locations by fluid outside the control volume on fluid inside will be due to pressure. Furthermore, when subsonic flow exits from a control volume into the atmosphere, atmospheric pressure prevails at the exit cross section. In Example 5.11, the flow was subsonic and so we set the exit flow pressure at the atmospheric level. The continuity equation (Eq. 5.12) allowed us to evaluate the fluid flow velocities  $w_1$  and  $w_2$  at sections (1) and (2).
6. The forces due to atmospheric pressure acting on the control surface may need consideration as indicated by Eq. 13 for the reaction force between the nozzle and the

*Linear momentum is a vector quantity.*

*A control volume diagram is similar to a free body diagram.*

fluid. When calculating the anchoring force,  $F_A$ , the forces due to atmospheric pressure on the control surface cancel each other (for example, after combining Eqs. 12 and 13 the atmospheric pressure forces are no longer involved) and gage pressures may be used.

7. The external forces have an algebraic sign, positive if the force is in the assigned positive coordinate direction and negative otherwise.
8. Only external forces acting on the contents of the control volume are considered in the linear momentum equation (Eq. 5.22). If the fluid alone is included in a control volume, reaction forces between the fluid and the surface or surfaces in contact with the fluid [wetted surface(s)] will need to be in Eq. 5.22. If the fluid and the wetted surface or surfaces are within the control volume, the reaction forces between fluid and wetted surface(s) do not appear in the linear momentum equation (Eq. 5.22) because they are internal, not external forces. The anchoring force that holds the wetted surface(s) in place is an external force, however, and must therefore be in Eq. 5.22.
9. The force required to anchor an object will generally exist in response to surface pressure and/or shear forces acting on the control surface, to a change in linear momentum flow through the control volume containing the object, and to the weight of the object and the fluid contained in the control volume. In Example 5.11 the nozzle anchoring force was required mainly because of pressure forces and partly because of a change in linear momentum flow associated with accelerating the fluid in the nozzle. The weight of the water and the nozzle contained in the control volume influenced the size of the anchoring force only slightly.

To further demonstrate the use of the linear momentum equation (Eq. 5.22), we consider another one-dimensional flow example before moving on to other facets of this important equation.

## EXAMPLE 5.12

Water flows through a horizontal,  $180^\circ$  pipe bend as illustrated in Fig. E5.12a. The flow cross-sectional area is constant at a value of  $0.1 \text{ ft}^2$  through the bend. The flow velocity everywhere in the bend is axial and  $50 \text{ ft/s}$ . The absolute pressures at the entrance and exit of the bend are  $30 \text{ psia}$  and  $24 \text{ psia}$ , respectively. Calculate the horizontal ( $x$  and  $y$ ) components of the anchoring force required to hold the bend in place.

## SOLUTION

Since we want to evaluate components of the anchoring force to hold the pipe bend in place, an appropriate control volume (see dashed line in Fig. E5.12a) contains the bend and the water in the bend at an instant. The horizontal forces acting on the contents of this control volume are identified in Fig. E5.12b. Note that the weight of the water is vertical (in the negative  $z$  direction) and does not contribute to the  $x$  and  $y$  components of the anchoring force. All of the horizontal normal and tangential forces exerted on the fluid and the pipe bend are resolved and combined into the two resultant components,  $F_{Ax}$  and  $F_{Ay}$ . These two forces act on the control volume contents, and thus for the  $x$  direction, Eq. 5.22 leads to

$$\int_{cs} u \rho \mathbf{V} \cdot \hat{\mathbf{n}} dA = F_{Ax} \quad (1)$$

At sections (1) and (2), the flow is in the  $y$  direction and therefore  $u = 0$  at both cross sections. There is no  $x$  direction momentum flow into or out of the control volume and we conclude from Eq. 1 that

$$F_{Ax} = 0. \quad (\text{Ans})$$

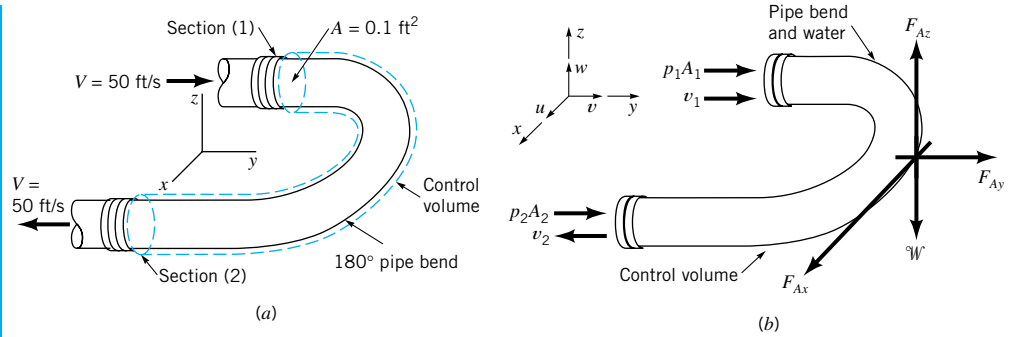


FIGURE E5.12

For the  $y$  direction, we get from Eq. 5.22

$$\int_{cs} v \rho \mathbf{V} \cdot \hat{\mathbf{n}} dA = F_{Ay} + p_1 A_1 + p_2 A_2 \quad (2)$$

For one-dimensional flow, the surface integral in Eq. 2 is easy to evaluate and Eq. 2 becomes

$$(+v_1)(-\dot{m}_1) + (-v_2)(+\dot{m}_2) = F_{Ay} + p_1 A_1 + p_2 A_2 \quad (3)$$

Note that the  $y$  component of velocity is positive at section (1) but is negative at section (2). Also, the mass flowrate term is negative at section (1) (flow in) and is positive at section (2) (flow out). From the continuity equation (Eq. 5.12), we get

$$\dot{m} = \dot{m}_1 = \dot{m}_2 \quad (4)$$

and thus Eq. 3 can be written as

$$-\dot{m}(v_1 + v_2) = F_{Ay} + p_1 A_1 + p_2 A_2 \quad (5)$$

Solving Eq. 5 for  $F_{Ay}$  we obtain

$$F_{Ay} = -\dot{m}(v_1 + v_2) - p_1 A_1 - p_2 A_2 \quad (6)$$

From the given data we can calculate  $\dot{m}$  from Eq. 5.6 as

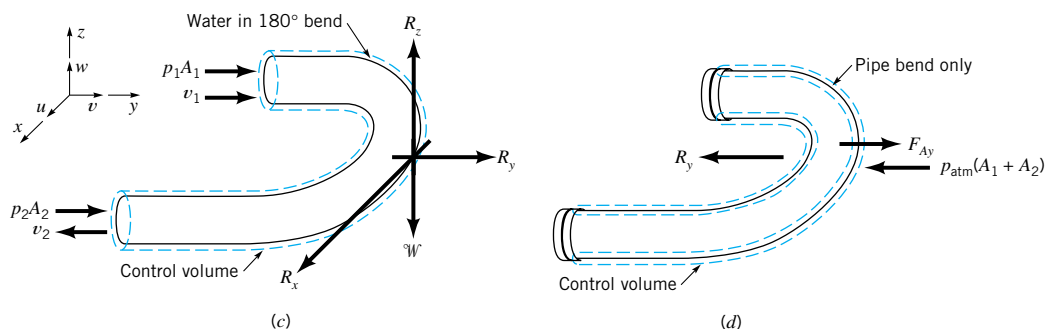
$$\dot{m} = \rho_1 A_1 v_1 = (1.94 \text{ slugs/ft}^3)(0.1 \text{ ft}^2)(50 \text{ ft/s}) = 9.70 \text{ slugs/s}$$

For determining the anchoring force,  $F_{Ay}$ , the effects of atmospheric pressure cancel and thus gage pressures for  $p_1$  and  $p_2$  are appropriate. By substituting numerical values of variables into Eq. 6, we get

$$\begin{aligned} F_{Ay} &= -(9.70 \text{ slugs/s})(50 \text{ ft/s} + 50 \text{ ft/s})\{1 \text{ lb}/[\text{slug} \cdot (\text{ft/s}^2)]\} \\ &\quad - (30 \text{ psia} - 14.7 \text{ psia})(144 \text{ in.}^2/\text{ft}^2)(0.1 \text{ ft}^2) \\ &\quad - (24 \text{ psia} - 14.7 \text{ psia})(144 \text{ in.}^2/\text{ft}^2)(0.1 \text{ ft}^2) \\ F_{Ay} &= -970 \text{ lb} - 220 \text{ lb} - 134 \text{ lb} = -1324 \text{ lb} \end{aligned} \quad (\text{Ans})$$

The negative sign for  $F_{Ay}$  is interpreted as meaning that the  $y$  component of the anchoring force is actually in the negative  $y$  direction, not the positive  $y$  direction as originally indicated in Fig. E5.12b.

As with Example 5.11, the anchoring force for the pipe bend is independent of the atmospheric pressure. However, the force that the bend puts on the fluid inside of it,  $R_y$ , depends on the atmospheric pressure. We can see this by using a control volume which



■ FIGURE E5.12 cont.

surrounds only the fluid within the bend as shown in Fig. E5.12c. Application of the momentum equation to this situation gives

$$R_y = -\dot{m}(v_1 + v_2) - p_1 A_1 - p_2 A_2$$

where  $p_1$  and  $p_2$  must be in terms of absolute pressure because the force between the fluid and the pipe wall,  $R_y$ , is the complete pressure effect (i.e., absolute pressure).

Thus, we obtain

$$\begin{aligned} R_y &= -(9.70 \text{ slugs/s})(50 \text{ ft/s} + 50 \text{ ft/s}) - (30 \text{ psia})(144 \text{ in.}^2/\text{ft}^2)(0.1 \text{ ft}^2) \\ &\quad - (24 \text{ psia})(144 \text{ in.}^2/\text{ft}^2)(0.1 \text{ ft}^2) \\ &= -1748 \text{ lb} \end{aligned} \quad (7)$$

We can use the control volume that includes just the pipe bend (without the fluid inside it) as shown in Fig. E5.12d to determine  $F_{Ay}$ , the anchoring force component in the  $y$  direction necessary to hold the bend stationary. The  $y$  component of the momentum equation applied to this control volume gives

$$F_{Ay} = R_y + p_{\text{atm}}(A_1 + A_2) \quad (8)$$

where  $R_y$  is given by Eq. 7. The  $p_{\text{atm}}(A_1 + A_2)$  term represents the net pressure force on the outside portion of the control volume. Recall that the pressure force on the inside of the bend is accounted for by  $R_y$ . By combining Eqs. 7 and 8, we obtain

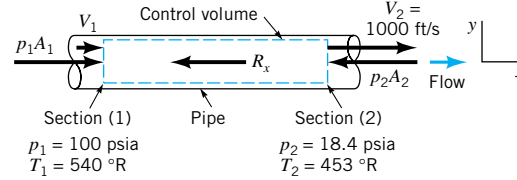
$$F_{Ay} = -1748 \text{ lb} + 14.7 \text{ lb/in.}^2 (0.1 \text{ ft}^2 + 0.1 \text{ ft}^2)(144 \text{ in.}^2/\text{ft}^2) = -1324 \text{ lb}$$

in agreement with the original answer obtained using the control volume of Fig. E5.12b.

In Example 5.12, the direction of flow entering the control volume was different from the direction of flow leaving the control volume by  $180^\circ$ . This change in flow direction only (the flow speed remained constant) resulted in a large portion of the reaction force exerted by the pipe on the water. This is in contrast to the small contribution of fluid acceleration to the anchoring force of Example 5.11. From Examples 5.11 and 5.12, we see that changes in flow speed and/or direction result in a reaction force. Other types of problems that can be solved with the linear momentum equation (Eq. 5.22) are illustrated in the following examples.

## EXAMPLE 5.13

Air flows steadily between two cross sections in a long, straight portion of 4-in. inside diameter pipe as indicated in Fig. E5.13, where the uniformly distributed temperature and pressure at each cross section are given. If the average air velocity at section (2) is 1000 ft/s, we found in Example 5.2 that the average air velocity at section (1) must be 219 ft/s. Assuming uniform velocity distributions at sections (1) and (2), determine the frictional force exerted by the pipe wall on the air flow between sections (1) and (2).



■ FIGURE E5.13

## SOLUTION

The control volume of Example 5.2 is appropriate for this problem. The forces acting on the air between sections (1) and (2) are identified in Fig. E5.13. The weight of air is considered negligibly small. The reaction force between the wetted wall of the pipe and the flowing air,  $R_x$ , is the frictional force sought. Application of the axial component of Eq. 5.22 to this control volume yields

$$\int_{cs} u \rho \mathbf{V} \cdot \hat{\mathbf{n}} dA = -R_x + p_1 A_1 - p_2 A_2 \quad (1)$$

The positive  $x$  direction is set as being to the right. Furthermore, for uniform velocity distributions (one-dimensional flow), Eq. 1 becomes

$$(+u_1)(-\dot{m}_1) + (+u_2)(+\dot{m}_2) = -R_x + p_1 A_1 - p_2 A_2 \quad (2)$$

From conservation of mass (Eq. 5.12) we get

$$\dot{m} = \dot{m}_1 = \dot{m}_2 \quad (3)$$

so that Eq. 2 becomes

$$\dot{m}(u_2 - u_1) = -R_x + A_2(p_1 - p_2) \quad (4)$$

Solving Eq. 4 for  $R_x$ , we get

$$R_x = A_2(p_1 - p_2) - \dot{m}(u_2 - u_1) \quad (5)$$

The equation of state gives

$$\rho_2 = \frac{p_2}{RT_2} \quad (6)$$

and the equation for area  $A_2$  is

$$A_2 = \frac{\pi D_2^2}{4} \quad (7)$$

Thus, from Eqs. 3, 6, and 7

$$\begin{aligned} \dot{m} &= \left( \frac{p_2}{RT_2} \right) \left( \frac{\pi D_2^2}{4} \right) u_2 = \frac{(18.4 \text{ psia})(144 \text{ in.}^2/\text{ft}^2)}{(1716 \text{ ft} \cdot \text{lb}/\text{slug} \cdot ^\circ\text{R})(453 ^\circ\text{R})} \\ &\quad \times \frac{\pi(4 \text{ in.})^2}{4(144 \text{ in.}^2/\text{ft}^2)} (1000 \text{ ft/s}) = 0.297 \text{ slugs/s} \end{aligned} \quad (8)$$

Thus, from Eqs. 5 and 8

$$R_x = \frac{\pi(4 \text{ in.})^2}{4}(100 \text{ psia} - 18.4 \text{ psia}) \\ - (0.297 \text{ slugs/s})(1000 \text{ ft/s} - 219 \text{ ft/s})[1 \text{ lb}/(\text{slug} \cdot \text{ft/s}^2)]$$

or

$$R_x = 1025 \text{ lb} - 232 \text{ lb} = 793 \text{ lb} \quad (\text{Ans})$$

Note that both the pressure and momentum contribute to the friction force,  $R_x$ . If the fluid flow were incompressible, then  $u_1 = u_2$  and there would be no momentum contribution to  $R_x$ .

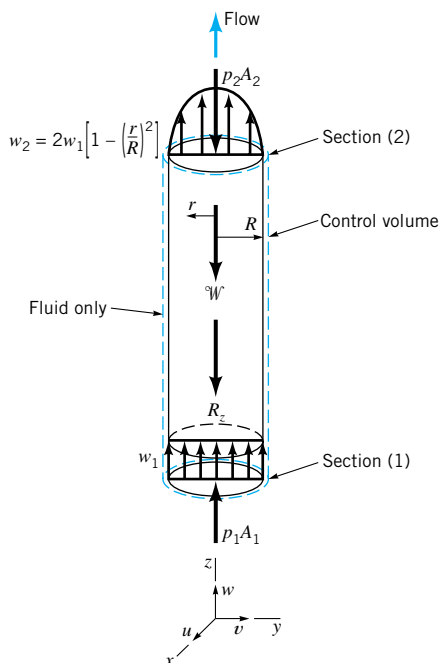
## EXAMPLE 5.14

If the flow of Example 5.4 is vertically upward, develop an expression for the fluid pressure drop that occurs between sections (1) and (2).

### SOLUTION

A control volume (see dashed lines in Fig. E5.4) that includes only fluid from section (1) to section (2) is selected. The forces acting on the fluid in this control volume are identified in Fig. E5.14. The application of the axial component of Eq. 5.22 to the fluid in this control volume results in

$$\int_{\text{CS}} w \rho \mathbf{V} \cdot \hat{\mathbf{n}} dA = p_1 A_1 - R_z - {}^w W - p_2 A_2 \quad (1)$$



■ FIGURE E5.14

where  $R_z$  is the resultant force of the wetted pipe wall on the fluid. Further, for uniform flow at section (1), and because the flow at section (2) is out of the control volume, Eq. 1 becomes

$$(+w_1)(-\dot{m}_1) + \int_{A_2} (+w_2)\rho(+w_2 dA_2) = p_1A_1 - R_z - \mathcal{W} - p_2A_2 \quad (2)$$

The positive direction is considered up. The surface integral over the cross-sectional area at section (2),  $A_2$ , is evaluated by using the parabolic velocity profile obtained in Example 5.4,  $w_2 = 2w_1[1 - (r/R)^2]$ , as

$$\int_{A_2} w_2\rho w_2 dA_2 = \rho \int_0^R w_2^2 2\pi r dr = 2\pi\rho \int_0^R (2w_1)^2 \left[1 - \left(\frac{r}{R}\right)^2\right]^2 r dr$$

or

$$\int_{A_2} w_2\rho w_2 dA_2 = 4\pi\rho w_1^2 \frac{R^2}{3} \quad (3)$$

Combining Eqs. 2 and 3 we obtain

$$-w_1^2\rho\pi R^2 + \frac{4}{3}w_1^2\rho\pi R^2 = p_1A_1 - R_z - \mathcal{W} - p_2A_2 \quad (4)$$

Solving Eq. 4 for the pressure drop from section (1) to section (2),  $p_1 - p_2$ , we obtain

$$p_1 - p_2 = \frac{\rho w_1^2}{3} + \frac{R_z}{A_1} + \frac{\mathcal{W}}{A_1} \quad (\text{Ans})$$

We see that the drop in pressure from section (1) to section (2) occurs because of the following:

1. The change in momentum flow between the two sections associated with going from a uniform velocity profile to a parabolic velocity profile.
2. Pipe wall friction.
3. The weight of the water column; a hydrostatic pressure effect.

If the velocity profiles had been identically parabolic at sections (1) and (2), the momentum flowrate at each section would have been identical, a condition we call “fully developed” flow. Then, the pressure drop,  $p_1 - p_2$ , would be due only to pipe wall friction and the weight of the water column. If in addition to being fully developed, the flow involved negligible weight effects (for example, horizontal flow of liquids or the flow of gases in any direction) the drop in pressure between any two sections,  $p_1 - p_2$ , would be a result of pipe wall friction only.

Note that although the average velocity is the same at section (1) as it is at section (2) ( $\bar{V}_1 = \bar{V}_2 = w_1$ ), the momentum flux across section (1) is not the same as it is across section (2). If it were, the left-hand side of Eq. (4) would be zero. For this nonuniform flow the momentum flux can be written in terms of the average velocity,  $\bar{V}$ , and the *momentum coefficient*,  $\beta$ , as

$$\beta = \frac{\int w\rho\mathbf{V} \cdot \hat{\mathbf{n}} dA}{\rho\bar{V}^2A}$$

Hence the momentum flux can be written as

$$\int_{cs} w\rho\mathbf{V} \cdot \hat{\mathbf{n}} dA = -\beta_1 w_1^2\rho\pi R^2 + \beta_2 w_1^2\rho\pi R^2$$

where  $\beta_1 = 1$  ( $\beta = 1$  for uniform flow) and  $\beta_2 = 4/3$  ( $\beta > 1$  for nonuniform flow).



## EXAMPLE 5.15

A static thrust stand as sketched in Fig. E5.15 is to be designed for testing a jet engine. The following conditions are known for a typical test: Intake air velocity = 200 m/s; exhaust gas velocity = 500 m/s; intake cross-sectional area = 1 m<sup>2</sup>; intake static pressure = -22.5 kPa = 78.5 kPa; (abs); intake static temperature = 268 K; exhaust static pressure = 0 kPa = 101 kPa (abs). Estimate the nominal thrust for which to design.

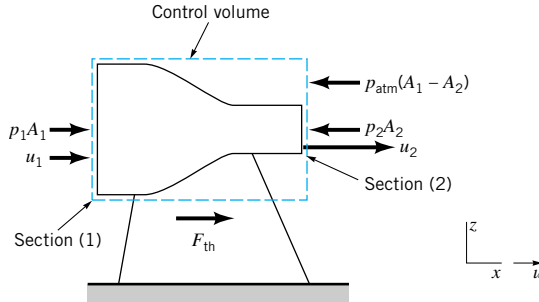


FIGURE E5.15

## SOLUTION

The cylindrical control volume outlined with a dashed line in Fig. E5.15 is selected. The external forces acting in the axial direction are also shown. Application of the momentum equation (Eq. 5.22) to the contents of this control volume yields

$$\int_{cs} u \rho \mathbf{V} \cdot \hat{\mathbf{n}} dA = p_1 A_1 + F_{th} - p_2 A_2 - p_{atm} (A_1 - A_2) \quad (1)$$

where the pressures are absolute. Thus, for one-dimensional flow, Eq. 1 becomes

$$(+u_1)(-\dot{m}_1) + (+u_2)(+\dot{m}_2) = (p_1 - p_{atm})A_1 - (p_2 - p_{atm})A_2 + F_{th} \quad (2)$$

The positive direction is to the right. The conservation of mass equation (Eq. 5.12) leads to

$$\dot{m} = \dot{m}_1 = \rho_1 A_1 u_1 = \dot{m}_2 = \rho_2 A_2 u_2 \quad (3)$$

Combining Eqs. 2 and 3 and using gage pressure we obtain

$$\dot{m}(u_2 - u_1) = p_1 A_1 - p_2 A_2 + F_{th} \quad (4)$$

Solving Eq. 4 for the thrust force,  $F_{th}$ , we obtain

$$F_{th} = -p_1 A_1 + p_2 A_2 + \dot{m}(u_2 - u_1) \quad (5)$$

We need to determine the mass flowrate,  $\dot{m}$ , to calculate  $F_{th}$ , and to calculate  $\dot{m} = \rho_1 A_1 u_1$ , we need  $\rho_1$ . From the ideal gas equation of state

$$\rho_1 = \frac{p_1}{RT_1} = \frac{(78.5 \text{ kPa})(1000 \text{ Pa/kPa})[1(\text{N/m}^2)/\text{Pa}]}{(286.9 \text{ J/kg} \cdot \text{K})(268 \text{ K})(1 \text{ N} \cdot \text{m/J})} = 1.02 \text{ kg/m}^3$$

Thus,

$$\dot{m} = \rho_1 A_1 u_1 = (1.02 \text{ kg/m}^3)(1 \text{ m}^2)(200 \text{ m/s}) = 204 \text{ kg/s} \quad (6)$$

Finally, combining Eqs. 5 and 6 and substituting given data with  $p_2 = 0$ , we obtain

$$F_{th} = -(1 \text{ m}^2)(-22.5 \text{ kPa})(1000 \text{ Pa/kPa})[1(\text{N/m}^2)/\text{Pa}] + (204 \text{ kg/s})(500 \text{ m/s} - 200 \text{ m/s})[1 \text{ N}/(\text{kg} \cdot \text{m/s}^2)]$$

and

$$F_{th} = 22,500 \text{ N} + 61,200 \text{ N} = 83,700 \text{ N} \quad (\text{Ans})$$

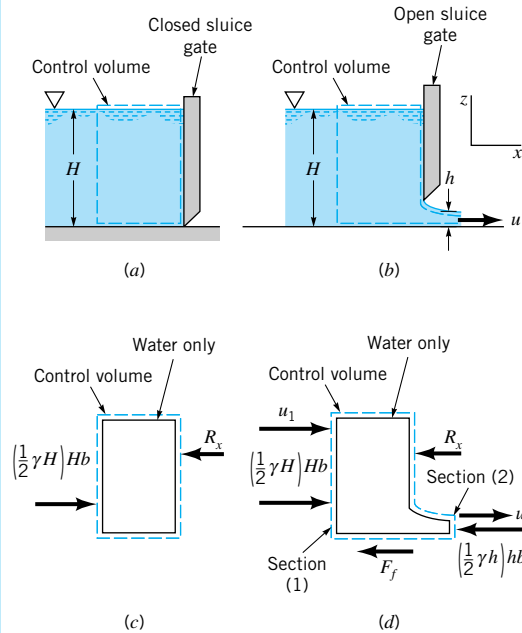
The force of the thrust stand on the engine is directed toward the right. Conversely, the engine pushes to the left on the thrust stand (or aircraft).

## EXAMPLE 5.16

A sluice gate across a channel of width  $b$  is shown in the closed and open positions in Figs. E5.16a and E5.16b. Is the anchoring force required to hold the gate in place larger when the gate is closed or when it is open?

## SOLUTION

We will answer this question by comparing expressions for the horizontal reaction force,  $R_x$ , between the gate and the water when the gate is closed and when the gate is open. The control volume used in each case is indicated with dashed lines in Figs. E5.16a and E5.16b.



■ FIGURE E5.16

When the gate is closed, the horizontal forces acting on the contents of the control volume are identified in Fig. E5.16c. Application of [Eq. 5.22](#) to the contents of this control volume yields

$$\int_{cs} u \rho \mathbf{V} \cdot \hat{\mathbf{n}} dA = \frac{1}{2} \gamma H^2 b - R_x \quad (1)$$

Note that the hydrostatic pressure force,  $\gamma H^2 b/2$ , is used. From Eq. 1, the force exerted on the water by the gate (which is equal to the force necessary to hold the gate stationary) is

$$R_x = \frac{1}{2} \gamma H^2 b \quad (2)$$

which is equal in magnitude to the hydrostatic force exerted on the gate by the water.

When the gate is open, the horizontal forces acting on the contents of the control volume are shown in Fig. E5.16d. Application of Eq. 5.22 to the contents of this control volume leads to

$$\int_{cs} u \rho \mathbf{V} \cdot \hat{\mathbf{n}} dA = \frac{1}{2} \gamma H^2 b - R_x - \frac{1}{2} \gamma h^2 b - F_f \quad (3)$$

Note that we have assumed that the pressure distribution is hydrostatic in the water at sections (1) and (2) (see Section 3.4). Also, the frictional force between the channel bottom and the water is specified as  $F_f$ . The surface integral in Eq. 3 is nonzero only where there is flow across the control surface. With the assumption of uniform velocity distributions,

$$\int_{cs} u \rho \mathbf{V} \cdot \hat{\mathbf{n}} dA = (u_1) \rho (-u_1) H b + (+u_2) \rho (+u_2) h b \quad (4)$$

Thus, Eqs. 3 and 4 combine to form

$$-\rho u_1^2 H b + \rho u_2^2 h b = \frac{1}{2} \gamma H^2 b - R_x - \frac{1}{2} \gamma h^2 b - F_f \quad (5)$$

If  $H \gg h$ , the upstream velocity,  $u_1$ , is much less than  $u_2$  so that the contribution of the incoming momentum flow to the control surface integral can be neglected and from Eq. 5 we obtain

$$R_x = \frac{1}{2} \gamma H^2 b - \frac{1}{2} \gamma h^2 b - F_f - \rho u_2^2 h b \quad (6)$$

Comparing the expressions for  $R_x$  (Eqs. 2 and 6), we conclude that the reaction force between the gate and the water (and therefore the anchoring force required to hold the gate in place) is smaller when the gate is open than when it is closed. **(Ans)**

*The linear momentum equation can be written for a moving control volume.*

All of the linear momentum examples considered thus far have involved stationary and nondeforming control volumes which are thus inertial because there is no acceleration. A nondeforming control volume translating in a straight line at constant speed is also inertial because there is no acceleration. For a system and an inertial, moving, nondeforming control volume that are both coincident at an instant of time, the Reynolds transport theorem (Eq. 4.23) leads to

$$\frac{D}{Dt} \int_{sys} \mathbf{V} \rho dV = \frac{\partial}{\partial t} \int_{cv} \mathbf{V} \rho dV + \int_{cs} \mathbf{V} \rho \mathbf{W} \cdot \hat{\mathbf{n}} dA \quad (5.23)$$

When we combine Eq. 5.23 with Eqs. 5.19 and 5.20, we get

$$\frac{\partial}{\partial t} \int_{cv} \mathbf{V} \rho dV + \int_{cs} \mathbf{V} \rho \mathbf{W} \cdot \hat{\mathbf{n}} dA = \sum \mathbf{F}_{\text{contents of the control volume}} \quad (5.24)$$

When the equation relating absolute, relative, and control volume velocities (Eq. 5.14) is used with Eq. 5.24, the result is

$$\frac{\partial}{\partial t} \int_{cv} (\mathbf{W} + \mathbf{V}_{cv}) \rho dV + \int_{cs} (\mathbf{W} + \mathbf{V}_{cv}) \rho \mathbf{W} \cdot \hat{\mathbf{n}} dA = \sum \mathbf{F}_{\text{contents of the control volume}} \quad (5.25)$$

For a constant control volume velocity,  $\mathbf{V}_{cv}$ , and steady flow in the control volume reference frame,

$$\frac{\partial}{\partial t} \int_{cv} (\mathbf{W} + \mathbf{V}_{cv}) \rho dV = 0 \quad (5.26)$$

Also, for this inertial, nondeforming control volume

$$\int_{cs} (\mathbf{W} + \mathbf{V}_{cv}) \rho \mathbf{W} \cdot \hat{\mathbf{n}} dA = \int_{cs} \mathbf{W} \rho \mathbf{W} \cdot \hat{\mathbf{n}} dA + \mathbf{V}_{cv} \int_{cs} \rho \mathbf{W} \cdot \hat{\mathbf{n}} dA \quad (5.27)$$

For steady flow (on an instantaneous or time-average basis), Eq. 5.15 gives

$$\int_{cs} \rho \mathbf{W} \cdot \hat{\mathbf{n}} dA = 0 \quad (5.28)$$

Combining Eqs. 5.25, 5.26, 5.27, and 5.28, we conclude that the linear momentum equation for an inertial, moving, nondeforming control volume that involves steady (instantaneous or time-average) flow is

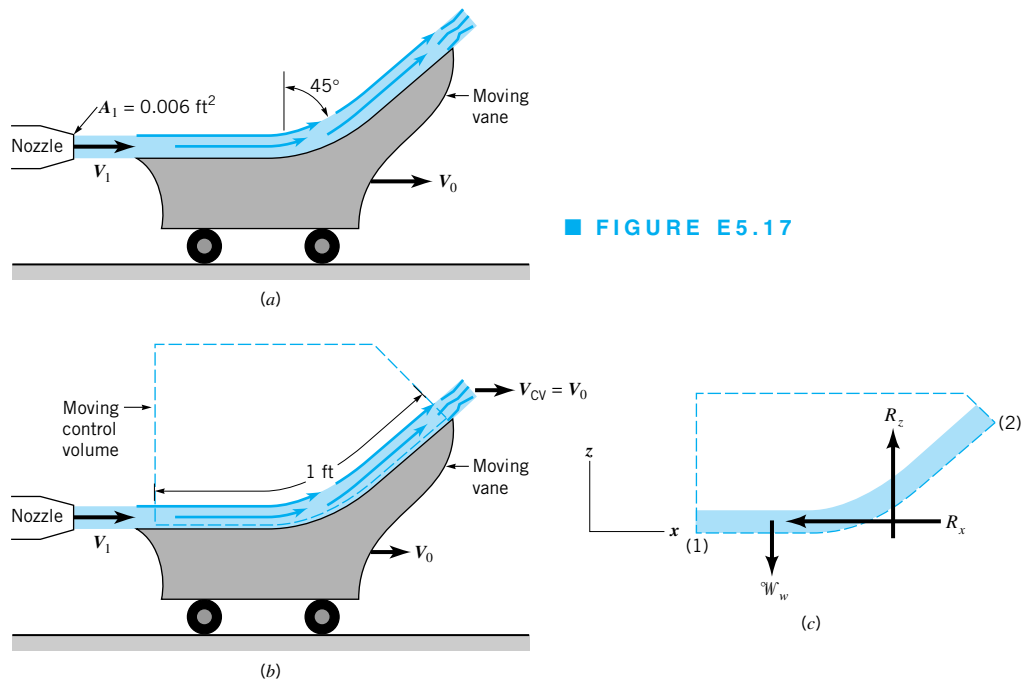
*The linear momentum equation for a moving control volume involves the relative velocity.*

$$\int_{cs} \mathbf{W} \rho \mathbf{W} \cdot \hat{\mathbf{n}} dA = \sum \mathbf{F}_{\text{contents of the control volume}} \quad (5.29)$$

Example 5.17 illustrates the use of Eq. 5.29.

## EXAMPLE 5.17

A vane on wheels moves with constant velocity  $\mathbf{V}_0$  when a stream of water having a nozzle exit velocity of  $\mathbf{V}_1$  is turned  $45^\circ$  by the vane as indicated in Fig. E5.17a. Note that this is the same moving vane considered in Section 4.4.6 earlier. Determine the magnitude and direction of the force,  $\mathbf{F}$ , exerted by the stream of water on the vane surface. The speed of the water jet leaving the nozzle is 100 ft/s, and the vane is moving to the right with a constant speed of 20 ft/s.



■ FIGURE E5.17

## SOLUTION

To determine the magnitude and direction of the force,  $\mathbf{F}$ , exerted by the water on the vane, we apply Eq. 5.29 to the contents of the moving control volume shown in Fig. E5.17*b*. The forces acting on the contents of this control volume are indicated in Fig. E5.17*c*. Note that since the ambient pressure is atmospheric, all pressure forces cancel each other out. Equation 5.29 is applied to the contents of the moving control volume in component directions. For the  $x$  direction (positive to the right), we get

$$\int_{cs} W_x \rho \mathbf{W} \cdot \hat{\mathbf{n}} dA = -R_x$$

or

$$(+W_1)(-\dot{m}_1) + (+W_2 \cos 45^\circ)(\dot{m}_2) = -R_x \quad (1)$$

where

$$\dot{m}_1 = \rho_1 W_1 A_1 \quad \text{and} \quad \dot{m}_2 = \rho_2 W_2 A_2.$$

For the vertical or  $z$  direction (positive up) we get

$$\int_{cs} W_z \rho \mathbf{W} \cdot \hat{\mathbf{n}} dA = R_z - \dot{\mathcal{W}}_w$$

or

$$(+W_2 \sin 45^\circ)(\dot{m}_2) = R_z - \dot{\mathcal{W}}_w \quad (2)$$

We assume for simplicity, that the water flow is frictionless and that the change in water elevation across the vane is negligible. Thus, from the Bernoulli equation (Eq. 3.7) we conclude that the speed of the water relative to the moving control volume,  $W$ , is constant or

$$W_1 = W_2$$

The relative speed of the stream of water entering the control volume,  $W_1$ , is

$$W_1 = V_1 - V_0 = 100 \text{ ft/s} - 20 \text{ ft/s} = 80 \text{ ft/s} = W_2$$

The water density is constant so that

$$\rho_1 = \rho_2 = 1.94 \text{ slugs/ft}^3$$

Application of the conservation of mass principle to the contents of the moving control volume (Eq. 5.16) leads to

$$\dot{m}_1 = \rho_1 W_1 A_1 = \rho_2 W_2 A_2 = \dot{m}_2$$

Combining results we get

$$R_x = \rho W_1^2 A_1 (1 - \cos 45^\circ)$$

or

$$R_x = (1.94 \text{ slugs/ft}^3)(80 \text{ ft/s})^2(0.006 \text{ ft}^2)(1 - \cos 45^\circ)$$

$$R_x = 21.8 \text{ lb}$$

Also,

$$R_z = \rho W_1^2 (\sin 45^\circ) A_1 + \dot{\mathcal{W}}_w$$

where

$$\mathcal{W}_w = \rho g A_1 \ell$$

Thus,

$$\begin{aligned} R_z &= (1.94 \text{ slugs/ft}^3)(80 \text{ ft/s})^2(\sin 45^\circ)(0.006 \text{ ft}^2) \\ &\quad + (62.4 \text{ lb/ft}^3)(0.006 \text{ ft}^2)(1 \text{ ft}) \\ &= 52.6 \text{ lb} + 0.37 \text{ lb} = 53 \text{ lb} \end{aligned}$$

Combining the components we get

$$R = \sqrt{R_x^2 + R_z^2} = [(21.8 \text{ lb})^2 + (53 \text{ lb})^2]^{1/2} = 57.3 \text{ lb}$$

The angle of  $\mathbf{R}$  from the  $x$  direction,  $\alpha$ , is

$$\alpha = \tan^{-1} \frac{R_z}{R_x} = \tan^{-1} (53 \text{ lb}/21.8 \text{ lb}) = 67.6^\circ$$

The force of the water on the vane is equal in magnitude but opposite in direction from  $\mathbf{R}$ ; thus it points to the right and down at an angle of  $67.6^\circ$  from the  $x$  direction and is equal in magnitude to 57.3 lb. (Ans)

It should be clear from the preceding examples that fluid flows can lead to a reaction force in the following ways:

1. Linear momentum flow variation in direction and/or magnitude.
2. Fluid pressure forces.
3. Fluid friction forces.
4. Fluid weight.

The selection of a control volume is an important matter. An appropriate control volume can make a problem solution straightforward.

### 5.2.3 Derivation of the Moment-of-Momentum Equation<sup>2</sup>

In many engineering problems, the moment of a force with respect to an axis, namely, *torque*, is important. Newton's second law of motion has already led to a useful relationship between forces and linear momentum flow. The linear momentum equation can also be used to solve problems involving torques. However, by forming the moment of the linear momentum and the resultant force associated with each particle of fluid with respect to a point in an inertial coordinate system, we will develop a *moment-of-momentum equation* that relates *torques* and *angular momentum flow* for the contents of a control volume. When torques are important, the moment-of-momentum equation is often more convenient to use than the linear momentum equation.

Application of Newton's second law of motion to a particle of fluid yields

$$\frac{D}{Dt} (\mathbf{V} \rho \delta \mathcal{V}) = \delta \mathbf{F}_{\text{particle}} \quad (5.30)$$

*The moment-of-momentum equation involves torques and angular momentum.*

<sup>2</sup>This section may be omitted, along with Sections 5.2.4 and 5.3.5, without loss of continuity in the text material. However, these sections are recommended for those interested in Chapter 12.

*The angular momentum equation is derived from Newton's second law.*

where  $\mathbf{V}$  is the particle velocity measured in an inertial reference system,  $\rho$  is the particle density,  $\delta\mathcal{V}$  is the infinitesimally small particle volume, and  $\delta\mathbf{F}_{\text{particle}}$  is the resultant external force acting on the particle. If we form the moment of each side of Eq. 5.30 with respect to the origin of an inertial coordinate system, we obtain

$$\mathbf{r} \times \frac{D}{Dt} (\mathbf{V} \rho \delta\mathcal{V}) = \mathbf{r} \times \delta\mathbf{F}_{\text{particle}} \quad (5.31)$$

where  $\mathbf{r}$  is the position vector from the origin of the inertial coordinate system to the fluid particle (Fig. 5.3). We note that

$$\frac{D}{Dt} [(\mathbf{r} \times \mathbf{V}) \rho \delta\mathcal{V}] = \frac{D\mathbf{r}}{Dt} \times \mathbf{V} \rho \delta\mathcal{V} + \mathbf{r} \times \frac{D(\mathbf{V} \rho \delta\mathcal{V})}{Dt} \quad (5.32)$$

and

$$\frac{D\mathbf{r}}{Dt} = \mathbf{V} \quad (5.33)$$

Thus, since

$$\mathbf{V} \times \mathbf{V} = 0 \quad (5.34)$$

by combining Eqs. 5.31, 5.32, 5.33, and 5.34, we obtain the expression

$$\frac{D}{Dt} [(\mathbf{r} \times \mathbf{V}) \rho \delta\mathcal{V}] = \mathbf{r} \times \delta\mathbf{F}_{\text{particle}} \quad (5.35)$$

Equation 5.35 is valid for every particle of a system. For a system (collection of fluid particles), we need to use the sum of both sides of Eq. 5.35 to obtain

$$\int_{\text{sys}} \frac{D}{Dt} [(\mathbf{r} \times \mathbf{V}) \rho d\mathcal{V}] = \sum (\mathbf{r} \times \mathbf{F})_{\text{sys}} \quad (5.36)$$

where

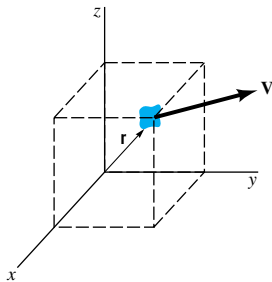
$$\sum \mathbf{r} \times \delta\mathbf{F}_{\text{particle}} = \sum (\mathbf{r} \times \mathbf{F})_{\text{sys}} \quad (5.37)$$

We note that

$$\frac{D}{Dt} \int_{\text{sys}} (\mathbf{r} \times \mathbf{V}) \rho d\mathcal{V} = \int_{\text{sys}} \frac{D}{Dt} [(\mathbf{r} \times \mathbf{V}) \rho d\mathcal{V}] \quad (5.38)$$

since the sequential order of differentiation and integration can be reversed without consequence. (Recall that the material derivative,  $D(\ )/Dt$ , denotes the time derivative following a given system; see [Section 4.2.1](#).) Thus, from Eqs. 5.36 and 5.38 we get

$$\frac{D}{Dt} \int_{\text{sys}} (\mathbf{r} \times \mathbf{V}) \rho d\mathcal{V} = \sum (\mathbf{r} \times \mathbf{F})_{\text{sys}} \quad (5.39)$$



■ FIGURE 5.3 Inertial coordinate system.

or

the time rate of change of the                      = sum of external torques  
moment-of-momentum of the system            acting on the system

For a control volume that is instantaneously coincident with the system, the torques acting on the system and on the control volume contents will be identical:

$$\sum (\mathbf{r} \times \mathbf{F})_{\text{sys}} = \sum (\mathbf{r} \times \mathbf{F})_{\text{cv}} \quad (5.40)$$

Further, for the system and the contents of the coincident control volume that is fixed and nondeforming, the Reynolds transport theorem (Eq. 4.19) leads to

$$\frac{D}{Dt} \int_{\text{sys}} (\mathbf{r} \times \mathbf{V}) \rho \, d\mathcal{V} = \frac{\partial}{\partial t} \int_{\text{cv}} (\mathbf{r} \times \mathbf{V}) \rho \, d\mathcal{V} + \int_{\text{cs}} (\mathbf{r} \times \mathbf{V}) \rho \mathbf{V} \cdot \hat{\mathbf{n}} \, dA \quad (5.41)$$

or

*For a system, the rate of change of moment-of-momentum equals the net torque.*

time rate of change    time rate of change    net rate of flow  
of the moment-of-    of the moment-of-    of the moment-of-  
momentum of the    = momentum of the    + momentum through  
system                    contents of the            the control surface  
                                  control volume

For a control volume that is fixed (and therefore inertial) and nondeforming, we combine Eqs. 5.39, 5.40, and 5.41 to obtain the moment-of-momentum equation:

$$\frac{\partial}{\partial t} \int_{\text{cv}} (\mathbf{r} \times \mathbf{V}) \rho \, d\mathcal{V} + \int_{\text{cs}} (\mathbf{r} \times \mathbf{V}) \rho \mathbf{V} \cdot \hat{\mathbf{n}} \, dA = \sum (\mathbf{r} \times \mathbf{F})_{\text{contents of the control volume}} \quad (5.42)$$

An important category of fluid mechanical problems that is readily solved with the help of the moment-of-momentum equation (Eq. 5.42) involves machines that rotate or tend to rotate around a single axis. Examples of these machines include rotary lawn sprinklers, ceiling fans, lawn mower blades, wind turbines, turbochargers, and gas turbine engines. As a class, these devices are often called turbomachines.

### 5.2.4 Application of the Moment-of-Momentum Equation<sup>3</sup>

We simplify our use of Eq. 5.42 in several ways:

1. We assume that flows considered are one-dimensional (uniform distributions of average velocity at any section).
2. We confine ourselves to steady or steady-in-the-mean cyclical flows. Thus,

$$\frac{\partial}{\partial t} \int_{\text{cv}} (\mathbf{r} \times \mathbf{V}) \rho \, d\mathcal{V} = 0$$

at any instant of time for steady flows or on a time-average basis for cyclical unsteady flows.

3. We work only with the component of Eq. 5.42 resolved along the axis of rotation.

Consider the rotating sprinkler sketched in Fig. 5.4. Because the direction and magnitude of the flow through the sprinkler from the inlet [section (1)] to the outlet [section (2)] of the arm

<sup>3</sup>This section may be omitted, along with Sections 5.2.3 and 5.3.5, without loss of continuity in the text material. However, these sections are recommended for those interested in Chapter 12.



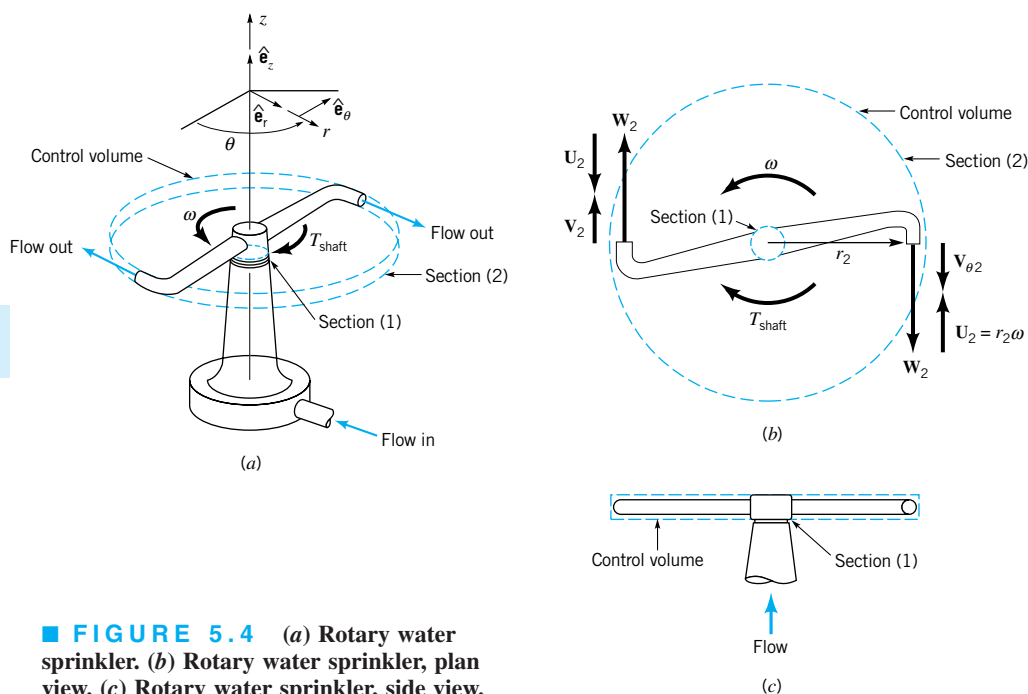
changes, the water exerts a torque on the sprinkler head causing it to tend to rotate or to actually rotate in the direction shown, much like a turbine rotor. In applying the moment-of-momentum equation (Eq. 5.42) to this flow situation, we elect to use the fixed and non-deforming control volume shown in Fig. 5.4. This disk-shaped control volume contains within its boundaries the spinning or stationary sprinkler head and the portion of the water flowing through the sprinkler contained in the control volume at an instant. The control surface cuts through the sprinkler head's solid material so that the shaft torque that resists motion can be clearly identified. When the sprinkler is rotating, the flow field in the stationary control volume is cyclical and unsteady, but steady in the mean. We proceed to use the axial component of the moment-of-momentum equation (Eq. 5.42) to analyze this flow.

The integrand of the moment-of-momentum flow term in Eq. 5.42,

$$\int_{cs} (\mathbf{r} \times \mathbf{V}) \rho \mathbf{V} \cdot \hat{\mathbf{n}} dA$$

*The component of angular momentum along the axis of rotation is often the most important.*

can be nonzero only where fluid is crossing the control surface. Everywhere else on the control surface this term will be zero because  $\mathbf{V} \cdot \hat{\mathbf{n}} = 0$ . Water enters the control volume axially through the hollow stem of the sprinkler at section (1). At this portion of the control surface, the component of  $\mathbf{r} \times \mathbf{V}$  resolved along the axis of rotation is zero because  $\mathbf{r} \times \mathbf{V}$  and the axis of rotation are perpendicular. Thus, there is no axial moment-of-momentum flow in at section (1). Water leaves the control volume through each of the two nozzle openings at section (2). For the exiting flow, the magnitude of the axial component of  $\mathbf{r} \times \mathbf{V}$  is  $r_2 V_{\theta 2}$ , where  $r_2$  is the radius from the axis of rotation to the nozzle centerline and  $V_{\theta 2}$  is the value of the tangential component of the velocity of the flow exiting each nozzle as observed from a frame of reference attached to the fixed and nondeforming control volume. The fluid velocity measured relative to a fixed control surface is an absolute velocity,  $\mathbf{V}$ . The velocity of the nozzle exit flow as viewed from the nozzle is called the relative velocity,  $\mathbf{W}$ . The absolute and relative velocities,  $\mathbf{V}$  and  $\mathbf{W}$ , are related by the vector relationship



■ **FIGURE 5.4** (a) Rotary water sprinkler. (b) Rotary water sprinkler, plan view. (c) Rotary water sprinkler, side view.



V5.5 Rotating lawn sprinkler

$$\mathbf{V} = \mathbf{W} + \mathbf{U} \quad (5.43)$$

where  $\mathbf{U}$  is the velocity of the moving nozzle as measured relative to the fixed control surface.

The cross product and the dot product involved in the moment-of-momentum flow term of Eq. 5.42,

$$\int_{cs} (\mathbf{r} \times \mathbf{V}) \rho \mathbf{V} \cdot \hat{\mathbf{n}} dA$$

can each result in a positive or negative value. For flow into the control volume,  $\mathbf{V} \cdot \hat{\mathbf{n}}$  is negative. For flow out,  $\mathbf{V} \cdot \hat{\mathbf{n}}$  is positive. The correct algebraic sign to assign the axis component of  $\mathbf{r} \times \mathbf{V}$  can be ascertained by using the right-hand rule. The positive direction along the axis of rotation is the direction the thumb of the right hand points when it is extended and the remaining fingers are curled around the rotation axis in the positive direction of rotation as illustrated in Fig. 5.5. The direction of the axial component of  $\mathbf{r} \times \mathbf{V}$  is similarly ascertained by noting the direction of the cross product of the radius from the axis of rotation,  $r\hat{\mathbf{e}}_r$ , and the tangential component of absolute velocity,  $V_\theta\hat{\mathbf{e}}_\theta$ . Thus, for the sprinkler of Fig. 5.4, we can state that

*The algebraic sign is obtained by the right-hand rule.*

$$\left[ \int_{cs} (\mathbf{r} \times \mathbf{V}) \rho \mathbf{V} \cdot \hat{\mathbf{n}} dA \right]_{\text{axial}} = (-r_2 V_{\theta 2})(+\dot{m}) \quad (5.44)$$

where, because of mass conservation,  $\dot{m}$  is the total mass flowrate through both nozzles. As was demonstrated in Example 5.7, the mass flowrate is the same whether the sprinkler rotates or not. The correct algebraic sign of the axial component of  $\mathbf{r} \times \mathbf{V}$  can be easily remembered in the following way: if  $\mathbf{V}_\theta$  and  $\mathbf{U}$  are in the same direction, use +; if  $\mathbf{V}_\theta$  and  $\mathbf{U}$  are in opposite directions, use −.

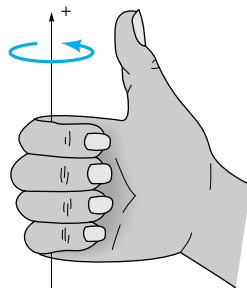
The torque term  $\left[ \sum (\mathbf{r} \times \mathbf{F})_{\text{contents of the control volume}} \right]$  of the moment-of-momentum equation (Eq. 5.42) is analyzed next. Confining ourselves to torques acting with respect to the axis of rotation only, we conclude that the shaft torque is important. The net torque with respect to the axis of rotation associated with normal forces exerted on the contents of the control volume will be very small if not zero. The net axial torque due to fluid tangential forces is also negligibly small for the control volume of Fig. 5.4. Thus, for the sprinkler of Fig. 5.4

$$\sum \left[ (\mathbf{r} \times \mathbf{F})_{\text{contents of the control volume}} \right]_{\text{axial}} = T_{\text{shaft}} \quad (5.45)$$

Note that we have entered  $T_{\text{shaft}}$  as a positive quantity in Eq. 5.45. This is equivalent to assuming that  $T_{\text{shaft}}$  is in the same direction as rotation.

For the sprinkler of Fig. 5.4, the axial component of the moment-of-momentum equation (Eq. 5.42) is, from Eqs. 5.44 and 5.45

$$-r_2 V_{\theta 2} \dot{m} = T_{\text{shaft}} \quad (5.46)$$



■ FIGURE 5.5 Right-hand rule convention.

We interpret  $T_{\text{shaft}}$  being a negative quantity from Eq. 5.46 to mean that the shaft torque actually opposes the rotation of the sprinkler arms as shown in Fig. 5.4. The shaft torque,  $T_{\text{shaft}}$ , opposes rotation in all turbine devices.

We could evaluate the shaft power,  $\dot{W}_{\text{shaft}}$ , associated with shaft torque,  $T_{\text{shaft}}$ , by forming the product of  $T_{\text{shaft}}$  and the rotational speed of the shaft,  $\omega$ . (We use the notation that  $W$  = work,  $(\cdot) = d(\cdot)/dt$ , and thus  $\dot{W}$  = power.) Thus, from Eq. 5.46 we get

$$\dot{W}_{\text{shaft}} = T_{\text{shaft}} \omega = -r_2 V_{\theta 2} \dot{m} \omega \quad (5.47)$$

Since  $r_2 \omega$  is the speed of each sprinkler nozzle,  $U$ , we can also state Eq. 5.47 in the form

$$\dot{W}_{\text{shaft}} = -U_2 V_{\theta 2} \dot{m} \quad (5.48)$$

Shaft work per unit mass,  $w_{\text{shaft}}$ , is equal to  $\dot{W}_{\text{shaft}}/\dot{m}$ . Dividing Eq. 5.48 by the mass flowrate,  $\dot{m}$ , we obtain

$$w_{\text{shaft}} = -U_2 V_{\theta 2} \quad (5.49)$$

Negative shaft work as in Eqs. 5.47, 5.48, and 5.49 is work out of the control volume, i.e., work done by the fluid on the rotor and thus its shaft.

The principles associated with this sprinkler example can be extended to handle most simplified turbomachine flows. The fundamental technique is not difficult. However, the geometry of some turbomachine flows is quite complicated.

Example 5.18 further illustrates how the axial component of the moment-of-momentum equation (Eq. 5.46) can be used.

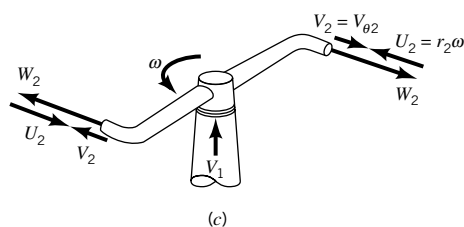
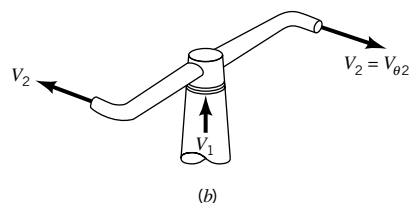
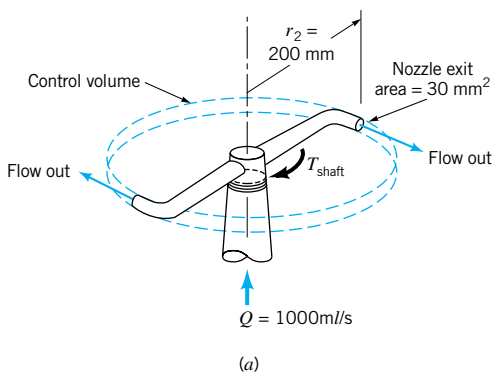


#### V5.6 Impulse-type lawn sprinkler

### EXAMPLE 5.18

Water enters a rotating lawn sprinkler through its base at the steady rate of 1000 ml/s as sketched in Fig. E5.18. The exit area of each of the two nozzles is 30 mm<sup>2</sup>, and the flow leaving each nozzle is in the tangential direction. The radius from the axis of rotation to the centerline of each nozzle is 200 mm.

- Determine the resisting torque required to hold the sprinkler head stationary.
- Determine the resisting torque associated with the sprinkler rotating with a constant speed of 500 rev/min.
- Determine the speed of the sprinkler if no resisting torque is applied.



■ FIGURE E5.18

## SOLUTION

To solve parts (a), (b), and (c) of this example we can use the same fixed and nondeforming, disk shaped control volume illustrated in Fig. 5.4. As is indicated in Fig. E.5.18a, the only axial torque considered is the one resisting motion,  $T_{\text{shaft}}$ .

When the sprinkler head is held stationary as specified in part (a) of this example problem, the velocities of the fluid entering and leaving the control volume are as shown in Fig. E.18b. Equation 5.46 applies to the contents of this control volume. Thus,

$$T_{\text{shaft}} = -r_2 V_{\theta 2} \dot{m} \quad (1)$$

Since the control volume is fixed and nondeforming and the flow exiting from each nozzle is tangential,

$$V_{\theta 2} = V_2 \quad (2)$$

Equations 1 and 2 give

$$T_{\text{shaft}} = -r_2 V_2 \dot{m} \quad (3)$$

In Example 5.7, we ascertained that  $V_2 = 16.7 \text{ m/s}$ . Thus, from Eq. 3,

$$T_{\text{shaft}} = - \frac{(200 \text{ mm})(16.7 \text{ m/s})(1000 \text{ ml/s})(10^{-3} \text{ m}^3/\text{liter})(999 \text{ kg/m}^3)[1 \text{ (N/kg)/(m/s}^2)]}{(1000 \text{ mm/m})(1000 \text{ ml/liter})}$$

or

$$T_{\text{shaft}} = -3.34 \text{ N} \cdot \text{m} \quad (\text{Ans})$$

When the sprinkler is rotating at a constant speed of 500 rpm, the flow field in the control volume is unsteady but cyclical. Thus, the flow field is steady in the mean. The velocities of the flow entering and leaving the control volume are as indicated in Fig. E5.18c. The absolute velocity of the fluid leaving each nozzle,  $V_2$ , is, from Eq. 5.43,

$$V_2 = W_2 - U_2 \quad (4)$$

where  $W_2 = 16.7 \text{ m/s}$  as determined in Example 5.7. The speed of the nozzle,  $U_2$ , is obtained from

$$U_2 = r_2 \omega \quad (5)$$

Application of the axial component of the moment-of-momentum equation (Eq. 5.46) leads again to Eq. 3. From Eqs. 4 and 5,

$$V_2 = 16.7 \text{ m/s} - r_2 \omega = 16.7 \text{ m/s} - \frac{(200 \text{ mm})(500 \text{ rev/min})(2\pi \text{ rad/rev})}{(1000 \text{ mm/m})(60 \text{ s/min})}$$

or

$$V_2 = 16.7 \text{ m/s} - 10.5 \text{ m/s} = 6.2 \text{ m/s}$$

Thus, using Eq. 3, we get

$$T_{\text{shaft}} = - \frac{(200 \text{ mm})(6.2 \text{ m/s})(1000 \text{ ml/s})(10^{-3} \text{ m}^3/\text{liter})(999 \text{ kg/m}^3)[1 \text{ (N/kg)/(m/s}^2)]}{(1000 \text{ mm/m})(1000 \text{ ml/liter})}$$

or

$$T_{\text{shaft}} = -1.24 \text{ N} \cdot \text{m} \quad (\text{Ans})$$

Note that the resisting torque associated with sprinkler head rotation is much less than the resisting torque that is required to hold the sprinkler stationary.

When no resisting torque is applied to the rotating sprinkler head, a maximum constant speed of rotation will occur as demonstrated below. Application of Eqs. 3, 4, and 5 to the contents of the control volume results in

$$T_{\text{shaft}} = -r_2(W_2 - r_2\omega)\dot{m} \quad (6)$$

For no resisting torque ( $T_{\text{shaft}} = 0$ ), Eq. 6 yields

$$\omega = \frac{W_2}{r_2} \quad (7)$$

In Example 5.7, we learned that the relative velocity of the fluid leaving each nozzle,  $W_2$ , is the same regardless of the speed of rotation of the sprinkler head,  $\omega$ , as long as the mass flowrate of the fluid,  $\dot{m}$ , remains constant. Thus, by using Eq. 7 we obtain

$$\omega = \frac{W_2}{r_2} = \frac{(16.7 \text{ m/s})(1000 \text{ mm/m})}{(200 \text{ mm})} = 83.5 \text{ rad/s}$$

or

$$\omega = \frac{(83.5 \text{ rad/s})(60 \text{ s/min})}{2\pi \text{ rad/rev}} = 797 \text{ rpm} \quad (\text{Ans})$$

For this condition ( $T_{\text{shaft}} = 0$ ), the water both enters and leaves the control volume with zero angular momentum.

In summary, we observe that the resisting torque associated with rotation is less than the torque required to hold a rotor stationary. Even in the absence of a resisting torque, the rotor maximum speed is finite.

When the moment-of-momentum equation (Eq. 5.42) is applied to a more general, one-dimensional flow through a rotating machine, we obtain

$$T_{\text{shaft}} = (-\dot{m}_{\text{in}})(\pm r_{\text{in}}V_{\theta\text{in}}) + \dot{m}_{\text{out}}(\pm r_{\text{out}}V_{\theta\text{out}}) \quad (5.50)$$

by applying the same kind of analysis used with the sprinkler of Fig. 5.4. The “−” is used with mass flowrate into the control volume,  $\dot{m}_{\text{in}}$ , and the “+” is used with mass flowrate out of the control volume,  $\dot{m}_{\text{out}}$ , to account for the sign of the dot product,  $\mathbf{V} \cdot \hat{\mathbf{n}}$ , involved. Whether “+” or “−” is used with the  $rV_\theta$  product depends on the direction of  $(\mathbf{r} \times \mathbf{V})_{\text{axial}}$ . A simple way to determine the sign of the  $rV_\theta$  product is to compare the direction of  $V_\theta$  and the blade speed,  $U$ . If  $V_\theta$  and  $U$  are in the same direction, then the  $rV_\theta$  product is positive. If  $V_\theta$  and  $U$  are in opposite directions, the  $rV_\theta$  product is negative. The sign of the shaft torque is “+” if  $T_{\text{shaft}}$  is in the same direction along the axis of rotation as  $\omega$ , and “−” otherwise.

The shaft power,  $\dot{W}_{\text{shaft}}$ , is related to shaft torque,  $T_{\text{shaft}}$ , by

$$\dot{W}_{\text{shaft}} = T_{\text{shaft}} \omega \quad (5.51)$$

Thus, using Eqs. 5.50 and 5.51 with a “+” sign for  $T_{\text{shaft}}$  in Eq. 5.50, we obtain

$$\dot{W}_{\text{shaft}} = (-\dot{m}_{\text{in}})(\pm r_{\text{in}}\omega V_{\theta\text{in}}) + \dot{m}_{\text{out}}(\pm r_{\text{out}}\omega V_{\theta\text{out}}) \quad (5.52)$$

or since  $r\omega = U$

$$\dot{W}_{\text{shaft}} = (-\dot{m}_{\text{in}})(\pm U_{\text{in}}V_{\theta\text{in}}) + \dot{m}_{\text{out}}(\pm U_{\text{out}}V_{\theta\text{out}}) \quad (5.53)$$

*The angular momentum equation is used to obtain torque and power for rotating machines.*

The “+” is used for the  $UV_\theta$  product when  $U$  and  $V_\theta$  are in the same direction; the “−” is used when  $U$  and  $V_\theta$  are in opposite directions. Also, since  $+T_{\text{shaft}}$  was used to obtain Eq. 5.53, when  $\dot{W}_{\text{shaft}}$  is positive, power is into the control volume (e.g., pump), and when  $\dot{W}_{\text{shaft}}$  is negative, power is out of the control volume (e.g., turbine).

The shaft work per unit mass,  $w_{\text{shaft}}$ , can be obtained from the shaft power,  $\dot{W}_{\text{shaft}}$ , by dividing Eq. 5.53 by the mass flowrate,  $\dot{m}$ . By conservation of mass,

$$\dot{m} = \dot{m}_{\text{in}} = \dot{m}_{\text{out}}$$

From Eq. 5.53, we obtain

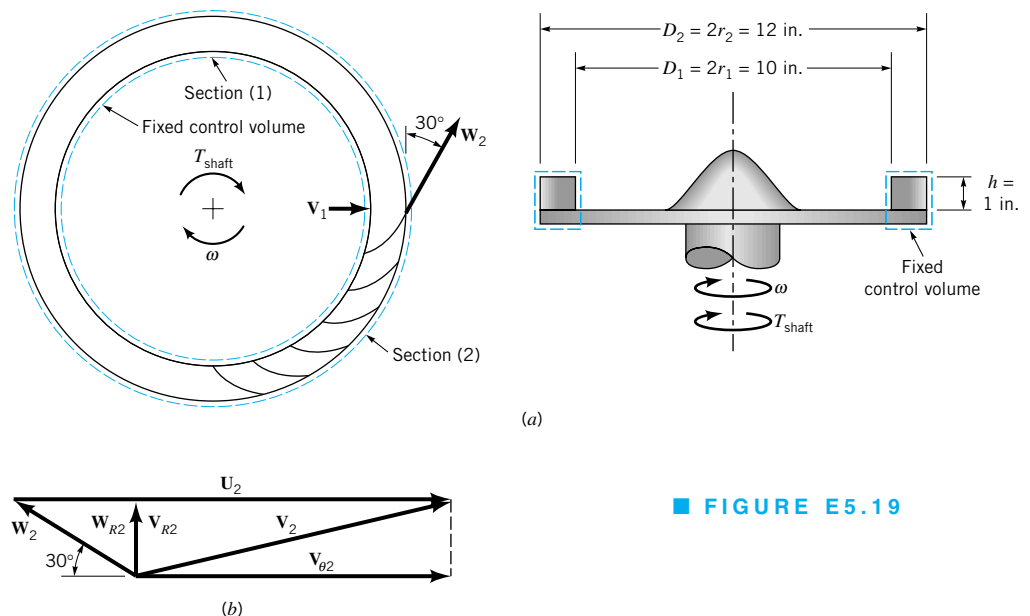
$$w_{\text{shaft}} = -(\pm U_{\text{in}} V_{\theta \text{in}}) + (\pm U_{\text{out}} V_{\theta \text{out}}) \quad (5.54)$$

The application of Eqs. 5.50, 5.53, and 5.54 is demonstrated in Example 5.19. More examples of the application of Eqs. 5.50, 5.53, and 5.54 are included in Chapter 12.

*The shaft work per unit mass involves only the blade velocity and the tangential fluid velocity.*

## EXAMPLE 5.19

An air fan has a bladed rotor of 12-in. outside diameter and 10-in. inside diameter as illustrated in Fig. E5.19a. The height of each rotor blade is constant at 1 in. from blade inlet to outlet. The flowrate is steady, on a time-average basis, at  $230 \text{ ft}^3/\text{min}$ , and the absolute velocity of the air at blade inlet,  $\mathbf{V}_1$ , is radial. The blade discharge angle is  $30^\circ$  from the tangential direction. If the rotor rotates at a constant speed of 1725 rpm, estimate the power required to run the fan.



■ FIGURE E5.19

## SOLUTION

We select a fixed and nondeforming control volume that includes the rotating blades and the fluid within the blade row at an instant, as shown with a dashed line in Fig. E5.19a. The flow within this control volume is cyclical, but steady in the mean. The only torque we consider is the driving shaft torque,  $T_{\text{shaft}}$ . This torque is provided by a motor. We assume that the entering and leaving flows are each represented by uniformly distributed velocities and

flow properties. Since shaft power is sought, Eq. 5.53 is appropriate. Application of Eq. 5.53 to the contents of the control volume in Fig. E5.19 gives

$$\dot{W}_{\text{shaft}} = (-\dot{m}_1)(\pm U_1 V_{\theta 1}) + \dot{m}_2(\pm U_2 V_{\theta 2}) \quad (1)$$

↗ 0 ( $\mathbf{V}_1$  is radial)

From Eq. 1 we see that to calculate fan power, we need mass flowrate,  $\dot{m}$ , rotor exit blade velocity,  $U_2$ , and fluid tangential velocity at blade exit,  $V_{\theta 2}$ . The mass flowrate,  $\dot{m}$ , is easily obtained from Eq. 5.6 as

$$\dot{m} = \rho Q = \frac{(2.38 \times 10^{-3} \text{ slug/ft}^3)(230 \text{ ft}^3/\text{min})}{(60 \text{ s/min})} = 0.00912 \text{ slug/s} \quad (2)$$

The rotor exit blade speed,  $U_2$ , is

$$U_2 = r_2 \omega = \frac{(6 \text{ in.})(1725 \text{ rpm})(2\pi \text{ rad/rev})}{(12 \text{ in./ft})(60 \text{ s/min})} = 90.3 \text{ ft/s} \quad (3)$$

To determine the fluid tangential speed at the fan rotor exit,  $V_{\theta 2}$ , we use Eq. 5.43 to get

$$\mathbf{V}_2 = \mathbf{W}_2 + \mathbf{U}_2 \quad (4)$$

The vector addition of Eq. 4 is shown in the form of a “velocity triangle” in Fig. E5.19b. From Fig. E5.19b, we can see that

$$V_{\theta 2} = U_2 - W_2 \cos 30^\circ \quad (5)$$

To solve Eq. 5 for  $V_{\theta 2}$  we need a value of  $W_2$ , in addition to the value of  $U_2$  already determined (Eq. 3). To get  $W_2$ , we recognize that

$$W_2 \sin 30^\circ = V_{R2} \quad (6)$$

where  $V_{R2}$  is the radial component of either  $\mathbf{W}_2$  or  $\mathbf{V}_2$ . Also, using Eq. 5.6, we obtain

$$\dot{m} = \rho A_2 V_{R2} \quad (7)$$

or since

$$A_2 = 2\pi r_2 h \quad (8)$$

where  $h$  is the blade height, Eqs. 7 and 8 combine to form

$$\dot{m} = \rho 2\pi r_2 h V_{R2} \quad (9)$$

Taking Eqs. 6 and 9 together we get

$$W_2 = \frac{\dot{m}}{\rho 2\pi r_2 h \sin 30^\circ} \quad (10)$$

Substituting known values into Eq. 10, we obtain

$$W_2 = \frac{(0.00912 \text{ slugs/s})(12 \text{ in./ft})(12 \text{ in./ft})}{(2.38 \times 10^{-3} \text{ slugs/ft}^3)2\pi(6 \text{ in.})(1 \text{ in.}) \sin 30^\circ} = 29.3 \text{ ft/s}$$

By using this value of  $W_2$  in Eq. 5 we get

$$\begin{aligned} V_{\theta 2} &= U_2 - W_2 \cos 30^\circ \\ &= 90.3 \text{ ft/s} - (29.3 \text{ ft/s})(0.866) = 64.9 \text{ ft/s} \end{aligned}$$

Equation 1 can now be used to obtain

$$\dot{W}_{\text{shaft}} = \dot{m} U_2 V_{\theta 2} = \frac{(0.00912 \text{ slug/s})(90.3 \text{ ft/s})(64.9 \text{ ft/s})}{[1 (\text{slug} \cdot \text{ft/s}^2)/\text{lb}][550 (\text{ft} \cdot \text{lb})/(\text{hp} \cdot \text{s})]}$$

or

$$\dot{W}_{\text{shaft}} = 0.0972 \text{ hp} \quad (\text{Ans})$$

Note that the “+” was used with the  $U_2 V_{\theta 2}$  product because  $U_2$  and  $V_{\theta 2}$  are in the same direction. This result, 0.0972 hp, is the power that needs to be delivered through the fan shaft for the given conditions. Ideally, all of this power would go into the flowing air. However, because of fluid friction, only some of this power will produce a useful effect (e.g., pressure rise) in the air. How much useful effect depends on the efficiency of the energy transfer between the fan blades and the fluid. This important aspect of energy transfer is covered in [Section 5.3.5](#).

## 5.3 First Law of Thermodynamics—The Energy Equation

### 5.3.1 Derivation of the Energy Equation

The first law of thermodynamics for a system is, in words

time rate of increase of the total stored energy of the system	=	net time rate of energy addition by heat transfer into the system	+	net time rate of energy addition by work transfer into the system
---	---	--	---	--

*The energy equation is a statement of the first law of thermodynamics.*

In symbolic form, this statement is

$$\frac{D}{Dt} \int_{\text{sys}} e \rho dV = \left( \sum \dot{Q}_{\text{in}} - \sum \dot{Q}_{\text{out}} \right)_{\text{sys}} + \left( \sum \dot{W}_{\text{in}} - \sum \dot{W}_{\text{out}} \right)_{\text{sys}}$$

or

$$\frac{D}{Dt} \int_{\text{sys}} e \rho dV = (\dot{Q}_{\text{net}} + \dot{W}_{\text{net}})_{\text{in, sys}} \quad (5.55)$$

Some of these variables deserve a brief explanation before proceeding further. The total stored energy per unit mass for each particle in the system,  $e$ , is related to the internal energy per unit mass,  $\tilde{u}$ , the kinetic energy per unit mass,  $V^2/2$ , and the potential energy per unit mass,  $gz$ , by the equation

$$e = \tilde{u} + \frac{V^2}{2} + gz \quad (5.56)$$

The net rate of heat transfer into the system is denoted with  $\dot{Q}_{\text{net in}}$ , and the net rate of work transfer into the system is labeled  $\dot{W}_{\text{net in}}$ . Heat transfer and work transfer are considered “+” going into the system and “−” coming out.

Equation 5.55 is valid for inertial and noninertial reference systems. We proceed to develop the control volume statement of the first law of thermodynamics. For the control volume that is coincident with the system at an instant of time

$$(\dot{Q}_{\text{net}} + \dot{W}_{\text{net}})_{\text{in, sys}} = (\dot{Q}_{\text{net}} + \dot{W}_{\text{net}})_{\text{in, coincident control volume}} \quad (5.57)$$

Furthermore, for the system and the contents of the coincident control volume that is fixed and nondeforming, the Reynolds transport theorem (Eq. 4.19 with the parameter  $b$  set equal to  $e$ ) allows us to conclude that

$$\frac{D}{Dt} \int_{\text{sys}} e \rho dV = \frac{\partial}{\partial t} \int_{\text{cv}} e \rho dV + \int_{\text{cs}} e \rho \mathbf{V} \cdot \hat{\mathbf{n}} dA \quad (5.58)$$



or in words,

the time rate of increase of the total stored energy of the system	=	the time rate of in- crease of the total stored energy of the contents of the control volume	+	the net rate of flow of the total stored energy out of the control volume through the control surface
--	---	---	---	---

Combining Eqs. 5.55, 5.57, and 5.58 we get the control volume formula for the first law of thermodynamics:

*The energy equation involves stored energy, heat transfer, and work.*

$$\frac{\partial}{\partial t} \int_{cv} e \rho dV + \int_{cs} e \rho \mathbf{V} \cdot \hat{\mathbf{n}} dA = (\dot{Q}_{\text{net}} + \dot{W}_{\text{net}})_{\text{in}} \quad (5.59)$$

The total stored energy per unit mass,  $e$ , in Eq. 5.59 is for fluid particles entering, leaving, and within the control volume. Further explanation of the heat transfer and work transfer involved in this equation follows.

The heat transfer rate,  $\dot{Q}$ , represents all of the ways in which energy is exchanged between the control volume contents and surroundings because of a temperature difference. Thus, radiation, conduction, and/or convection are possible. Heat transfer into the control volume is considered positive, heat transfer out is negative. In many engineering applications, the process is *adiabatic*; the heat transfer rate,  $\dot{Q}$ , is zero. The net heat transfer rate,  $\dot{Q}_{\text{net in}}$ , can also be zero when  $\sum \dot{Q}_{\text{in}} - \sum \dot{Q}_{\text{out}} = 0$ .

The work transfer rate,  $\dot{W}$ , also called *power*, is positive when work is done on the contents of the control volume by the surroundings. Otherwise, it is considered negative. Work can be transferred across the control surface in several ways. In the following paragraphs, we consider some important forms of work transfer.

In many instances, work is transferred across the control surface by a moving shaft. In rotary devices such as turbines, fans, and propellers, a rotating shaft transfers work across that portion of the control surface that slices through the shaft. Even in reciprocating machines like positive displacement internal combustion engines and compressors that utilize piston-in-cylinder arrangements, a rotating crankshaft is used. Since work is the dot product of force and related displacement, rate of work (or power) is the dot product of force and related displacement per unit time. For a rotating shaft, the power transfer,  $\dot{W}_{\text{shaft}}$ , is related to the shaft torque that causes the rotation,  $T_{\text{shaft}}$ , and the angular velocity of the shaft,  $\omega$ , by the relationship

$$\dot{W}_{\text{shaft}} = T_{\text{shaft}} \omega$$

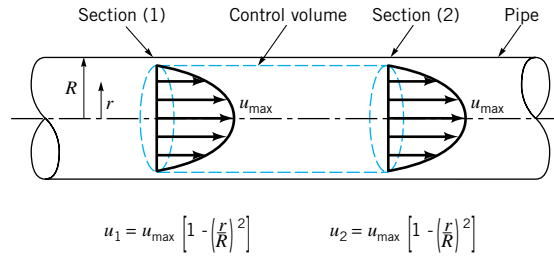
When the control surface cuts through the shaft material, the shaft torque is exerted by shaft material at the control surface. To allow for consideration of problems involving more than one shaft we use the notation

$$\dot{W}_{\text{shaft net in}} = \sum \dot{W}_{\text{shaft in}} - \sum \dot{W}_{\text{shaft out}} \quad (5.60)$$

Work transfer can also occur at the control surface when a force associated with fluid normal stress acts over a distance. Consider the simple pipe flow illustrated in Fig. 5.6 and the control volume shown. For this situation, the fluid normal stress,  $\sigma$ , is simply equal to the negative of fluid pressure,  $p$ , in all directions; that is,

$$\sigma = -p \quad (5.61)$$

This relationship can be used with varying amounts of approximation for many engineering problems (see [Chapter 6](#)).



■ **FIGURE 5.6** Simple, fully developed pipe flow.

The power transfer associated with normal stresses acting on a single fluid particle,  $\delta \dot{W}_{\text{normal stress}}$ , can be evaluated as the dot product of the normal stress force,  $\delta \mathbf{F}_{\text{normal stress}}$ , and the fluid particle velocity,  $\mathbf{V}$ , as

$$\delta \dot{W}_{\text{normal stress}} = \delta \mathbf{F}_{\text{normal stress}} \cdot \mathbf{V}$$

If the normal stress force is expressed as the product of local normal stress,  $\sigma = -p$ , and fluid particle surface area,  $\hat{\mathbf{n}} \delta A$ , the result is

$$\delta \dot{W}_{\text{normal stress}} = \sigma \hat{\mathbf{n}} \delta A \cdot \mathbf{V} = -p \hat{\mathbf{n}} \delta A \cdot \mathbf{V} = -p \mathbf{V} \cdot \hat{\mathbf{n}} \delta A$$

For all fluid particles on the control surface of Fig. 5.6 at the instant considered, power transfer due to fluid normal stress,  $\dot{W}_{\text{normal stress}}$ , is

$$\dot{W}_{\text{normal stress}} = \int_{\text{cs}} \sigma \mathbf{V} \cdot \hat{\mathbf{n}} dA = \int_{\text{cs}} -p \mathbf{V} \cdot \hat{\mathbf{n}} dA \quad (5.62)$$

Note that the value of  $\dot{W}_{\text{normal stress}}$  for particles on the wetted inside surface of the pipe is zero because  $\mathbf{V} \cdot \hat{\mathbf{n}}$  is zero there. Thus,  $\dot{W}_{\text{normal stress}}$  can be nonzero only where fluid enters and leaves the control volume. Although only a simple pipe flow was considered, Eq. 5.62 is quite general and the control volume used in this example can serve as a general model for other cases.

Work transfer can also occur at the control surface because of tangential stress forces. Rotating shaft work is transferred by tangential stresses in the shaft material. For a fluid particle, shear stress force power,  $\delta \dot{W}_{\text{tangential stress}}$ , can be evaluated as the dot product of tangential stress force,  $\delta \mathbf{F}_{\text{tangential stress}}$ , and the fluid particle velocity,  $\mathbf{V}$ . That is,

$$\delta \dot{W}_{\text{tangential stress}} = \delta \mathbf{F}_{\text{tangential stress}} \cdot \mathbf{V}$$

For the control volume of Fig. 5.6, the fluid particle velocity is zero everywhere on the wetted inside surface of the pipe. Thus, no tangential stress work is transferred across that portion of the control surface. Furthermore, where fluid crosses the control surface, the tangential stress force is perpendicular to the fluid particle velocity and therefore tangential stress work transfer is also zero there. In general, we select control volumes like the one of Fig. 5.6 and consider fluid tangential stress power transfer to be negligibly small.

Using the information we have developed about power, we can express the first law of thermodynamics for the contents of a control volume by combining Eqs. 5.59, 5.60, and 5.62 to obtain

$$\frac{\partial}{\partial t} \int_{\text{cv}} e \rho d\mathcal{V} + \int_{\text{cs}} e \rho \mathbf{V} \cdot \hat{\mathbf{n}} dA = \dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft net in}} - \int_{\text{cs}} p \mathbf{V} \cdot \hat{\mathbf{n}} dA \quad (5.63)$$

When the equation for total stored energy (Eq. 5.56) is considered with Eq. 5.63, we obtain the energy equation:

$$\frac{\partial}{\partial t} \int_{\text{cv}} e \rho d\mathcal{V} + \int_{\text{cs}} \left( \dot{u} + \frac{p}{\rho} + \frac{V^2}{2} + gz \right) \rho \mathbf{V} \cdot \hat{\mathbf{n}} dA = \dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft net in}} \quad (5.64)$$

*Work is transferred by rotating shafts, normal stresses, and tangential stresses.*

### 5.3.2 Application of the Energy Equation

In Eq. 5.64, the term  $\partial/\partial t \int_{\text{cv}} e \rho dV$  represents the time rate of change of the total stored energy,  $e$ , of the contents of the control volume. This term is zero when the flow is steady. This term is also zero in the mean when the flow is steady in the mean (cyclical).

In Eq. 5.64, the integrand of

$$\int_{\text{cs}} \left( \check{u} + \frac{p}{\rho} + \frac{V^2}{2} + gz \right) \rho \mathbf{V} \cdot \hat{\mathbf{n}} dA$$

can be nonzero only where fluid crosses the control surface ( $\mathbf{V} \cdot \hat{\mathbf{n}} \neq 0$ ). Otherwise,  $\mathbf{V} \cdot \hat{\mathbf{n}}$  is zero and the integrand is zero for that portion of the control surface. If the properties within parentheses,  $\check{u}$ ,  $p/\rho$ ,  $V^2/2$ , and  $gz$ , are all assumed to be uniformly distributed over the flow cross-sectional areas involved, the integration becomes simple and gives

$$\begin{aligned} \int_{\text{cs}} \left( \check{u} + \frac{p}{\rho} + \frac{V^2}{2} + gz \right) \rho \mathbf{V} \cdot \hat{\mathbf{n}} dA &= \sum_{\text{flow out}} \left( \check{u} + \frac{p}{\rho} + \frac{V^2}{2} + gz \right) \dot{m} \\ &\quad - \sum_{\text{flow in}} \left( \check{u} + \frac{p}{\rho} + \frac{V^2}{2} + gz \right) \dot{m} \end{aligned} \quad (5.65)$$

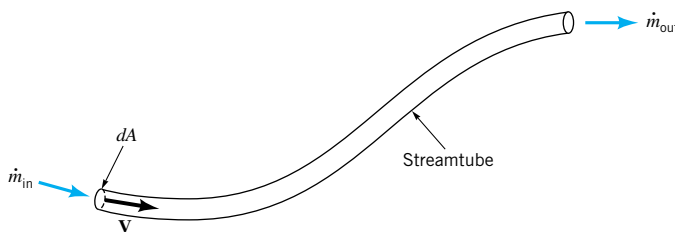
Furthermore, if there is only one stream entering and leaving the control volume, then

$$\begin{aligned} \int_{\text{cs}} \left( \check{u} + \frac{p}{\rho} + \frac{V^2}{2} + gz \right) \rho \mathbf{V} \cdot \hat{\mathbf{n}} dA &= \\ \left( \check{u} + \frac{p}{\rho} + \frac{V^2}{2} + gz \right)_{\text{out}} \dot{m}_{\text{out}} - \left( \check{u} + \frac{p}{\rho} + \frac{V^2}{2} + gz \right)_{\text{in}} \dot{m}_{\text{in}} \end{aligned} \quad (5.66)$$

*The energy equation can be simplified by using flow through a streamtube.*

Uniform flow as described above will occur in an infinitesimally small diameter streamtube as illustrated in Fig. 5.7. This kind of streamtube flow is representative of the steady flow of a particle of fluid along a pathline. We can also idealize actual conditions by disregarding nonuniformities in a finite cross section of flow. We call this one-dimensional flow and although such uniform flow rarely occurs in reality, the simplicity achieved with the one-dimensional approximation often justifies its use. More details about the effects of nonuniform distributions of velocities and other fluid flow variables are considered in Section 5.3.4 and in Chapters 8, 9, and 10.

If shaft work is involved, the flow must be unsteady, at least locally (see Refs. 1 and 2). The flow in any fluid machine that involves shaft work is unsteady within that machine. For example, the velocity and pressure at a fixed location near the rotating blades of a fan are unsteady. However, upstream and downstream of the machine, the flow may be steady. Most often shaft work is associated with flow that is unsteady in a recurring or cyclical way. On a time-average basis for flow that is one-dimensional, cyclical, and involves only one stream



■ FIGURE 5.7  
Streamtube flow.

of fluid entering and leaving the control volume, Eq. 5.64 can be simplified with the help of Eqs. 5.9 and 5.66 to form

$$\dot{m} \left[ \check{u}_{\text{out}} - \check{u}_{\text{in}} + \left( \frac{p}{\rho} \right)_{\text{out}} - \left( \frac{p}{\rho} \right)_{\text{in}} + \frac{V_{\text{out}}^2 - V_{\text{in}}^2}{2} + g(z_{\text{out}} - z_{\text{in}}) \right] = \dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft net in}} \quad (5.67)$$

We call Eq. 5.67 the *one-dimensional energy equation for steady-in-the-mean flow*. Note that Eq. 5.67 is valid for incompressible and compressible flows. Often, the fluid property called *enthalpy*,  $\check{h}$ , where

$$\check{h} = \check{u} + \frac{p}{\rho} \quad (5.68)$$

is used in Eq. 5.67. With enthalpy, the one-dimensional energy equation for steady-in-the-mean flow (Eq. 5.67) is

$$\dot{m} \left[ \check{h}_{\text{out}} - \check{h}_{\text{in}} + \frac{V_{\text{out}}^2 - V_{\text{in}}^2}{2} + g(z_{\text{out}} - z_{\text{in}}) \right] = \dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft net in}} \quad (5.69)$$

Equation 5.69 is often used for solving compressible flow problems. Examples 5.20 and 5.21 illustrate how Eqs. 5.67 and 5.69 can be used.

*The energy equation is sometimes written in terms of enthalpy.*

## EXAMPLE 5.20

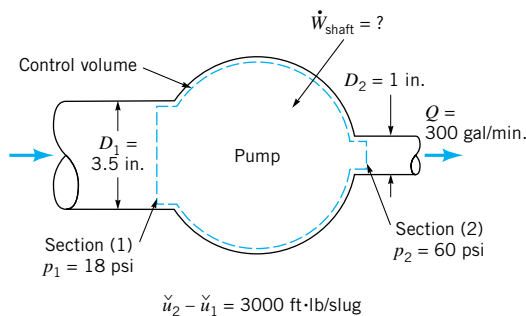
A pump delivers water at a steady rate of 300 gal/min as shown in Fig. E5.20. Just upstream of the pump [section (1)] where the pipe diameter is 3.5 in., the pressure is 18 psi. Just downstream of the pump [section (2)] where the pipe diameter is 1 in., the pressure is 60 psi. The change in water elevation across the pump is zero. The rise in internal energy of water,  $\check{u}_2 - \check{u}_1$ , associated with a temperature rise across the pump is 3000 ft · lb/slug. If the pumping process is considered to be adiabatic, determine the power (hp) required by the pump.

## SOLUTION

We include in our control volume the water contained in the pump between its entrance and exit sections. Application of Eq. 5.67 to the contents of this control volume on a time-average basis yields

$$\begin{aligned} \dot{m} \left[ \check{u}_2 - \check{u}_1 + \left( \frac{p}{\rho} \right)_2 - \left( \frac{p}{\rho} \right)_1 + \frac{V_2^2 - V_1^2}{2} + \cancel{g(z_2 - z_1)} \right] & \quad \begin{array}{l} 0 \text{ (no elevation change)} \\ \nearrow \end{array} \\ = \cancel{\dot{Q}_{\text{net in}}} + \dot{W}_{\text{shaft net in}} & \quad \begin{array}{l} 0 \text{ (adiabatic flow)} \\ \nearrow \end{array} \end{aligned} \quad (1)$$

We can solve directly for the power required by the pump,  $\dot{W}_{\text{shaft net in}}$ , from Eq. 1, after we first determine the mass flowrate,  $\dot{m}$ , the speed of flow into the pump,  $V_1$ , and the speed of the flow out of the pump,  $V_2$ . All other quantities in Eq. 1 are given in the problem statement.



■ FIGURE E5.20

From Eq. 5.6, we get

$$\dot{m} = \rho Q = \frac{(1.94 \text{ slugs/ft}^3)(300 \text{ gal/min})}{(7.48 \text{ gal/ft}^3)(60 \text{ s/min})} = 1.30 \text{ slugs/s} \quad (2)$$

Also from Eq. 5.6,

$$V = \frac{Q}{A} = \frac{Q}{\pi D^2/4}$$

so

$$V_1 = \frac{Q}{A_1} = \frac{(300 \text{ gal/min})4 (12 \text{ in./ft})^2}{(7.48 \text{ gal/ft}^3)(60 \text{ s/min})\pi(3.5 \text{ in.})^2} = 10.0 \text{ ft/s} \quad (3)$$

and

$$V_2 = \frac{Q}{A_2} = \frac{(300 \text{ gal/min})4 (12 \text{ in./ft})^2}{(7.48 \text{ gal/ft}^3)(60 \text{ s/min})\pi(1 \text{ in.})^2} = 123 \text{ ft/s} \quad (4)$$

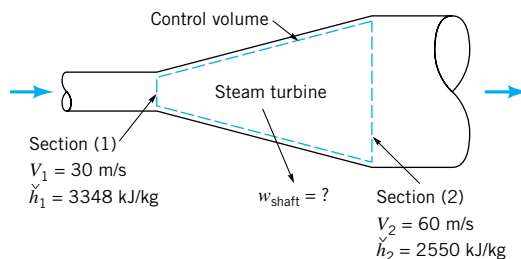
Substituting the values of Eqs. 2, 3, and 4 and values from the problem statement into Eq. 1 we obtain

$$\begin{aligned} \dot{W}_{\text{shaft net in}} &= (1.30 \text{ slugs/s}) \left[ (3000 \text{ ft} \cdot \text{lb/slug}) + \frac{(60 \text{ psi})(144 \text{ in.}^2/\text{ft}^2)}{(1.94 \text{ slugs/ft}^3)} \right. \\ &\quad \left. - \frac{(18 \text{ psi})(144 \text{ in.}^2/\text{ft}^2)}{(1.94 \text{ slugs/ft}^3)} + \frac{(123 \text{ ft/s})^2 - (10.0 \text{ ft/s})^2}{2[1 (\text{slug} \cdot \text{ft})/(\text{lb} \cdot \text{s}^2)]} \right] \\ &\quad \times \frac{1}{[550(\text{ft} \cdot \text{lb/s})/\text{hp}]} = 32.2 \text{ hp} \quad (\text{Ans}) \end{aligned}$$

Of the total 32.2 hp, internal energy change accounts for 7.09 hp, the pressure rise accounts for 7.37 hp, and the kinetic energy increase accounts for 17.8 hp.

## EXAMPLE 5.21

Steam enters a turbine with a velocity of 30 m/s and enthalpy,  $\check{h}_1$ , of 3348 kJ/kg (see Fig. E5.21). The steam leaves the turbine as a mixture of vapor and liquid having a velocity of 60 m/s and an enthalpy of 2550 kJ/kg. If the flow through the turbine is adiabatic and changes in elevation are negligible, determine the work output involved per unit mass of steam through-flow.



■ FIGURE E5.21

## SOLUTION

We use a control volume that includes the steam in the turbine from the entrance to the exit as shown in Fig. E5.21. Applying Eq. 5.69 to the steam in this control volume we get

$$\dot{m} \left[ \check{h}_2 - \check{h}_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) \right] = \dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft net in}} \quad (1)$$

0 (elevation change is negligible)  
0 (adiabatic flow)

The work output per unit mass of steam through-flow,  $w_{\text{shaft net in}}$ , can be obtained by dividing

Eq. 1 by the mass flow rate,  $\dot{m}$ , to obtain

$$w_{\text{shaft net in}} = \frac{\dot{W}_{\text{shaft net in}}}{\dot{m}} = \check{h}_2 - \check{h}_1 + \frac{V_2^2 - V_1^2}{2} \quad (2)$$

Since  $w_{\text{shaft net out}} = -w_{\text{shaft net in}}$ , we obtain

$$w_{\text{shaft net out}} = \check{h}_1 - \check{h}_2 + \frac{V_1^2 - V_2^2}{2}$$

or

$$w_{\text{shaft net out}} = 3348 \text{ kJ/kg} - 2550 \text{ kJ/kg} + \frac{[(30 \text{ m/s})^2 - (60 \text{ m/s})^2][1 \text{ J}/(\text{N} \cdot \text{m})]}{2[1 (\text{kg} \cdot \text{m})/(\text{N} \cdot \text{s}^2)](1000 \text{ J/kJ})}$$

Thus

$$w_{\text{shaft net out}} = 3348 \text{ kJ/kg} - 2550 \text{ kJ/kg} - 1.35 \text{ kJ/kg} = 797 \text{ kJ/kg} \quad (\text{Ans})$$

Note that in this particular example, the change in kinetic energy is small in comparison to the difference in enthalpy involved. This is often true in applications involving steam turbines. To determine the power output,  $\dot{W}_{\text{shaft}}$ , we must know the mass flowrate,  $\dot{m}$ .

If the flow is steady throughout, one-dimensional, and only one fluid stream is involved, then the shaft work is zero and the energy equation is

$$\dot{m} \left[ \check{u}_{\text{out}} - \check{u}_{\text{in}} + \left( \frac{p}{\rho} \right)_{\text{out}} - \left( \frac{p}{\rho} \right)_{\text{in}} + \frac{V_{\text{out}}^2 - V_{\text{in}}^2}{2} + g(z_{\text{out}} - z_{\text{in}}) \right] = \dot{Q}_{\text{net in}} \quad (5.70)$$

We call Eq. 5.70 the *one-dimensional, steady flow energy equation*. This equation is valid for incompressible and compressible flows. For compressible flows, enthalpy is most often used in the one-dimensional, steady flow energy equation and, thus, we have

$$\dot{m} \left[ \check{h}_{\text{out}} - \check{h}_{\text{in}} + \frac{V_{\text{out}}^2 - V_{\text{in}}^2}{2} + g(z_{\text{out}} - z_{\text{in}}) \right] = \dot{Q}_{\text{net in}} \quad (5.71)$$

An example of the application of Eq. 5.70 follows.

## EXAMPLE 5.22

A 500-ft waterfall involves steady flow from one large body of water to another. Determine the temperature change associated with this flow.

### SOLUTION

To solve this problem we consider a control volume consisting of a small cross sectional streamtube from the nearly motionless surface of the upper body of water to the nearly motionless surface of the lower body of water as is sketched in Fig. E5.22. We need to determine  $T_2 - T_1$ . This temperature change is related to the change of internal energy of the water,  $\check{u}_2 - \check{u}_1$ , by the relationship

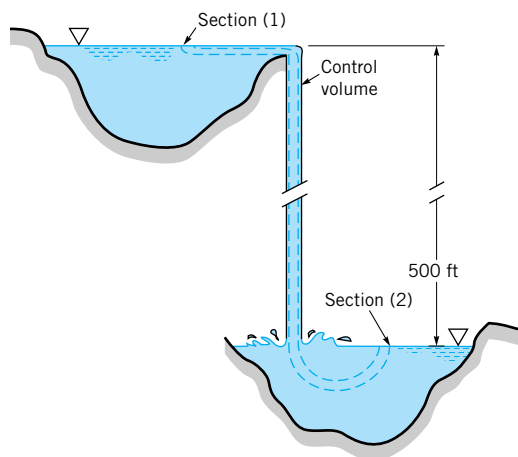
$$T_2 - T_1 = \frac{\check{u}_2 - \check{u}_1}{\check{c}} \quad (1)$$

where  $\check{c} = 1 \text{ Btu}/(\text{lbm} \cdot ^\circ\text{R})$  is the specific heat of water. The application of Eq. 5.70 to the contents of this control volume leads to

$$\dot{m} \left[ \check{u}_2 + \check{u}_1 + \left( \frac{p}{\rho} \right)_2 - \left( \frac{p}{\rho} \right)_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) \right] = \dot{Q}_{\text{net in}} \quad (2)$$

We assume that the flow is adiabatic. Thus  $\dot{Q}_{\text{net in}} = 0$ . Also,

$$\left( \frac{p}{\rho} \right)_1 = \left( \frac{p}{\rho} \right)_2 \quad (3)$$



■ FIGURE E5.22

because the flow is incompressible and atmospheric pressure prevails at sections (1) and (2). Furthermore,

$$V_1 = V_2 = 0 \quad (4)$$

because the surface of each large body of water is considered motionless. Thus, Eqs. 1 through 4 combine to yield

$$T_2 - T_1 = \frac{g(z_1 - z_2)}{\check{c}}$$

or

$$T_2 - T_1 = \frac{(32.2 \text{ ft/s}^2)(500 \text{ ft})}{[1 \text{ Btu}/(\text{lbm} \cdot ^\circ\text{R})][32.2 (\text{lbm} \cdot \text{ft})/(\text{lb} \cdot \text{s}^2)](778 \text{ ft} \cdot \text{lb/Btu})} = 0.643 ^\circ\text{R} \quad (\text{Ans})$$

Note that it takes a considerable change of potential energy to produce even a small increase in temperature.

A form of the energy equation that is most often used to solve incompressible flow problems is developed in the next section.

### 5.3.3 Comparison of the Energy Equation with the Bernoulli Equation

When the one-dimensional energy equation for steady-in-the-mean flow, Eq. 5.67, is applied to a flow that is steady, Eq. 5.67 becomes the one-dimensional, steady-flow energy equation, Eq. 5.70. The only difference between Eq. 5.67 and Eq. 5.70 is that shaft power,  $\dot{W}_{\text{shaft net in}}$ , is zero if the flow is steady throughout the control volume (fluid machines involve locally unsteady flow). If in addition to being steady, the flow is incompressible, we get from Eq. 5.70

$$\dot{m} \left[ \check{u}_{\text{out}} - \check{u}_{\text{in}} + \frac{p_{\text{out}}}{\rho} - \frac{p_{\text{in}}}{\rho} + \frac{V_{\text{out}}^2 - V_{\text{in}}^2}{2} + g(z_{\text{out}} - z_{\text{in}}) \right] = \dot{Q}_{\text{net in}} \quad (5.72)$$

Dividing Eq. 5.72 by the mass flowrate,  $\dot{m}$ , and rearranging terms we obtain

$$\frac{p_{\text{out}}}{\rho} + \frac{V_{\text{out}}^2}{2} + gz_{\text{out}} = \frac{p_{\text{in}}}{\rho} + \frac{V_{\text{in}}^2}{2} + gz_{\text{in}} - (\check{u}_{\text{out}} - \check{u}_{\text{in}} - q_{\text{net in}}) \quad (5.73)$$

where

$$q_{\text{net in}} = \frac{\dot{Q}_{\text{net in}}}{\dot{m}}$$

is the heat transfer rate per mass flowrate, or heat transfer per unit mass. Note that Eq. 5.73 involves energy per unit mass and is applicable to one-dimensional flow of a single stream of fluid between two sections or flow along a streamline between two sections.

If the steady, incompressible flow we are considering also involves negligible viscous effects (frictionless flow), then the Bernoulli equation, Eq. 3.7, can be used to describe what happens between two sections in the flow as

$$p_{\text{out}} + \frac{\rho V_{\text{out}}^2}{2} + \gamma z_{\text{out}} = p_{\text{in}} + \frac{\rho V_{\text{in}}^2}{2} + \gamma z_{\text{in}} \quad (5.74)$$

*With certain assumptions, the energy equation and the Bernoulli equation are similar.*



where  $\gamma = \rho g$  is the specific weight of the fluid. To get Eq. 5.74 in terms of energy per unit mass, so that it can be compared directly with Eq. 5.73, we divide Eq. 5.74 by density,  $\rho$ , and obtain

$$\frac{p_{\text{out}}}{\rho} + \frac{V_{\text{out}}^2}{2} + gz_{\text{out}} = \frac{p_{\text{in}}}{\rho} + \frac{V_{\text{in}}^2}{2} + gz_{\text{in}} \quad (5.75)$$

A comparison of Eqs. 5.73 and 5.75 prompts us to conclude that

$$\check{u}_{\text{out}} - \check{u}_{\text{in}} - q_{\text{net in}} = 0 \quad (5.76)$$

when the steady incompressible flow is frictionless. For steady incompressible flow with friction, we learn from experience that

$$\check{u}_{\text{out}} - \check{u}_{\text{in}} - q_{\text{net in}} > 0 \quad (5.77)$$

In Eqs. 5.73 and 5.75, we can consider the combination of variables

$$\frac{p}{\rho} + \frac{V^2}{2} + gz$$

*A loss of useful energy occurs because of friction.*

as equal to *useful* or *available energy*. Thus, from inspection of Eqs. 5.73 and 5.75, we can conclude that  $\check{u}_{\text{out}} - \check{u}_{\text{in}} - q_{\text{net in}}$  represents the *loss* of useful or available energy that occurs in an incompressible fluid flow because of friction. In equation form we have

$$\check{u}_{\text{out}} - \check{u}_{\text{in}} - q_{\text{net in}} = \text{loss} \quad (5.78)$$

For a frictionless flow, Eqs. 5.73 and 5.75 tell us that loss equals zero.

It is often convenient to express Eq. 5.73 in terms of loss as

$$\frac{p_{\text{out}}}{\rho} + \frac{V_{\text{out}}^2}{2} + gz_{\text{out}} = \frac{p_{\text{in}}}{\rho} + \frac{V_{\text{in}}^2}{2} + gz_{\text{in}} - \text{loss} \quad (5.79)$$

An example of the application of Eq. 5.79 follows.

## EXAMPLE 5.23

Compare the volume flowrates associated with two different vent configurations, a cylindrical hole in the wall having a diameter of 120 mm and the same diameter cylindrical hole in the wall but with a well-rounded entrance (see Fig. E5.23). The room pressure is held constant at 1.0 kPa above atmospheric pressure. Both vents exhaust into the atmosphere. As discussed in Section 8.4.2, the loss in available energy associated with flow through the cylindrical vent from the room to the vent exit is  $0.5V_2^2/2$  where  $V_2$  is the uniformly distributed exit velocity of air. The loss in available energy associated with flow through the rounded entrance vent from the room to the vent exit is  $0.05V_2^2/2$ , where  $V_2$  is the uniformly distributed exit velocity of air.

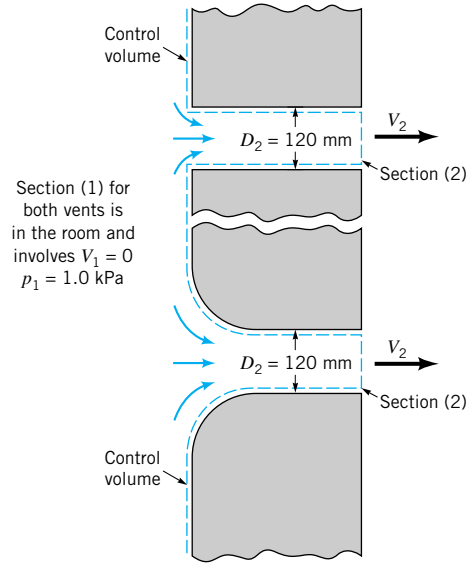
## SOLUTION

We use the control volume for each vent sketched in Fig. E5.23. What is sought is the flowrate,  $Q = A_2 V_2$ , where  $A_2$  is the vent exit cross-sectional area, and  $V_2$  is the uniformly distributed exit velocity. For both vents, application of Eq. 5.79 leads to

$$\frac{p_2}{\rho} + \frac{V_2^2}{2} + \overbrace{gz_2}^{0 \text{ (no elevation change)}} = \frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1 - \text{loss}_2$$

$\swarrow$   
 $0 \text{ } (V_1 \approx 0)$

(1)



■ FIGURE E5.23

where  ${}_1\text{loss}_2$  is the loss between sections (1) and (2). Solving Eq. 1 for  $V_2$  we get

$$V_2 = \sqrt{2 \left[ \left( \frac{p_1 - p_2}{\rho} \right) - {}_1\text{loss}_2 \right]} \quad (2)$$

Since

$${}_1\text{loss}_2 = K_L \frac{V_2^2}{2} \quad (3)$$

where  $K_L$  is the loss coefficient ( $K_L = 0.5$  and  $0.05$  for the two vent configurations involved), we can combine Eqs. 2 and 3 to get

$$V_2 = \sqrt{2 \left[ \left( \frac{p_1 - p_2}{\rho} \right) - K_L \frac{V_2^2}{2} \right]} \quad (4)$$

Solving Eq. 4 for  $V_2$  we obtain

$$V_2 = \sqrt{\frac{p_1 - p_2}{\rho[(1 + K_L)/2]}} \quad (5)$$

Therefore, for flowrate,  $Q$ , we obtain

$$Q = A_2 V_2 = \frac{\pi D_2^2}{4} \sqrt{\frac{p_1 - p_2}{\rho[(1 + K_L)/2]}} \quad (6)$$

For the rounded entrance cylindrical vent, Eq. 6 gives

$$Q = \frac{\pi(120 \text{ mm})^2}{4(1000 \text{ mm/m})^2} \sqrt{\frac{(1.0 \text{ kPa})(1000 \text{ Pa/kPa})[1(\text{N/m}^2)/(\text{Pa})]}{(1.23 \text{ kg/m}^3)[(1 + 0.05)/2][1(\text{N} \cdot \text{s}^2)/(\text{kg} \cdot \text{m})]}}$$

or

$$Q = 0.445 \text{ m}^3/\text{s}$$

(Ans)

For the cylindrical vent, Eq. 6 gives us

$$Q = \frac{\pi(120 \text{ mm})^2}{4(1000 \text{ mm/m})^2} \sqrt{\frac{(1.0 \text{ kPa})(1000 \text{ Pa/kPa})[1(\text{N/m}^2)/(\text{Pa})]}{(1.23 \text{ kg/m}^3)[(1 + 0.5)/2][1(\text{N} \cdot \text{s}^2)/(\text{kg} \cdot \text{m})]}}$$

or

$$Q = 0.372 \text{ m}^3/\text{s} \quad (\text{Ans})$$

Note that the rounded entrance vent passes more air than the cylindrical vent because the loss associated with the rounded entrance vent is less than with the cylindrical vent.

An important group of fluid mechanics problems involves one-dimensional, incompressible, steady-in-the-mean flow with friction and shaft work. Included in this category are constant density flows through pumps, blowers, fans, and turbines. For this kind of flow, Eq. 5.67 becomes

$$\dot{m} \left[ \check{u}_{\text{out}} - \check{u}_{\text{in}} + \frac{p_{\text{out}}}{\rho} - \frac{p_{\text{in}}}{\rho} + \frac{V_{\text{out}}^2 - V_{\text{in}}^2}{2} + g(z_{\text{out}} - z_{\text{in}}) \right] = \dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft net in}} \quad (5.80)$$

Dividing Eq. 5.80 by mass flowrate and using the work per unit mass,  $w_{\text{shaft net in}} = \dot{W}_{\text{shaft net in}}/\dot{m}$ , we obtain

$$\frac{p_{\text{out}}}{\rho} + \frac{V_{\text{out}}^2}{2} + gz_{\text{out}} = \frac{p_{\text{in}}}{\rho} + \frac{V_{\text{in}}^2}{2} + gz_{\text{in}} + w_{\text{shaft net in}} - (\check{u}_{\text{out}} - \check{u}_{\text{in}} - q_{\text{net in}}) \quad (5.81)$$

If the flow is steady throughout, Eq. 5.81 becomes identical to Eq. 5.73, and the previous observation that  $\check{u}_{\text{out}} - \check{u}_{\text{in}} - q_{\text{net in}}$  equals the loss of available energy is valid. Thus, we conclude that Eq. 5.81 can be expressed as

$$\frac{p_{\text{out}}}{\rho} + \frac{V_{\text{out}}^2}{2} + gz_{\text{out}} = \frac{p_{\text{in}}}{\rho} + \frac{V_{\text{in}}^2}{2} + gz_{\text{in}} + w_{\text{shaft net in}} - \text{loss} \quad (5.82)$$

The mechanical energy equation can be written in terms of energy per unit mass.



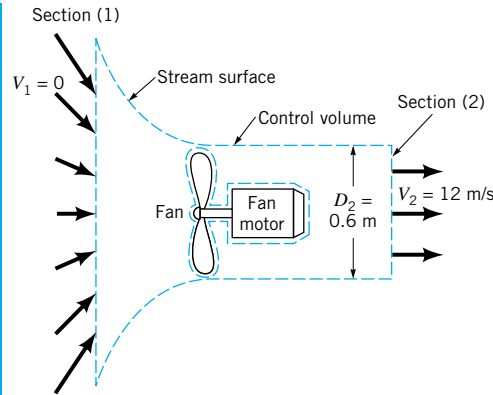
V5.7 Energy transfer

This is a form of the energy equation for steady-in-the-mean flow that is often used for incompressible flow problems. It is sometimes called the *mechanical energy* equation or the *extended Bernoulli* equation. Note that Eq. 5.82 involves energy per unit mass ( $\text{ft} \cdot \text{lb}/\text{slug} = \text{ft}^2/\text{s}^2$  or  $\text{N} \cdot \text{m} = \text{m}^2/\text{s}^2$ ).

According to Eq. 5.82, when the shaft work is into the control volume, as for example with a pump, a larger amount of loss will result in more shaft work being required for the same rise in available energy. Similarly, when the shaft work is out of the control volume (e.g., a turbine), a larger loss will result in less shaft work out for the same drop in available energy. Designers spend a great deal of effort on minimizing losses in fluid flow components. The following examples demonstrate why losses should be kept as small as possible in fluid systems.

## EXAMPLE 5.24

An axial-flow ventilating fan driven by a motor that delivers 0.4 kW of power to the fan blades produces a 0.6-m-diameter axial stream of air having a speed of 12 m/s. The flow upstream of the fan involves negligible speed. Determine how much of the work to the air actually produces a useful effect, that is, a rise in available energy and estimate the fluid mechanical efficiency of this fan.



■ FIGURE E5.24

## SOLUTION

We select a fixed and nondeforming control volume as is illustrated in Fig. E5.24. The application of Eq. 5.82 to the contents of this control volume leads to

$$w_{\text{shaft net in}} - \text{loss} = \underbrace{\left( \frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2 \right) - \left( \frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1 \right)}_{\substack{0 \text{ (atmospheric pressures cancel)} \\ 0 \text{ (no elevation change)}}} \quad (1)$$

where  $w_{\text{shaft net in}} - \text{loss}$  is the amount of work added to the air that produces a useful effect. Equation 1 leads to

$$w_{\text{shaft net in}} - \text{loss} = \frac{V_2^2}{2} = \frac{(12 \text{ m/s})^2}{2[1(\text{kg} \cdot \text{m})/(\text{N} \cdot \text{s}^2)]} = 72.0 \text{ N} \cdot \text{m/kg} \quad (2) \quad \text{(Ans)}$$

A reasonable estimate of *efficiency*,  $\eta$ , would be the ratio of amount of work that produces a useful effect, Eq. 2, to the amount of work delivered to the fan blades. That is,

$$\eta = \frac{w_{\text{shaft net in}} - \text{loss}}{w_{\text{shaft net in}}} \quad (3)$$

To calculate the efficiency we need a value of  $w_{\text{shaft net in}}$  which is related to the power delivered to the blades,  $\dot{W}_{\text{shaft net in}}$ . We note that

$$w_{\text{shaft net in}} = \frac{\dot{W}_{\text{shaft net in}}}{\dot{m}} \quad (4)$$

where the mass flowrate,  $\dot{m}$ , is (from Eq. 5.6)

$$\dot{m} = \rho AV = \rho \frac{\pi D_2^2}{4} V_2 \quad (5)$$

For fluid density,  $\rho$ , we use  $1.23 \text{ kg/m}^3$  (standard air), and thus from Eqs. 4 and 5 we obtain

$$w_{\text{shaft net in}} = \frac{\dot{W}_{\text{shaft net in}}}{(\rho \pi D_2^2 / 4) V_2} = \frac{(0.4 \text{ kW})[1000 (\text{N} \cdot \text{m})/(\text{s} \cdot \text{kW})]}{(1.23 \text{ kg/m}^3)[(\pi)(0.6 \text{ m})^2/4](12 \text{ m/s})}$$

or

$$w_{\text{shaft net in}} = 95.8 \text{ N} \cdot \text{m/kg} \quad (6)$$

From Eqs. 2, 3, and 6 we obtain

$$\eta = \frac{72.0 \text{ N} \cdot \text{m/kg}}{95.8 \text{ N} \cdot \text{m/kg}} = 0.752 \quad (\text{Ans})$$

Note that only 75% of the power that was delivered to the air resulted in a useful effect, and thus 25% of the shaft power is lost to air friction.

If Eq. 5.82, which involves energy per unit mass, is multiplied by fluid density,  $\rho$ , we obtain

$$p_{\text{out}} + \frac{\rho V_{\text{out}}^2}{2} + \gamma z_{\text{out}} = p_{\text{in}} + \frac{\rho V_{\text{in}}^2}{2} + \gamma z_{\text{in}} + \rho w_{\text{shaft net in}} - \rho(\text{loss}) \quad (5.83)$$

where  $\gamma = \rho g$  is the specific weight of the fluid. Equation 5.83 involves *energy per unit volume* and the units involved are identical with those used for pressure ( $\text{ft} \cdot \text{lb}/\text{ft}^3 = \text{lb}/\text{ft}^2$  or  $\text{N} \cdot \text{m}/\text{m}^3 = \text{N}/\text{m}^2$ ).

If Eq. 5.82 is divided by the acceleration of gravity,  $g$ , we get

$$\frac{p_{\text{out}}}{\gamma} + \frac{V_{\text{out}}^2}{2g} + z_{\text{out}} = \frac{p_{\text{in}}}{\gamma} + \frac{V_{\text{in}}^2}{2g} + z_{\text{in}} + h_s - h_L \quad (5.84)$$

where

$$h_s = w_{\text{shaft net in}}/g = \frac{\dot{W}_{\text{shaft net in}}}{\dot{m}g} = \frac{\dot{W}_{\text{shaft net in}}}{\gamma Q} \quad (5.85)$$

and  $h_L = \text{loss}/g$ . Equation 5.84 involves *energy per unit weight* ( $\text{ft} \cdot \text{lb}/\text{lb} = \text{ft}$  or  $\text{N} \cdot \text{m}/\text{N} = \text{m}$ ). In [Section 3.7](#), we introduced the notion of “head,” which is energy per unit weight. Units of length (e.g., ft, m) are used to quantify the amount of head involved. If a turbine is in the control volume, the notation  $h_s = -h_T$  (with  $h_T > 0$ ) is sometimes used, particularly in the field of hydraulics. For a pump in the control volume,  $h_s = h_p$ . The quantity  $h_T$  is termed the *turbine head* and  $h_p$  is the *pump head*. The loss term,  $h_L$  is often referred to as *head loss*. The turbine head is often written as

$$h_T = -(h_s + h_L)_T$$

where the subscript  $T$  refers to the turbine component of the contents of the control volume only. The quantity  $h_T$  is the actual head drop across the turbine and is the sum of the shaft work head out of the turbine and the head loss within the turbine. When a pump is in the control volume,

$$h_p = (h_s - h_L)_P$$

is often used where  $h_p$  is the actual head rise across the pump and is equal to the difference between the shaft work head into the pump and the head loss within the pump. Notice that the  $h_L$  used for the turbine and the pump is the head loss within that component only. When  $h_s$  is used in Eq. 5.84,  $h_L$  involves all losses including those within the turbine or compressor. When  $h_T$  or  $h_p$  is used for  $h_s$ , then  $h_L$  includes all losses except those associated with the turbine or pump flows.



#### V5.8 Water plant aerator

*The energy equation written in terms of energy per unit weight involves heads.*

# EXAMPLE 5.25

The pump shown in Fig. E5.25 adds 10 horsepower to the water as it pumps  $2 \text{ ft}^3/\text{s}$  from the lower lake to the upper lake. The elevation difference between the lake surfaces is 30 ft. Determine the head loss and power loss associated with this flow.

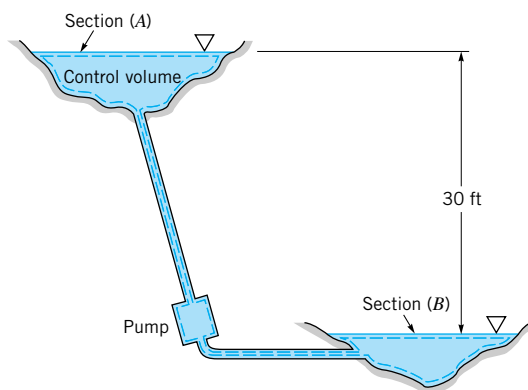


FIGURE E5.25

## SOLUTION

The energy equation (Eq. 5.84) for this flow is

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B + h_s - h_L \quad (1)$$

where points  $A$  and  $B$  (corresponding to “out” and “in” in Eq. 5.84) are located on the lake surfaces. Thus,  $p_A = p_B = 0$  and  $V_A = V_B = 0$  so that Eq. 1 becomes

$$h_L = h_s + z_B - z_A \quad (2)$$

where  $z_B = 0$  and  $z_A = 30 \text{ ft}$ . The pump head is obtained from Eq. 5.85 as

$$\begin{aligned} h_s &= \dot{W}_{\text{shaft net in}} / \gamma Q \\ &= (10 \text{ hp})(550 \text{ ft} \cdot \text{lb/s/hp}) / (62.4 \text{ lb/ft}^3)(2 \text{ ft}^3/\text{s}) \\ &= 44.1 \text{ ft} \end{aligned}$$

Hence, from Eq. 2,

$$h_L = 44.1 \text{ ft} - 30 \text{ ft} = 14.1 \text{ ft} \quad (\text{Ans})$$

Note that in this example the purpose of the pump is to lift the water (a 30-ft head) and overcome the head loss (a 14.1 ft head); it does not, overall, alter the water’s pressure or velocity.

The power lost due to friction can be obtained from Eq. 5.85 as

$$\begin{aligned} \dot{W}_{\text{loss}} &= \gamma Q h_L = (62.4 \text{ lb/ft}^3)(2 \text{ ft}^3/\text{s})(14.1 \text{ ft}) \\ &= 1760 \text{ ft} \cdot \text{lb/s} \quad (1 \text{ hp}/550 \text{ ft} \cdot \text{lb/s}) \\ &= 3.20 \text{ hp} \quad (\text{Ans}) \end{aligned}$$

The remaining  $10 \text{ hp} - 3.20 \text{ hp} = 6.80 \text{ hp}$  that the pump adds to the water is used to lift the water from the lower to the upper lake. This energy is not “lost,” but it is stored as potential energy.

A comparison of the energy equation and the Bernoulli equation has led to the concept of loss of available energy in incompressible fluid flows with friction. In Chapter 8, we discuss in detail some methods for estimating loss in incompressible flows with friction. In [Section 5.4](#) and [Chapter 11](#), we demonstrate that loss of available energy is also an important factor to consider in compressible flows with friction.

### 5.3.4 Application of the Energy Equation to Nonuniform Flows

The forms of the energy equation discussed in Sections 5.3.2 and 5.3.3 are applicable to one-dimensional flows, flows that are approximated with uniform velocity distributions where fluid crosses the control surface.

If the velocity profile at any section where flow crosses the control surface is not uniform, inspection of the energy equation for a control volume, Eq. 5.64, suggests that the integral

$$\int_{cs} \frac{V^2}{2} \rho \mathbf{V} \cdot \hat{\mathbf{n}} dA$$

will require special attention. The other terms of Eq. 5.64 can be accounted for as already discussed in Sections 5.3.2 and 5.3.3.

For one stream of fluid entering and leaving the control volume, we can define the relationship

$$\int_{cs} \frac{V^2}{2} \rho \mathbf{V} \cdot \hat{\mathbf{n}} dA = \dot{m} \left( \frac{\alpha_{out} \bar{V}_{out}^2}{2} - \frac{\alpha_{in} \bar{V}_{in}^2}{2} \right)$$

where  $\alpha$  is the *kinetic energy coefficient* and  $\bar{V}$  is the average velocity defined earlier in Eq. 5.7. From the above we can conclude that

$$\frac{\dot{m} \alpha \bar{V}^2}{2} = \int_A \frac{V^2}{2} \rho \mathbf{V} \cdot \hat{\mathbf{n}} dA$$

for flow through surface area  $A$  of the control surface. Thus,

$$\alpha = \frac{\int_A (V^2/2) \rho \mathbf{V} \cdot \hat{\mathbf{n}} dA}{\dot{m} \bar{V}^2/2} \quad (5.86)$$

It can be shown that for any velocity profile,  $\alpha \geq 1$ , with  $\alpha = 1$  only for uniform flow. For nonuniform velocity profiles, the energy equation on an energy per unit mass basis for the incompressible flow of one stream of fluid through a control volume that is steady in the mean is

$$\frac{p_{out}}{\rho} + \frac{\alpha_{out} \bar{V}_{out}^2}{2} + gz_{out} = \frac{p_{in}}{\rho} + \frac{\alpha_{in} \bar{V}_{in}^2}{2} + gz_{in} + w_{shaft \text{ net in}} - \text{loss} \quad (5.87)$$

On an energy per unit volume basis we have

$$p_{out} + \frac{\rho \alpha_{out} \bar{V}_{out}^2}{2} + \gamma z_{out} = p_{in} + \frac{\rho \alpha_{in} \bar{V}_{in}^2}{2} + \gamma z_{in} + \rho w_{shaft \text{ net in}} - \rho(\text{loss}) \quad (5.88)$$

*The kinetic energy coefficient is used to account for non-uniform flows.*

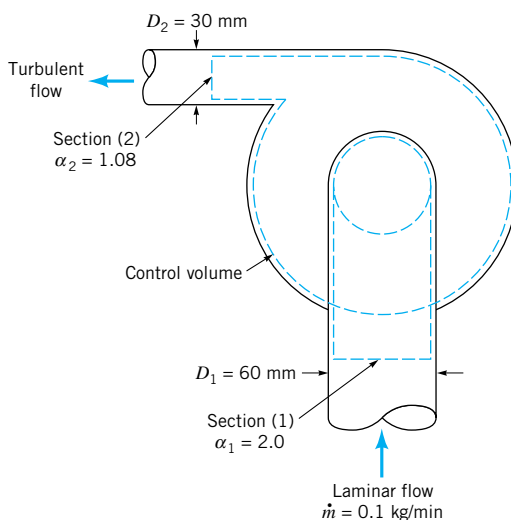
and on an energy per unit weight or head basis we have

$$\frac{p_{\text{out}}}{\gamma} + \frac{\alpha_{\text{out}} \bar{V}_{\text{out}}^2}{2g} + z_{\text{out}} = \frac{p_{\text{in}}}{\gamma} + \frac{\alpha_{\text{in}} \bar{V}_{\text{in}}^2}{2g} + z_{\text{in}} + \frac{w_{\text{shaft net in}}}{g} - h_L \quad (5.89)$$

The following examples illustrate the use of the kinetic energy coefficient.

## EXAMPLE 5.26

The small fan shown in Fig. E5.26 moves air at a mass flowrate of 0.1 kg/min. Upstream of the fan, the pipe diameter is 60 mm, the flow is laminar, the velocity distribution is parabolic, and the kinetic energy coefficient,  $\alpha_1$ , is equal to 2.0. Downstream of the fan, the pipe diameter is 30 mm, the flow is turbulent, the velocity profile is quite uniform, and the kinetic energy coefficient,  $\alpha_2$ , is equal to 1.08. If the rise in static pressure across the fan is 0.1 kPa and the fan motor draws 0.14 W, compare the value of loss calculated: (a) assuming uniform velocity distributions, (b) considering actual velocity distributions.



■ FIGURE E5.26

## SOLUTION

Application of Eq. 5.87 to the contents of the control volume shown in Fig. E5.26 leads to

$$\frac{p_2}{\rho} + \frac{\alpha_2 \bar{V}_2^2}{2} + \cancel{gz_2} = \frac{p_1}{\rho} + \frac{\alpha_1 \bar{V}_1^2}{2} + \cancel{gz_1} - \text{loss} + w_{\text{shaft net in}} \quad (1)$$

0 (change in  $gz$  is negligible)

or solving Eq. 1 for loss we get

$$\text{loss} = w_{\text{shaft net in}} - \left( \frac{p_2 - p_1}{\rho} \right) + \frac{\alpha_1 \bar{V}_1^2}{2} - \frac{\alpha_2 \bar{V}_2^2}{2} \quad (2)$$

To proceed further, we need values of  $w_{\text{shaft net in}}$ ,  $\bar{V}_1$ , and  $\bar{V}_2$ . These quantities can be obtained as follows. For shaft work



$$w_{\text{shaft net in}} = \frac{\text{power to fan motor}}{\dot{m}}$$

or

$$w_{\text{shaft net in}} = \frac{(0.14 \text{ W})[(1 \text{ N} \cdot \text{m/s})/\text{W}]}{0.1 \text{ kg/min}} (60 \text{ s/min})[1 (\text{kg} \cdot \text{m})/(\text{N} \cdot \text{s}^2)] = 84.0 \text{ N} \cdot \text{m/kg} \quad (3)$$

For the average velocity at section (1),  $\bar{V}_1$ , from Eq. 5.11 we obtain

$$\begin{aligned} \bar{V}_1 &= \frac{\dot{m}}{\rho A_1} \\ &= \frac{\dot{m}}{\rho(\pi D_1^2/4)} \\ &= \frac{(0.1 \text{ kg/min})}{(1.23 \text{ kg/m}^3)[\pi(60 \text{ mm})^2/4][(60 \text{ s/min})/(1000 \text{ mm/m})^2]} \\ &= 0.479 \text{ m/s} \end{aligned} \quad (4)$$

For the average velocity at section (2),  $\bar{V}_2$ ,

$$\bar{V}_2 = \frac{(0.1 \text{ kg/min})}{(1.23 \text{ kg/m}^3)[\pi(30 \text{ mm})^2/4][(60 \text{ s/min})/(1000 \text{ mm/m})^2]} = 1.92 \text{ m/s} \quad (5)$$

(a) For the assumed uniform velocity profiles ( $\alpha_1 = \alpha_2 = 1.0$ ), Eq. 2 yields

$$\text{loss} = w_{\text{shaft net in}} - \left( \frac{p_2 - p_1}{\rho} \right) + \frac{\bar{V}_1^2}{2} - \frac{\bar{V}_2^2}{2} \quad (6)$$

Using Eqs. 3, 4, and 5 and the pressure rise given in the problem statement, Eq. 6 gives

$$\begin{aligned} \text{loss} &= 84.0 \frac{\text{N} \cdot \text{m}}{\text{kg}} - \frac{(0.1 \text{ kPa})(1000 \text{ Pa/kPa})(1 \text{ N/m}^2/\text{Pa})}{1.23 \text{ kg/m}^3} \\ &\quad + \frac{(0.479 \text{ m/s})^2}{2[1 (\text{kg} \cdot \text{m})/(\text{N} \cdot \text{s}^2)]} - \frac{(1.92 \text{ m/s})^2}{2[1 (\text{kg} \cdot \text{m})/(\text{N} \cdot \text{s}^2)]} \end{aligned}$$

or

$$\begin{aligned} \text{loss} &= 84.0 \text{ N} \cdot \text{m/kg} - 81.3 \text{ N} \cdot \text{m/kg} + 0.115 \text{ N} \cdot \text{m/kg} - 1.84 \text{ N} \cdot \text{m/kg} \\ &= 0.975 \text{ N} \cdot \text{m/kg} \end{aligned} \quad (\text{Ans})$$

(b) For the actual velocity profiles ( $\alpha_1 = 2, \alpha_2 = 1.08$ ), Eq. 1 gives

$$\text{loss} = w_{\text{shaft net in}} - \left( \frac{p_2 - p_1}{\rho} \right) + \alpha_1 \frac{\bar{V}_1^2}{2} - \alpha_2 \frac{\bar{V}_2^2}{2} \quad (7)$$

If we use Eqs. 3, 4, and 5 and the given pressure rise, Eq. 7 yields

$$\begin{aligned} \text{loss} &= 84 \text{ N} \cdot \text{m/kg} - \frac{(0.1 \text{ kPa})(1000 \text{ Pa/kPa})(1 \text{ N/m}^2/\text{Pa})}{1.23 \text{ kg/m}^3} \\ &\quad + \frac{2(0.479 \text{ m/s})^2}{2[1 (\text{kg} \cdot \text{m})/(\text{N} \cdot \text{s}^2)]} - \frac{1.08(1.92 \text{ m/s})^2}{2[1 (\text{kg} \cdot \text{m})/(\text{N} \cdot \text{s}^2)]} \end{aligned}$$

or

$$\begin{aligned} \text{loss} &= 84.0 \text{ N} \cdot \text{m/kg} - 81.3 \text{ N} \cdot \text{m/kg} + 0.230 \text{ N} \cdot \text{m/kg} - 1.99 \text{ N} \cdot \text{m/kg} \\ &= 0.940 \text{ N} \cdot \text{m/kg} \end{aligned} \quad (\text{Ans})$$

The difference in loss calculated assuming uniform velocity profiles and actual velocity profiles is not large compared to  $w_{\text{shaft net in}}$  for this fluid flow situation.

## EXAMPLE 5.27

Apply Eq. 5.87 to the flow situation of Example 5.14 and develop an expression for the fluid pressure drop that occurs between sections (1) and (2). By comparing the equation for pressure drop obtained presently with the result of Example 5.14, obtain an expression for loss between sections (1) and (2).

## SOLUTION

Application of Eq. 5.87 to the flow of Example 5.14 (see Fig. E5.14) leads to

$$\frac{p_2}{\rho} + \frac{\alpha_2 \bar{w}_2^2}{2} + gz_2 = \frac{p_1}{\rho} + \frac{\alpha_1 \bar{w}_1^2}{2} + gz_1 - \text{loss} + w_{\text{shaft net in}} \quad (1)$$

0 (no shaft work)

Solving Eq. 1 for the pressure drop,  $p_1 - p_2$ , we obtain

$$p_1 - p_2 = \rho \left[ \frac{\alpha_2 \bar{w}_2^2}{2} - \frac{\alpha_1 \bar{w}_1^2}{2} + g(z_2 - z_1) + \text{loss} \right] \quad (2)$$

Since the fluid velocity at section (1),  $w_1$ , is uniformly distributed over cross-sectional area  $A_1$ , the corresponding kinetic energy coefficient,  $\alpha_1$ , is equal to 1.0. The kinetic energy coefficient at section (2),  $\alpha_2$ , needs to be determined from the velocity profile distribution given in Example 5.4. Using Eq. 5.86 we get

$$\alpha_2 = \frac{\int_{A_2} \rho w_2^3 dA_2}{\dot{m} \bar{w}_2^2} \quad (3)$$

Substituting the parabolic velocity profile equation into Eq. 3 we obtain

$$\alpha_2 = \frac{\rho \int_0^R (2w_1)^3 [1 - (r/R)^2]^3 2\pi r dr}{(\rho A_2 \bar{w}_2) \bar{w}_2^2}$$

From conservation of mass, since  $A_1 = A_2$

$$w_1 = \bar{w}_2 \quad (4)$$

Then, substituting Eq. 4 into Eq. 3, we obtain

$$\alpha_2 = \frac{\rho 8 \bar{w}_2^3 2\pi \int_0^R [1 - (r/R)^2]^3 r dr}{\rho \pi R^2 \bar{w}_2^3}$$

or

$$\alpha_2 = \frac{16}{R^2} \int_0^R [1 - 3(r/R)^2 + 3(r/R)^4 - (r/R)^6] r dr = 2 \quad (5)$$

Now we combine Eqs. 2 and 5 to get

$$p_1 - p_2 = \rho \left[ \frac{2.0\bar{w}_2^2}{2} - \frac{1.0\bar{w}_1^2}{2} + g(z_2 - z_1) + \text{loss} \right] \quad (6)$$

However, from conservation of mass  $\bar{w}_2 = \bar{w}_1 = \bar{w}$  so that Eq. 6 becomes

$$p_1 - p_2 = \frac{\rho\bar{w}^2}{2} + \rho g(z_2 - z_1) + \rho(\text{loss}) \quad (7)$$

The term associated with change in elevation,  $\rho g(z_2 - z_1)$ , is equal to the weight per unit cross-sectional area,  $\mathcal{W}/A$ , of the water contained between sections (1) and (2) at any instant,

$$\rho g(z_2 - z_1) = \frac{\mathcal{W}}{A} \quad (8)$$

Thus, combining Eqs. 7 and 8 we get

$$p_1 - p_2 = \frac{\rho\bar{w}^2}{2} + \frac{\mathcal{W}}{A} + \rho(\text{loss}) \quad (9)$$

The pressure drop between sections (1) and (2) is due to:

1. The change in kinetic energy between sections (1) and (2) associated with going from a uniform velocity profile to a parabolic velocity profile.
2. The weight of the water column, that is, hydrostatic pressure effect.
3. Viscous loss.

Comparing Eq. 9 for pressure drop with the one obtained in Example 5.14 (i.e., the answer of Example 5.14) we obtain

$$\frac{\rho\bar{w}^2}{2} + \frac{\mathcal{W}}{A} + \rho(\text{loss}) = \frac{\rho\bar{w}^2}{3} + \frac{R_z}{A} + \frac{\mathcal{W}}{A} \quad (10)$$

or

$$\text{loss} = \frac{R_z}{\rho A} - \frac{\bar{w}^2}{6} \quad (\text{Ans})$$

We conclude that while some of the pipe wall friction force,  $R_z$ , resulted in loss of available energy, a portion of this friction,  $\rho A \bar{w}^2/6$ , led to the velocity profile change.

### 5.3.5 Combination of the Energy Equation and the Moment-of-Momentum Equation<sup>4</sup>

If Eq. 5.82 is used for one-dimensional incompressible flow through a turbomachine, we can use Eq. 5.54, developed in Section 5.2.4 from the moment-of-momentum equation (Eq. 5.42), to evaluate shaft work. This application of both Eqs. 5.54 and 5.82 allows us to ascertain the

<sup>4</sup>This section may be omitted without loss of continuity in the text material. This section should not be considered without prior study of Sections 5.2.3 and 5.2.4. All of these sections are recommended for those interested in Chapter 12.

amount of loss that occurs in incompressible turbomachine flows as is demonstrated in Example 5.28.

## EXAMPLE 5.28

For the fan of **Example 5.19**, show that only some of the shaft power into the air is converted into a useful effect. Develop a meaningful efficiency equation and a practical means for estimating lost shaft energy.

### SOLUTION

We use the same control volume used in Example 5.19. Application of Eq. 5.82 to the contents of this control volume yields

$$\frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2 = \frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1 + w_{\text{shaft net in}} - \text{loss} \quad (1)$$

As in Example 5.26, we can see with Eq. 1 that a “useful effect” in this fan can be defined as

$$\text{useful effect} = w_{\text{shaft net in}} - \text{loss} = \left( \frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2 \right) - \left( \frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1 \right) \quad (2) \quad (\text{Ans})$$

In other words, only a portion of the shaft work delivered to the air by the fan blades is used to increase the available energy of the air; the rest is lost because of fluid friction.

A meaningful efficiency equation would involve the ratio of shaft work converted into a useful effect (Eq. 2) to shaft work into the air,  $w_{\text{shaft net in}}$ . Thus, we can express efficiency,  $\eta$ , as

$$\eta = \frac{w_{\text{shaft net in}} - \text{loss}}{w_{\text{shaft net in}}} \quad (3)$$

However, when Eq. 5.54, which was developed from the moment-of-momentum equation (Eq. 5.42), is applied to the contents of the control volume of Fig. E5.19, we obtain

$$w_{\text{shaft net in}} = +U_2 V_{\theta 2} \quad (4)$$

Combining Eqs. 2, 3, and 4, we obtain

$$\eta = \frac{[(p_2/\rho) + (V_2^2/2) + gz_2] - [(p_1/\rho) + (V_1^2/2) + gz_1]}{U_2 V_{\theta 2}} \quad (5) \quad (\text{Ans})$$

Equation 5 provides us with a practical means to evaluate the efficiency of the fan of Example 5.19.

Combining Eqs. 2 and 4, we obtain

$$\text{loss} = U_2 V_{\theta 2} - \left[ \left( \frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2 \right) - \left( \frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1 \right) \right] \quad (6) \quad (\text{Ans})$$

Equation 6 provides us with a useful method of evaluating the loss due to fluid friction in the fan of Example 5.19 in terms of fluid mechanical variables that can be measured.

## 5.4 Second Law of Thermodynamics—Irreversible Flow<sup>5</sup>

The second law of thermodynamics affords us with a means to formalize the inequality

$$\check{u}_2 - \check{u}_1 - q_{\text{net, in}} \geq 0 \quad (5.90)$$

*The second law of thermodynamics formalizes the notion of loss.*

for steady, incompressible, one-dimensional flow with friction (see Eq. 5.73). In this section we continue to develop the notion of loss of useful or available energy for flow with friction. Minimization of loss of available energy in any flow situation is of obvious engineering importance.

### 5.4.1 Semi-infinitesimal Control Volume Statement of the Energy Equation

If we apply the one-dimensional, steady flow energy equation, Eq. 5.70, to the contents of a control volume that is infinitesimally thin as illustrated in Fig 5.8, the result is

$$\dot{m} \left[ d\check{u} + d\left(\frac{p}{\rho}\right) + d\left(\frac{V^2}{2}\right) + g(dz) \right] = \delta \dot{Q}_{\text{net, in}} \quad (5.91)$$

For all pure substances including common engineering working fluids, such as air, water, oil, and gasoline, the following relationship is valid (see, for example, Ref. 3).

$$T ds = d\check{u} + pd\left(\frac{1}{\rho}\right) \quad (5.92)$$

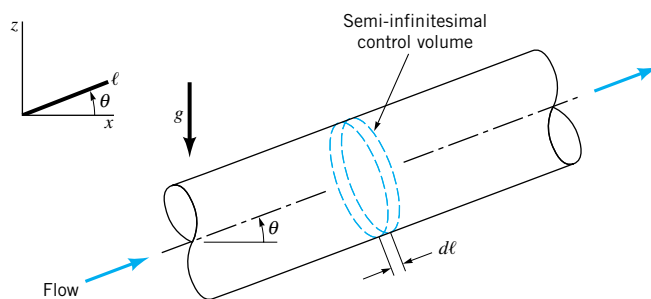
where  $T$  is the absolute temperature and  $s$  is the *entropy* per unit mass.

Combining Eqs. 5.91 and 5.92 we get

$$\dot{m} \left[ T ds - pd\left(\frac{1}{\rho}\right) + d\left(\frac{p}{\rho}\right) + d\left(\frac{V^2}{2}\right) + g dz \right] = \delta \dot{Q}_{\text{net, in}}$$

or, dividing through by  $\dot{m}$  and letting  $\delta q_{\text{net, in}} = \delta \dot{Q}_{\text{net, in}} / \dot{m}$ , we obtain

$$\frac{dp}{\rho} + d\left(\frac{V^2}{2}\right) + g dz = -(T ds - \delta q_{\text{net, in}}) \quad (5.93)$$



■ **FIGURE 5.8** Semi-infinitesimal control volume.

<sup>5</sup>This entire section may be omitted without loss of continuity in the text material.

### 5.4.2 Semi-infinitesimal Control Volume Statement of the Second Law of Thermodynamics

A general statement of the second law of thermodynamics is

$$\frac{D}{Dt} \int_{\text{sys}} s \rho dV \geq \sum \left( \frac{\delta \dot{Q}_{\text{net}}}{T} \right)_{\text{sys}} \quad (5.94)$$

*The second law of thermodynamics involves entropy, heat transfer, and temperature.*

or in words,

the time rate of increase of the      sum of the ratio of net heat  
entropy of a system       $\geq$  transfer rate into system to  
absolute temperature for each  
particle of mass in the system  
receiving heat from  
surroundings

The right-hand side of Eq. 5.94 is identical for the system and control volume at the instant when system and control volume are coincident; thus,

$$\sum \left( \frac{\delta \dot{Q}_{\text{net}}}{T} \right)_{\text{sys}} = \sum \left( \frac{\delta \dot{Q}_{\text{net}}}{T} \right)_{\text{cv}} \quad (5.95)$$

With the help of the Reynolds transport theorem (Eq. 4.19) the system time derivative can be expressed for the contents of the coincident control volume that is fixed and nondeforming. Using Eq. 4.19, we obtain

$$\frac{D}{Dt} \int_{\text{sys}} s \rho dV = \frac{\partial}{\partial t} \int_{\text{cv}} s \rho dV + \int_{\text{cs}} s \rho \mathbf{V} \cdot \hat{\mathbf{n}} dA \quad (5.96)$$

For a fixed, nondeforming control volume, Eqs. 5.94, 5.95, and 5.96 combine to give

$$\frac{\partial}{\partial t} \int_{\text{cv}} s \rho dV + \int_{\text{cs}} s \rho \mathbf{V} \cdot \hat{\mathbf{n}} dA \geq \sum \left( \frac{\delta \dot{Q}_{\text{net}}}{T} \right)_{\text{cv}} \quad (5.97)$$

At any instant for steady flow

$$\frac{\partial}{\partial t} \int_{\text{cv}} s \rho dV = 0 \quad (5.98)$$

If the flow consists of only one stream through the control volume and if the properties are uniformly distributed (one-dimensional flow), Eqs. 5.97 and 5.98 lead to

$$\dot{m}(s_{\text{out}} - s_{\text{in}}) \geq \sum \frac{\delta \dot{Q}_{\text{net}}}{T} \quad (5.99)$$

For the infinitesimally thin control volume of Fig. 5.8, Eq. 5.99 yields

$$\dot{m} ds \geq \sum \frac{\delta \dot{Q}_{\text{net}}}{T} \quad (5.100)$$

If all of the fluid in the infinitesimally thin control volume is considered as being at a uniform temperature,  $T$ , then from Eq. 5.100 we get

$$T ds \geq \delta q_{\text{net}}_{\text{in}}$$

or

$$T ds - \delta q_{\text{net in}} \geq 0 \quad (5.101)$$

The equality is for any reversible (frictionless) process; the inequality is for all irreversible (friction) processes.

### 5.4.3 Combination of the Equations of the First and Second Laws of Thermodynamics

Combining Eqs. 5.93 and 5.101, we conclude that

$$-\left[ \frac{dp}{\rho} + d\left(\frac{V^2}{2}\right) + g dz \right] \geq 0 \quad (5.102)$$

The equality is for any steady, reversible (frictionless) flow, an important example being flow for which the Bernoulli equation (Eq. 3.7) is applicable. The inequality is for all steady, irreversible (friction) flows. The actual amount of the inequality has physical significance. It represents the extent of loss of useful or available energy which occurs because of irreversible flow phenomena including viscous effects. Thus, Eq. 5.102 can be expressed as

$$-\left[ \frac{dp}{\rho} + d\left(\frac{V^2}{2}\right) + g dz \right] = \delta(\text{loss}) = (T ds - \delta q_{\text{net in}}) \quad (5.103)$$

*The irreversible flow loss is zero for frictionless flow.*

The irreversible flow loss is zero for a frictionless flow and greater than zero for a flow with frictional effects. Note that when the flow is frictionless, Eq. 5.103 multiplied by density,  $\rho$ , is identical to Eq. 3.5. Thus, for steady frictionless flow, Newton's second law of motion (see Section 3.1) and the first and second laws of thermodynamics lead to the same differential equation,

$$\frac{dp}{\rho} + d\left(\frac{V^2}{2}\right) + g dz = 0 \quad (5.104)$$

If some shaft work is involved, then the flow must be at least locally unsteady in a cyclical way and the appropriate form of the energy equation for the contents of an infinitesimally thin control volume can be developed starting with Eq. 5.67. The resulting equation is

$$-\left[ \frac{dp}{\rho} + d\left(\frac{V^2}{2}\right) + g dz \right] = \delta(\text{loss}) - \delta w_{\text{shaft net in}} \quad (5.105)$$

Equations 5.103 and 5.105 are valid for incompressible and compressible flows. If we combine Eqs. 5.92 and 5.103, we obtain

$$d\ddot{u} + p d\left(\frac{1}{\rho}\right) - \delta q_{\text{net in}} = \delta(\text{loss}) \quad (5.106)$$

For incompressible flow,  $d(1/\rho) = 0$  and, thus, from Eq. 5.106,

$$d\ddot{u} - \delta q_{\text{net in}} = \delta(\text{loss}) \quad (5.107)$$

Applying Eq. 5.107 to a finite control volume, we obtain

$$\check{u}_{\text{out}} - \check{u}_{\text{in}} - q_{\text{net in}} = \text{loss}$$

which is the same conclusion we reached earlier (see Eq. 5.78) for incompressible flows.

For compressible flow,  $d(1/\rho) \neq 0$ , and thus when we apply Eq. 5.106 to a finite control volume we obtain

$$\check{u}_{\text{out}} - \check{u}_{\text{in}} + \int_{\text{in}}^{\text{out}} p d\left(\frac{1}{\rho}\right) - q_{\text{net in}} = \text{loss} \quad (5.108)$$

indicating that  $u_{\text{out}} - u_{\text{in}} - q_{\text{net in}}$  is not equal to loss.

#### 5.4.4 Application of the Loss Form of the Energy Equation

Steady flow along a pathline in an incompressible and frictionless flow field provides a simple application of the loss form of the energy equation (Eq. 5.105). We start with Eq. 5.105 and integrate it term by term from one location on the pathline, section (1), to another one downstream, section (2). Note that because the flow is frictionless,  $\text{loss} = 0$ . Also, because the flow is steady throughout,  $w_{\text{shaft net in}} = 0$ . Since the flow is incompressible, the density is constant. The control volume in this case is an infinitesimally small diameter streamtube (Fig. 5.7). The resultant equation is

$$\frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2 = \frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1 \quad (5.109)$$

which is identical to the Bernoulli equation (Eq. 3.7) already discussed in Chapter 3.

If the frictionless and steady pathline flow of the fluid particle considered above was compressible, application of Eq. 5.105 would yield

$$\int_1^2 \frac{dp}{\rho} + \frac{V_2^2}{2} + gz_2 = \frac{V_1^2}{2} + gz_1 \quad (5.110)$$

To carry out the integration required,  $\int_1^2 (dp/\rho)$ , a relationship between fluid density,  $\rho$ , and pressure,  $p$ , must be known. If the frictionless compressible flow we are considering is adiabatic and involves the flow of an ideal gas, it is shown in Section 11.1 that

$$\frac{p}{\rho^k} = \text{constant} \quad (5.111)$$

where  $k = c_p/c_v$  is the ratio of gas specific heats,  $c_p$  and  $c_v$ , which are properties of the fluid. Using Eq. 5.111 we get

$$\int_1^2 \frac{dp}{\rho} = \frac{k}{k-1} \left( \frac{p_2}{\rho_2} - \frac{p_1}{\rho_1} \right) \quad (5.112)$$

Thus, Eqs. 5.110 and 5.112 lead to

$$\frac{k}{k-1} \frac{p_2}{\rho_2} + \frac{V_2^2}{2} + gz_2 = \frac{k}{k-1} \frac{p_1}{\rho_1} + \frac{V_1^2}{2} + gz_1 \quad (5.113)$$

Note that this equation is identical to Eq. 3.24. An example application of Eqs. 5.109 and 5.113 follows.

*Zero loss is associated with the Bernoulli equation.*



# EXAMPLE 5.29

Air steadily expands adiabatically and without friction from stagnation conditions of 100 psia and 520 °R to 14.7 psia. Determine the velocity of the expanded air assuming (a) incompressible flow, (b) compressible flow.

## SOLUTION

- (a) If the flow is considered incompressible, the Bernoulli equation, Eq. 5.109, can be applied to flow through an infinitesimal cross-sectional streamtube, like the one in Fig. 5.7, from the stagnation state (1) to the expanded state (2). From Eq. 109 we get

$$\frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2 = \frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1 \quad (1)$$

0 (1 is the stagnation state)

0 (changes in  $gz$  are negligible for air flow)

or

$$V_2 = \sqrt{2 \left( \frac{p_1 - p_2}{\rho} \right)}$$

We can calculate the density at state (1) by assuming that air behaves like an ideal gas,

$$\rho = \frac{p_1}{RT_1} = \frac{(100 \text{ psia})(144 \text{ in.}^2/\text{ft}^2)}{(1716 \text{ ft} \cdot \text{lb}/\text{slug} \cdot ^\circ\text{R})(520 ^\circ\text{R})} = 0.0161 \text{ slug}/\text{ft}^3 \quad (2)$$

Thus,

$$V_2 = \sqrt{\frac{2(100 \text{ psia} - 14.7 \text{ psia})(144 \text{ in.}^2/\text{ft}^2)}{(0.016 \text{ slug}/\text{ft}^3)[1 (\text{lb} \cdot \text{s}^2)/(\text{slug} \cdot \text{ft})]}} = 1240 \text{ ft/s} \quad (\text{Ans})$$

The assumption of incompressible flow is not valid in this case since for air a change from 100 psia to 14.7 psia would undoubtedly result in a significant density change.

- (b) If the flow is considered compressible, Eq. 5.113 can be applied to the flow through an infinitesimal cross-sectional control volume, like the one in Fig. 5.7, from the stagnation state (1) to the expanded state (2). We obtain

$$\frac{k}{k-1} \frac{p_2}{\rho_2} + \frac{V_2^2}{2} + gz_2 = \frac{k}{k-1} \frac{p_1}{\rho_1} + \frac{V_1^2}{2} + gz_1 \quad (3)$$

0 (1 is the stagnation state)

0 (changes in  $gz$  are negligible for air flow)

or

$$V_2 = \sqrt{\frac{2k}{k-1} \left( \frac{p_1}{\rho_1} - \frac{p_2}{\rho_2} \right)} \quad (4)$$

Given in the problem statement are values of  $p_1$  and  $p_2$ . A value of  $\rho_1$  was calculated earlier (Eq. 2). To determine  $\rho_2$  we need to make use of a property relationship for reversible (frictionless) and adiabatic flow of an ideal gas that is derived in Chapter 11; namely,

$$\frac{p}{\rho^k} = \text{constant} \quad (5)$$

where  $k = 1.4$  for air. Solving Eq. 5 for  $\rho_2$  we get

$$\rho_2 = \rho_1 \left( \frac{p_2}{p_1} \right)^{1/k}$$

or

$$\rho_2 = (0.0161 \text{ slug/ft}^3) \left[ \frac{14.7 \text{ psia}}{100 \text{ psia}} \right]^{1/1.4} = 0.00409 \text{ slug/ft}^3$$

Then, from Eq. 4,

$$V_2 = \sqrt{\frac{(2)(1.4) \left( \frac{100 \text{ psia}}{0.0161 \text{ slug/ft}^3} - \frac{14.7 \text{ psia}}{0.00409 \text{ slug/ft}^3} \right) (144 \text{ in.}^2/\text{ft}^2)}{1(\text{slug} \cdot \text{ft})/(\text{lb} \cdot \text{s}^2)}}$$

or

$$V_2 = 1620 \text{ ft/s} \quad (\text{Ans})$$

A considerable difference exists between the air velocities calculated assuming incompressible and compressible flow.

## Key Words and Topics

In the E-book, click on any key word or topic to go to that subject.

Available energy  
Average velocity  
Conservation of mass  
Continuity equation  
Control surface  
Control volume  
Deforming control volume  
Energy equation

First law of thermodynamics  
Head loss  
Heat transfer rate  
Kinetic energy coefficient  
Linear momentum equation  
Loss  
Mass flowrate  
Moment-of-momentum equation  
Moving control volume  
Newton's second law

Pump head  
Relative velocity  
Second law of thermodynamics  
Shaft head  
Shaft power  
Shaft torque  
Shaft work  
Turbine head  
Volume flowrate

## References

1. Eck, B., *Technische Stromungslehre*, Springer-Verlag, Berlin, Germany, 1957.
2. Dean, R. C., "On the Necessity of Unsteady Flow in Fluid Machines," *ASME Journal of Basic Engineering* 81D; 24–28, March 1959.
3. Moran, M. J., and Shapiro, H. N., *Fundamentals of Engineering Thermodynamics*, 4th Ed., Wiley, New York, 2000.

## Review Problems

In the E-book, [click here](#) to go to a set of review problems complete with answers and detailed solutions.

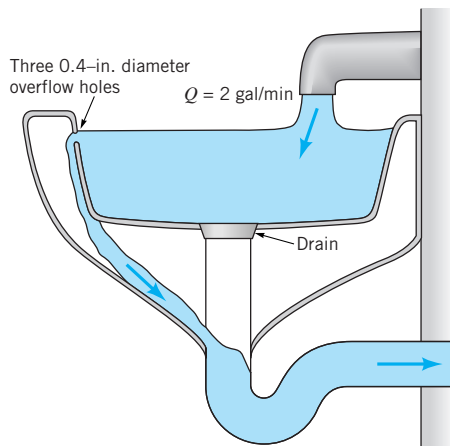
## Problems

**Note:** Unless otherwise indicated, use the values of fluid properties found in the tables on the inside of the front cover. Problems designated with an (\*) are intended to be solved with the aid of a programmable calculator or a computer. Problems des-

ignated with a (†) are "open-ended" problems and require critical thinking in that to work them one must make various assumptions and provide the necessary data. There is not a unique answer to these problems.

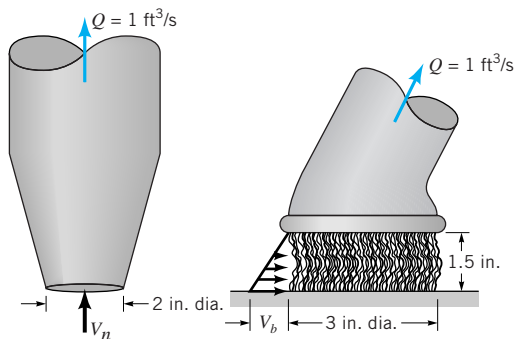
In the E-book, answers to the even-numbered problems can be obtained by clicking on the problem number. In the E-book, access to the videos that accompany problems can be obtained by clicking on the “video” segment (i.e., **Video 5.3**) of the problem statement. The lab-type problems can be accessed by clicking on the “click here” segment of the problem statement.

**5.1** Water flows into a sink as shown in **Video V5.1** and Fig. P5.1 at a rate of 2 gallons per minute. Determine the average velocity through each of the three 0.4-in. diameter overflow holes if the drain is closed and the water level in the sink remains constant.



■ FIGURE P5.1

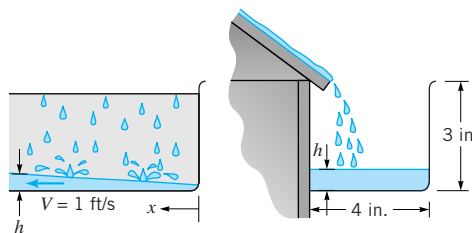
**5.2** Various types of attachments can be used with the shop vac shown in **Video V5.2**. Two such attachments are shown in Fig. P5.2—a nozzle and a brush. The flowrate is  $1 \text{ ft}^3/\text{s}$ . (a) Determine the average velocity through the nozzle entrance,  $V_n$ . (b) Assume the air enters the brush attachment in a radial direction all around the brush with a velocity profile that varies linearly from 0 to  $V_b$  along the length of the bristles as shown in the figure. Determine the value of  $V_b$ .



■ FIGURE P5.2

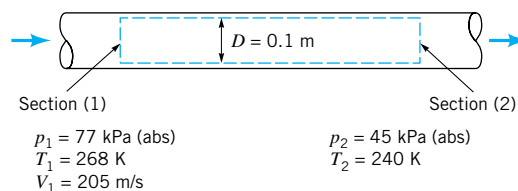
**5.3** Water flows into a rain gutter on a house as shown in Fig. P5.3 and in **Video V10.3** at a rate of  $0.0040 \text{ ft}^3/\text{s}$  per foot of length of the gutter. At the beginning of the gutter ( $x = 0$ ),

the water depth is zero. (a) If the water flows with a velocity of  $1.0 \text{ ft/s}$  throughout the entire gutter, determine an equation for the water depth,  $h$ , as a function of location,  $x$ . (b) At what location will the gutter overflow?



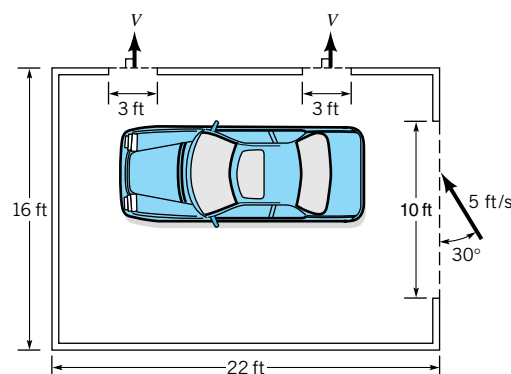
■ FIGURE P5.3

**5.4** Air flows steadily between two cross sections in a long, straight section of  $0.1\text{-m}$  inside-diameter pipe. The static temperature and pressure at each section are indicated in Fig. P5.4. If the average air velocity at section (1) is  $205 \text{ m/s}$ , determine the average air velocity at section (2).



■ FIGURE P5.4

**5.5** The wind blows through a  $7 \text{ ft} \times 10 \text{ ft}$  garage door opening with a speed of  $5 \text{ ft/s}$  as shown in Fig. P5.5. Determine the average speed,  $V$ , of the air through the two  $3 \text{ ft} \times 4 \text{ ft}$  openings in the windows.

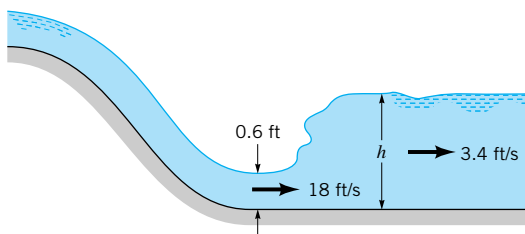


■ FIGURE P5.5

**5.6** A hydroelectric turbine passes 4 million gal/min through its blades. If the average velocity of the flow in the circular cross-sectional conduit leading to the turbine is not to exceed  $30 \text{ ft/s}$ , determine the minimum allowable diameter of the conduit.

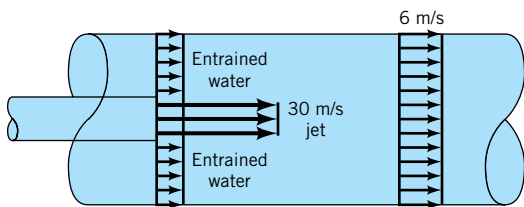
**5.7** The cross-sectional area of the test section of a large water tunnel is  $100 \text{ ft}^2$ . For a test velocity of  $50 \text{ ft/s}$ , what volume flowrate capacity in  $\text{gal/min}$  is needed?

**5.8** A hydraulic jump (see [Video V10.5](#)) is in place downstream from a spill-way as indicated in Fig. P5.8. Upstream of the jump, the depth of the stream is  $0.6 \text{ ft}$  and the average stream velocity is  $18 \text{ ft/s}$ . Just downstream of the jump, the average stream velocity is  $3.4 \text{ ft/s}$ . Calculate the depth of the stream,  $h$ , just downstream of the jump.



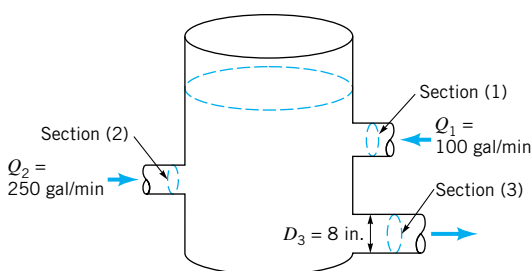
■ FIGURE P5.8

**5.9** A water jet pump (see Fig. P5.9) involves a jet cross-sectional area of  $0.01 \text{ m}^2$ , and a jet velocity of  $30 \text{ m/s}$ . The jet is surrounded by entrained water. The total cross-sectional area associated with the jet and entrained streams is  $0.075 \text{ m}^2$ . These two fluid streams leave the pump thoroughly mixed with an average velocity of  $6 \text{ m/s}$  through a cross-sectional area of  $0.075 \text{ m}^2$ . Determine the pumping rate (i.e., the entrained fluid flowrate) involved in  $\text{liters/s}$ .



■ FIGURE P5.9

**5.10** Water enters a cylindrical tank through two pipes at rates of  $250$  and  $100 \text{ gal/min}$  (see Fig. P5.10). If the level of the water in the tank remains constant, calculate the average velocity of the flow leaving the tank through an  $8\text{-in.}$  inside-diameter pipe.

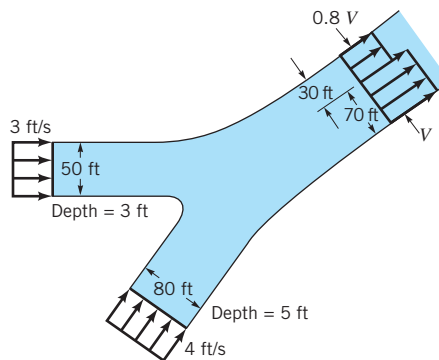


■ FIGURE P5.10

**5.11** At cruise conditions, air flows into a jet engine at a steady rate of  $65 \text{ lbm/s}$ . Fuel enters the engine at a steady rate of  $0.60 \text{ lbm/s}$ . The average velocity of the exhaust gases is  $1500 \text{ ft/s}$  relative to the engine. If the engine exhaust effective cross-sectional area is  $3.5 \text{ ft}^2$ , estimate the density of the exhaust gases in  $\text{lbm/ft}^3$ .

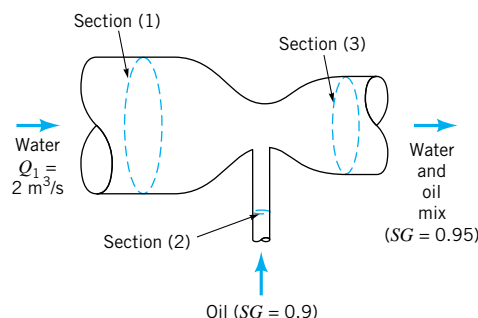
**5.12** Air at standard atmospheric conditions is drawn into a compressor at the steady rate of  $30 \text{ m}^3/\text{min}$ . The compressor pressure ratio,  $p_{\text{exit}}/p_{\text{inlet}}$ , is  $10$  to  $1$ . Through the compressor  $p/\rho^n$  remains constant with  $n = 1.4$ . If the average velocity in the compressor discharge pipe is not to exceed  $30 \text{ m/s}$ , calculate the minimum discharge pipe diameter required.

**5.13** Two rivers merge to form a larger river as shown in Fig. P5.13. At a location downstream from the junction (before the two streams completely merge), the nonuniform velocity profile is as shown and the depth is  $6 \text{ ft}$ . Determine the value of  $V$ .



■ FIGURE P5.13

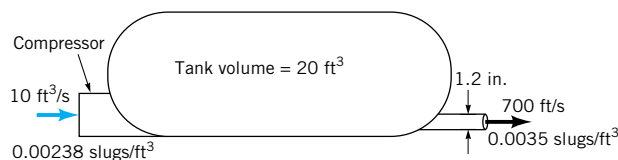
**5.14** Oil having a specific gravity of  $0.9$  is pumped as illustrated in Fig. P5.14 with a water jet pump (see [Video V3.6](#)). The water volume flowrate is  $2 \text{ m}^3/\text{s}$ . The water and oil mixture has an average specific gravity of  $0.95$ . Calculate the rate, in  $\text{m}^3/\text{s}$ , at which the pump moves oil.



■ FIGURE P5.14

**5.15** Air at standard conditions enters the compressor shown in Fig. P5.15 at a rate of  $10 \text{ ft}^3/\text{s}$ . It leaves the tank through a  $1.2\text{-in.}$ -diameter pipe with a density of  $0.0035 \text{ slugs/ft}^3$  and a uniform speed of  $700 \text{ ft/s}$ . (a) Determine the rate (slugs/s) at which the mass of air in the tank is increasing

or decreasing. **(b)** Determine the average time rate of change of air density within the tank.



■ FIGURE P5.15

**5.16** An appropriate turbulent pipe flow velocity profile is

$$\mathbf{V} = u_c \left( \frac{R - r}{R} \right)^{1/n} \hat{\mathbf{i}}$$

where  $u_c$  = centerline velocity,  $r$  = local radius,  $R$  = pipe radius, and  $\hat{\mathbf{i}}$  = unit vector along pipe centerline. Determine the ratio of average velocity,  $\bar{u}$ , to centerline velocity,  $u_c$ , for **(a)**  $n = 4$ , **(b)**  $n = 6$ , **(c)**  $n = 8$ , **(d)**  $n = 10$ .

**5.17** The velocity and temperature profiles for one circular cross section in laminar pipe flow of air with heat transfer are

$$\mathbf{V} = u_c \left[ 1 - \left( \frac{r}{R} \right)^2 \right] \hat{\mathbf{i}}$$

where the unit vector  $\hat{\mathbf{i}}$  is along the pipe axis, and

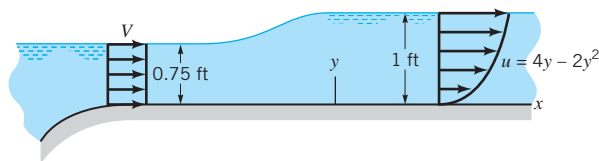
$$T = T_c \left[ 1 + \frac{1}{2} \left( \frac{r}{R} \right)^2 - \frac{1}{4} \left( \frac{r}{R} \right)^4 \right]$$

The subscript  $c$  refers to centerline value,  $r$  = local radius,  $R$  = pipe radius, and  $T$  = local temperature. Show how you would evaluate the mass flowrate through this cross-sectional area.

**\*5.18** To measure the mass flowrate of air through a 6-in. inside-diameter pipe, local velocity data are collected at different radii from the pipe axis (see Table). Determine the mass flowrate corresponding to the data listed below.

$r$ (in.)	Axial Velocity (ft/s)
0	30
0.2	29.71
0.4	29.39
0.6	29.06
0.8	28.70
1.0	28.31
1.2	27.89
1.4	27.42
1.6	26.90
1.8	26.32
2.0	25.64
2.2	24.84
2.4	23.84
2.6	22.50
2.8	20.38
2.9	18.45
2.95	16.71
2.98	14.66
3.00	0

**5.19** As shown in Fig. P5.19, at the entrance to a 3-ft-wide channel the velocity distribution is uniform with a velocity  $V$ . Further downstream the velocity profile is given by  $u = 4y - 2y^2$ , where  $u$  is in ft/s and  $y$  is in ft. Determine the value of  $V$ .

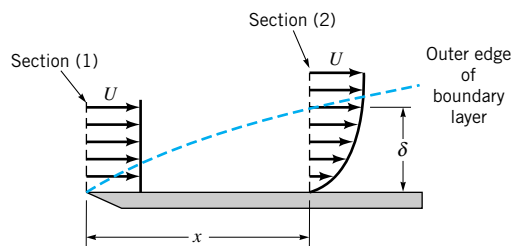


■ FIGURE P5.19

**5.20** Flow of a viscous fluid over a flat plate surface results in the development of a region of reduced velocity adjacent to the wetted surface as depicted in Fig. P5.20. This region of reduced flow is called a boundary layer. At the leading edge of the plate, the velocity profile may be considered uniformly distributed with a value  $U$ . All along the outer edge of the boundary layer, the fluid velocity component parallel to the plate surface is also  $U$ . If the  $x$  direction velocity profile at section (2) is

$$\frac{u}{U} = \left( \frac{y}{\delta} \right)^{1/7}$$

develop an expression for the volume flowrate through the edge of the boundary layer from the leading edge to a location downstream at  $x$  where the boundary layer thickness is  $\delta$ .



■ FIGURE P5.20

† **5.21** Estimate the rate (in gallons per hour) that your car uses gasoline when it is being driven on an interstate highway. Determine how long it would take to empty a 12-oz soft drink container at this flowrate. List all assumptions and show calculations.

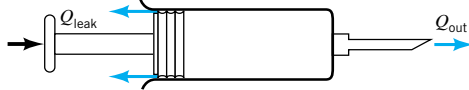
**5.22** How many hours would it take to fill a cylindrical-shaped swimming pool having a diameter of 10 m to a depth of 1.5 m with water from a garden hose if the flowrate is 1.0 liter/s?

**5.23** The Hoover Dam backs up the Colorado River and creates Lake Mead, which is approximately 115 miles long and has a surface area of approximately 225 square miles. (See [Video V2.3](#).) If during flood conditions the Colorado River flows into the lake at a rate of 45,000 cfs and the outflow from the dam is 8,000 cfs, how many feet per 24-hour day will the lake level rise?

**5.24** Storm sewer backup causes your basement to flood at the steady rate of 1 in. of depth per hour. The basement floor area is 1500 ft². What capacity (gal/min) pump would you rent to **(a)** keep the water accumulated in your basement at a con-

stant level until the storm sewer is blocked off, and **(b)** reduce the water accumulation in your basement at a rate of 3 in./hr even while the backup problem exists?

**5.25** A hypodermic syringe (see Fig. P5.25) is used to apply a vaccine. If the plunger is moved forward at the steady rate of 20 mm/s and if vaccine leaks past the plunger at 0.1 of the volume flowrate out the needle opening, calculate the average velocity of the needle exit flow. The inside diameters of the syringe and the needle are 20 mm and 0.7 mm.



■ FIGURE P5.25

† **5.26** Estimate the maximum flowrate of rainwater (during a heavy rain) that you would expect from the downspout connected to the gutters of your house. List all assumptions and show all calculations.

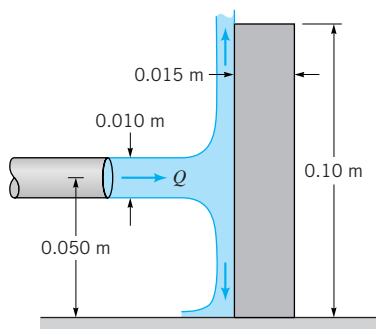
**5.27** It takes you 1 min to fill your car's fuel tank with 13.5 gallons of gasoline. What is the approximate average velocity of the gasoline leaving the nozzle at this pump?

**5.28** A gas flows steadily through a duct of varying cross-sectional area. If the gas density is assumed to be uniformly distributed at any cross section, show that the conservation of mass principle leads to

$$\frac{d\rho}{\rho} + \frac{d\bar{V}}{\bar{V}} + \frac{dA}{A} = 0$$

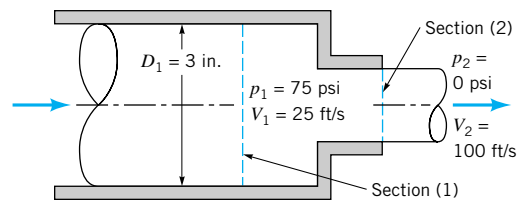
where  $\rho$  = gas density,  $\bar{V}$  = average speed of gas, and  $A$  = cross-sectional area.

**5.29** A 10-mm diameter jet of water is deflected by a homogeneous rectangular block (15 mm by 200 mm by 100 mm) that weighs 6 N as shown in Video V5.4 and Fig. P5.29. Determine the minimum volume flowrate needed to tip the block.



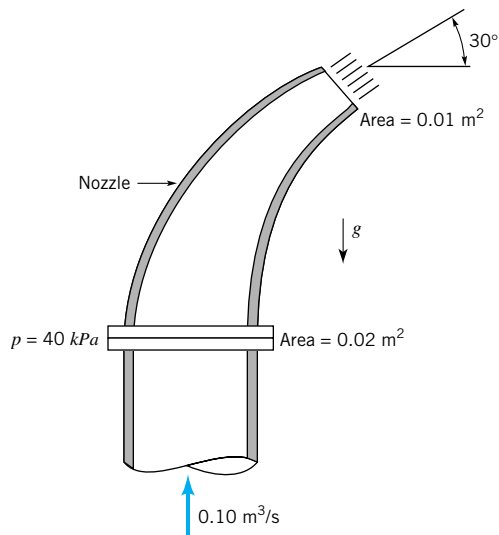
■ FIGURE P5.29

**5.30** Water enters the horizontal, circular cross-sectional, sudden contraction nozzle sketched in Fig. P5.30 at section (1) with a uniformly distributed velocity of 25 ft/s and a pressure of 75 psi. The water exits from the nozzle into the atmosphere at section (2) where the uniformly distributed velocity is 100 ft/s. Determine the axial component of the anchoring force required to hold the contraction in place.



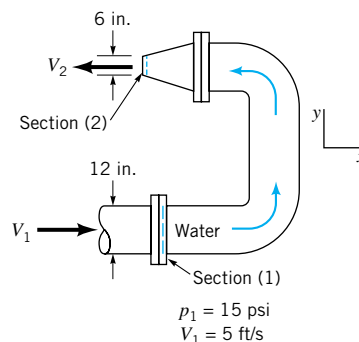
■ FIGURE P5.30

**5.31** A nozzle is attached to a vertical pipe and discharges water into the atmosphere as shown in Fig. P5.31. When the discharge is  $0.1 \text{ m}^3/\text{s}$ , the gage pressure at the flange is 40 kPa. Determine the vertical component of the anchoring force required to hold the nozzle in place. The nozzle has a weight of 200 N, and the volume of water in the nozzle is  $0.012 \text{ m}^3$ . Is the anchoring force directed upward or downward?



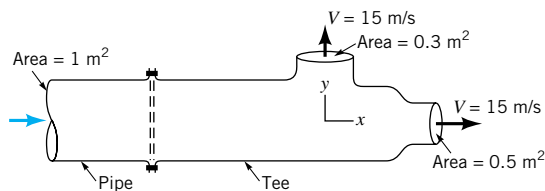
■ FIGURE P5.31

**5.32** Determine the magnitude and direction of the  $x$  and  $y$  components of the anchoring force required to hold in place the horizontal  $180^\circ$  elbow and nozzle combination shown in Fig. P5.32. Also determine the magnitude and direction of the  $x$  and  $y$  components of the reaction force exerted by the  $180^\circ$  elbow and nozzle on the flowing water.



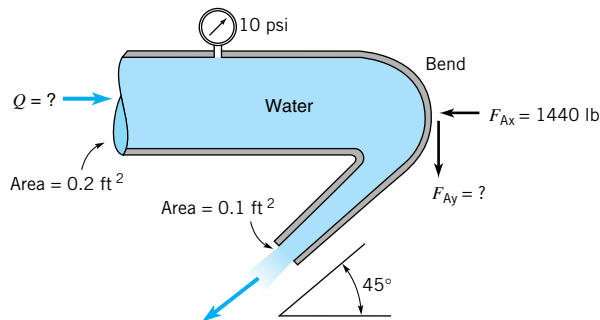
■ FIGURE P5.32

**5.33** Water flows as two free jets from the tee attached to the pipe shown in Fig. P5.33. The exit speed is 15 m/s. If viscous effects and gravity are negligible, determine the  $x$  and  $y$  components of the force that the pipe exerts on the tee.



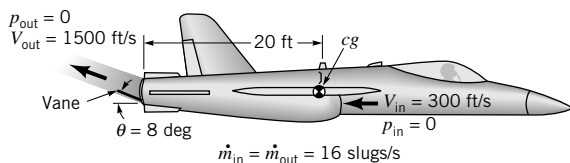
■ FIGURE P5.33

**5.34** Water flows through a horizontal bend and discharges into the atmosphere as shown in Fig. P5.34. When the pressure gage reads 10 psi, the resultant  $x$  direction anchoring force,  $F_{Ax}$ , in the horizontal plane required to hold the bend in place is shown on the figure. Determine the flowrate through the bend and the  $y$  direction anchoring force,  $F_{Ay}$ , required to hold the bend in place. The flow is not frictionless.



■ FIGURE P5.34

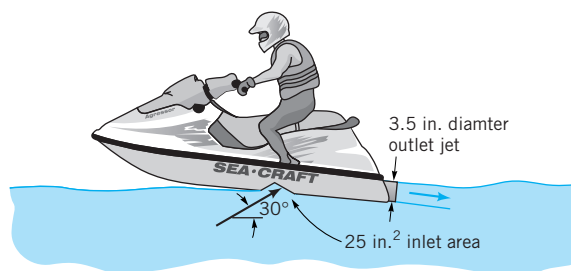
**5.35** Thrust vector control is a new technique that can be used to greatly improve the maneuverability of military fighter aircraft. It consists of using a set of vanes in the exit of a jet engine to deflect the exhaust gases as shown in Fig. P5.35. (a) Determine the pitching moment (the moment tending to rotate the nose of the aircraft up) about the aircraft's mass center (cg) for the conditions indicated in the figure. (b) By how much is the thrust (force along the centerline of the aircraft) reduced for the case indicated compared to normal flight when the exhaust is parallel to the centerline?



■ FIGURE P5.35

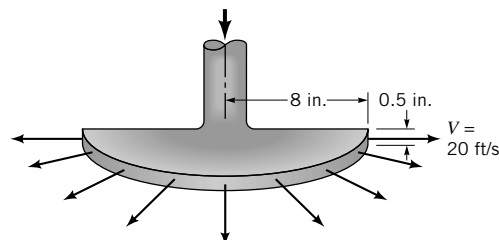
**5.36** The thrust developed to propel the jet ski shown in Video V9.7 and Fig. P5.36 is a result of water pumped through

the vehicle and exiting as a high-speed water jet. For the conditions shown in the figure, what flowrate is needed to produce a 300 lb thrust? Assume the inlet and outlet jets of water are free jets.



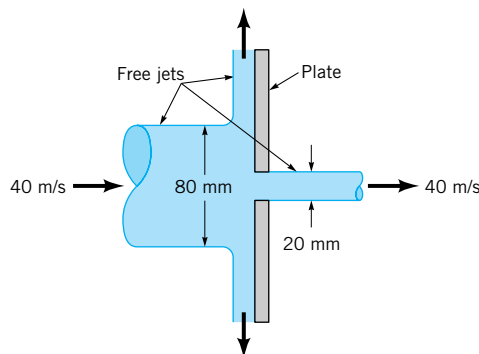
■ FIGURE P5.36

**5.37** Water is sprayed radially outward over 180° as indicated in Fig. P5.37. The jet sheet is in the horizontal plane. If the jet velocity at the nozzle exit is 20 ft/s, determine the direction and magnitude of the resultant horizontal anchoring force required to hold the nozzle in place.



■ FIGURE P5.37

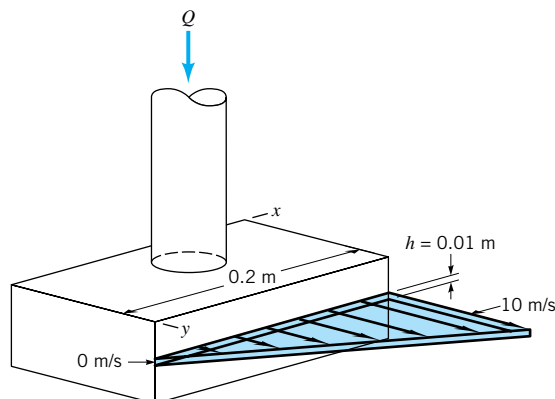
**5.38** A circular plate having a diameter of 300 mm is held perpendicular to an axisymmetric horizontal jet of air having a velocity of 40 m/s and a diameter of 80 mm as shown in Fig. P5.38. A hole at the center of the plate results in a discharge jet of air having a velocity of 40 m/s and a diameter of 20 mm. Determine the horizontal component of force required to hold the plate stationary.



■ FIGURE P5.38



**5.39** A sheet of water of uniform thickness ( $h = 0.01$  m) flows from the device shown in Fig. P5.39. The water enters vertically through the inlet pipe and exits horizontally with a speed that varies linearly from 0 to 10 m/s along the 0.2-m length of the slit. Determine the  $y$  component of anchoring force necessary to hold this device stationary.



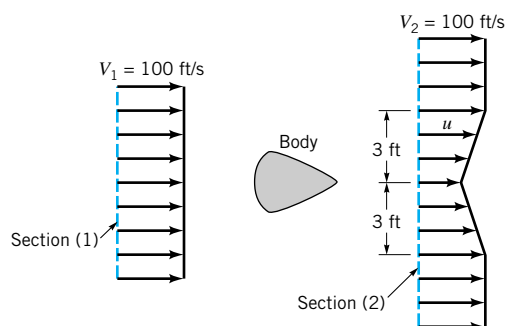
■ FIGURE P5.39

**5.40** The results of a wind tunnel test to determine the drag on a body (see Fig. P5.40) are summarized below. The upstream [section (1)] velocity is uniform at 100 ft/s. The static pressures are given by  $p_1 = p_2 = 14.7$  psia. The downstream velocity distribution, which is symmetrical about the centerline, is given by

$$u = 100 - 30 \left( 1 - \frac{|y|}{3} \right) \quad |y| \leq 3 \text{ ft}$$

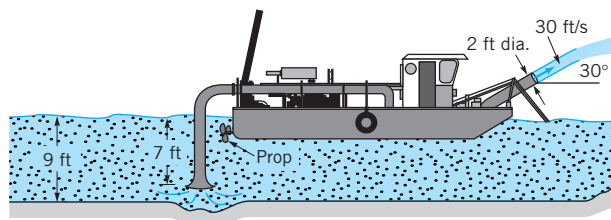
$$u = 100 \quad |y| > 3 \text{ ft}$$

where  $u$  is the velocity in ft/s and  $y$  is the distance on either side of the centerline in feet (see Fig. P5.40). Assume that the body shape does not change in the direction normal to the paper. Calculate the drag force (reaction force in  $x$  direction) exerted on the air by the body per unit length normal to the plane of the sketch.



■ FIGURE P5.40

**5.41** The hydraulic dredge shown in Fig. P5.41 is used to dredge sand from a river bottom. Estimate the thrust needed from the propeller to hold the boat stationary. Assume the specific gravity of the sand/water mixture is  $SG = 1.2$ .

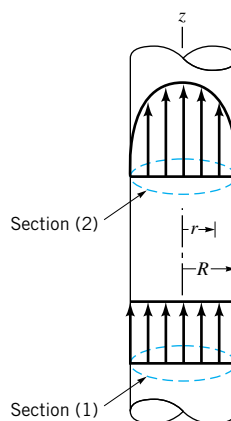


■ FIGURE P5.41

**5.42** Water flows vertically upward in a circular cross-sectional pipe as shown in Fig. P5.42. At section (1), the velocity profile over the cross-sectional area is uniform. At section (2), the velocity profile is

$$\mathbf{V} = w_c \left( \frac{R - r}{R} \right)^{1/7} \hat{\mathbf{k}}$$

where  $\mathbf{V}$  = local velocity vector,  $w_c$  = centerline velocity in the axial direction,  $R$  = pipe radius, and  $r$  = radius from pipe axis. Develop an expression for the fluid pressure drop that occurs between sections (1) and (2).

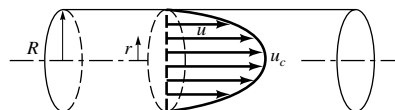


■ FIGURE P5.42

**5.43** In a laminar pipe flow that is fully developed, the axial velocity profile is parabolic. That is,

$$u = u_c \left[ 1 - \left( \frac{r}{R} \right)^2 \right]$$

as is illustrated in Fig. P5.43. Compare the axial direction momentum flowrate calculated with the average velocity,  $\bar{u}$ , with the axial direction momentum flowrate calculated with the nonuniform velocity distribution taken into account.



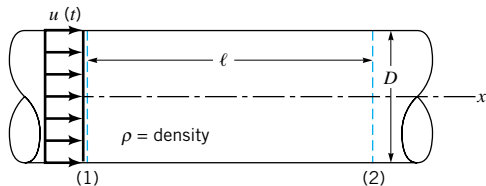
■ FIGURE P5.43

**\*5.44** For the pipe (6-in. inside diameter) air flow data of Problem 5.18, calculate the rate of flow of axial direction mo-



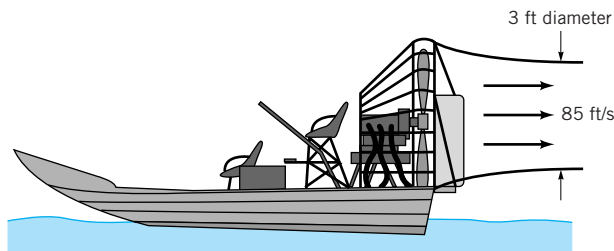
mentum. How large would the error be if the average axial velocity were used to calculate axial direction momentum flow?

**5.45** Consider unsteady flow in the constant diameter, horizontal pipe shown in Fig. P5.45. The velocity is uniform throughout the entire pipe, but it is a function of time:  $\mathbf{V} = u(t)\hat{\mathbf{i}}$ . Use the  $x$  component of the unsteady momentum equation to determine the pressure difference  $p_1 - p_2$ . Discuss how this result is related to  $F_x = ma_x$ .



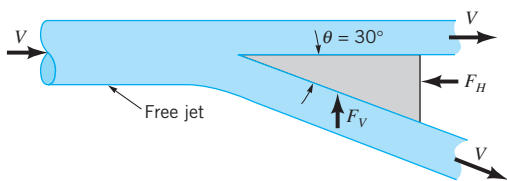
■ FIGURE P5.45

**5.46** The propeller on a swamp boat produces a jet of air having a diameter of 3 ft as illustrated in Fig. P5.46. The ambient air temperature is 80 °F, and the axial velocity of the flow is 85 ft/s relative to the boat. What propulsive forces are produced by the propeller when the boat is stationary and when the boat moves forward with a constant velocity of 20 ft/s?



■ FIGURE P5.46

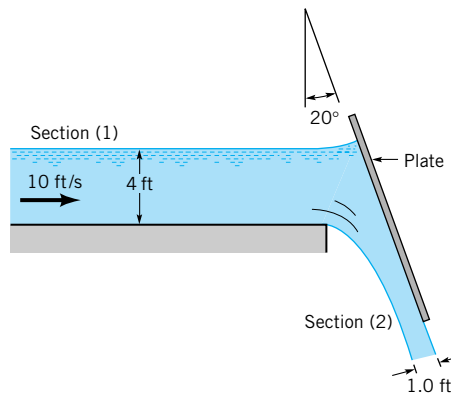
**5.47** A free jet of fluid strikes a wedge as shown in Fig. P5.47. Of the total flow, a portion is deflected 30°; the remainder is not deflected. The horizontal and vertical components of force needed to hold the wedge stationary are  $F_H$  and  $F_V$ , respectively. Gravity is negligible, and the fluid speed remains constant. Determine the force ratio,  $F_H/F_V$ .



■ FIGURE P5.47

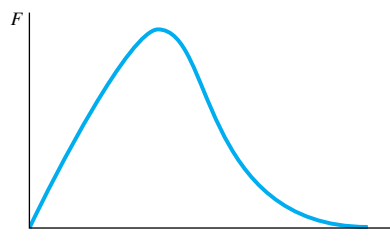
**5.48** Water flows from a two-dimensional open channel and is diverted by an inclined plate as illustrated in Fig. P5.48. When the velocity at section (1) is 10 ft/s, what horizontal force (per unit width) is required to hold the plate in position? At section

(1) the pressure distribution is hydrostatic, and the fluid acts as a free jet at section (2). Neglect friction.



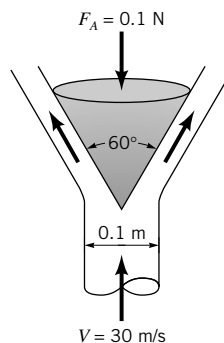
■ FIGURE P5.48

† **5.49** When a baseball player catches a ball, the force of the ball on her glove is as shown as a function of time in Fig. P5.49. Describe how this situation is similar to the force generated by the deflection of a jet of water by a vane. Note: Consider many baseballs being caught in quick succession.



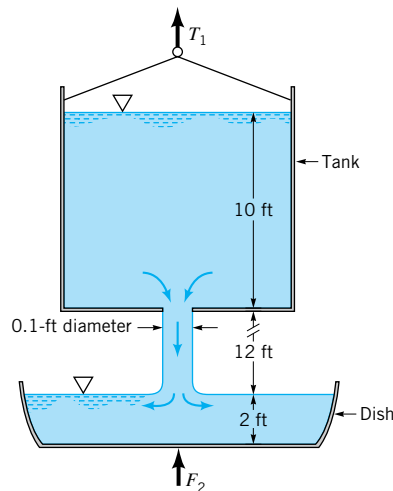
■ FIGURE P5.49

**5.50** A vertical, circular cross-sectional jet of air strikes a conical deflector as indicated in Fig. P5.50. A vertical anchoring force of 0.1 N is required to hold the deflector in place. Determine the mass (kg) of the deflector. The magnitude of velocity of the air remains constant.



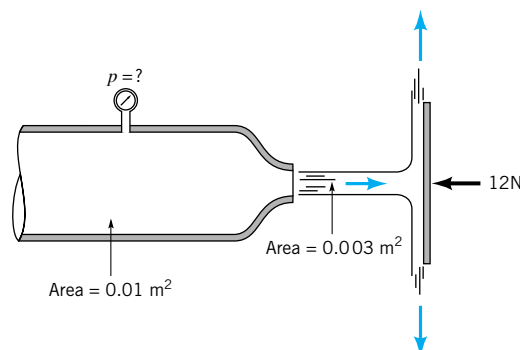
■ FIGURE P5.50

**5.51** Water flows from a large tank into a dish as shown in Fig. P5.51. (a) If at the instant shown the tank and the water in it weigh  $W_1$  lb, what is the tension,  $T_1$ , in the cable supporting the tank? (b) If at the instant shown the dish and the water in it weigh  $W_2$  lb, what is the force,  $F_2$ , needed to support the dish?



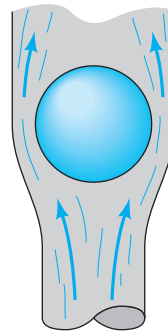
■ FIGURE P5.51

**5.52** Air flows into the atmosphere from a nozzle and strikes a vertical plate as shown in Fig. P5.52. A horizontal force of 12 N is required to hold the plate in place. Determine the reading on the pressure gage. Assume the flow to be incompressible and frictionless.



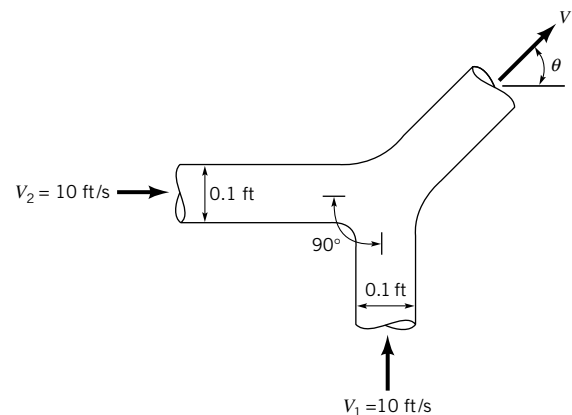
■ FIGURE P5.52

† **5.53** A table tennis ball “balances” on a jet of air as shown in Video V3.1 and in Fig. P5.53. Explain why the ball sits at the height it does and why the ball does not “roll off” the jet.



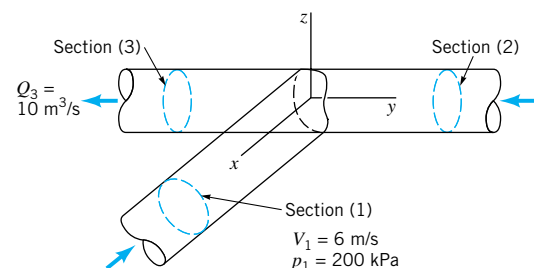
■ FIGURE P5.53

**5.54** Two water jets of equal size and speed strike each other as shown in Fig. P5.54. Determine the speed,  $V$ , and direction,  $\theta$ , of the resulting combined jet. Gravity is negligible.



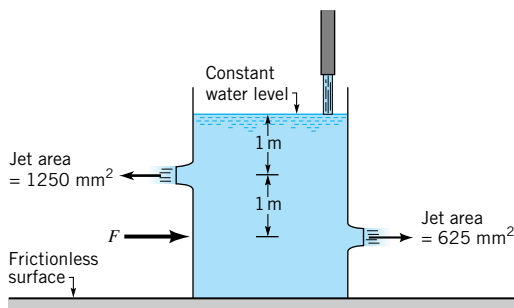
■ FIGURE P5.54

**5.55** Assuming frictionless, incompressible, one-dimensional flow of water through the horizontal tee connection sketched in Fig. P5.55, estimate values of the  $x$  and  $y$  components of the force exerted by the tee on the water. Each pipe has an inside diameter of 1 m.



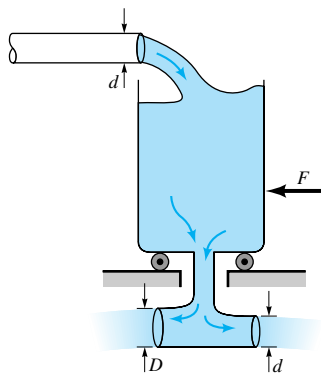
■ FIGURE P5.55

**5.56** Water is added to the tank shown in Fig. P5.56 through a vertical pipe to maintain a constant (water) level. The tank is placed on a horizontal plane which has a frictionless surface. Determine the horizontal force,  $F$ , required to hold the tank stationary. Neglect all losses.



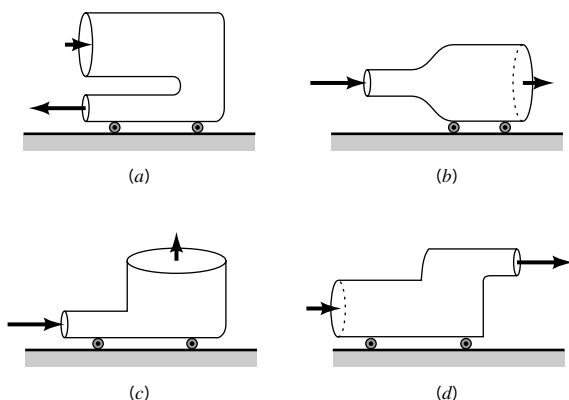
■ FIGURE P5.56

**5.57** Water flows steadily into and out of a tank that sits on frictionless wheels as shown in Fig. P5.57. Determine the diameter  $D$  so that the tank remains motionless if  $F = 0$ .



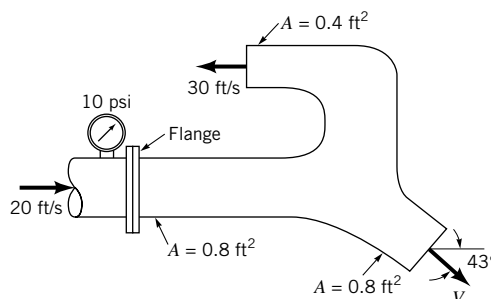
■ FIGURE P5.57

**5.58** The four devices shown in Fig. P5.58 rest on frictionless wheels, are restricted to move in the  $x$  direction only and are initially held stationary. The pressure at the inlets and outlets of each is atmospheric, and the flow is incompressible. The contents of each device is not known. When released, which devices will move to the right and which to the left? Explain.



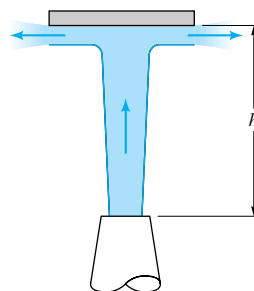
■ FIGURE P5.58

**5.59** Water discharges into the atmosphere through the device shown in Fig. P5.59. Determine the  $x$  component of force at the flange required to hold the device in place. Neglect the effect of gravity and friction.



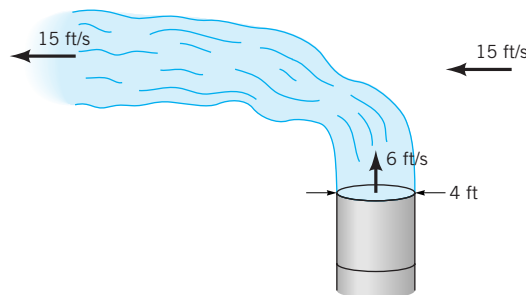
■ FIGURE P5.59

**5.60** A vertical jet of water leaves a nozzle at a speed of  $10 \text{ m/s}$  and a diameter of  $20 \text{ mm}$ . It suspends a plate having a mass of  $1.5 \text{ kg}$  as indicated in Fig. P5.60. What is the vertical distance  $h$ ?



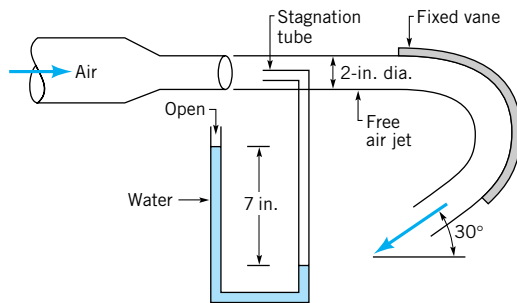
■ FIGURE P5.60

**5.61** Exhaust (assumed to have the properties of standard air) leaves the 4-ft-diameter chimney shown in Video V5.3 and Fig. P5.61 with a speed of  $6 \text{ ft/s}$ . Because of the wind, after a few diameters downstream the exhaust flows in a horizontal direction with the speed of the wind,  $15 \text{ ft/s}$ . Determine the horizontal component of the force that the blowing wind puts on the exhaust gases.



■ FIGURE P5.61

**5.62** Air discharges from a 2-in.-diameter nozzle and strikes a curved vane, which is in a vertical plane as shown in Fig. P5.62. A stagnation tube connected to a water U-tube manometer is located in the free air jet. Determine the horizontal component of the force that the air jet exerts on the vane. Neglect the weight of the air and all friction.

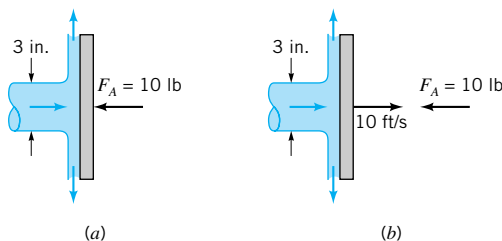


■ FIGURE P5.62

† **5.63** Water from a garden hose is sprayed against your car to rinse dirt from it. Estimate the force that the water exerts on the car. List all assumptions and show calculations.

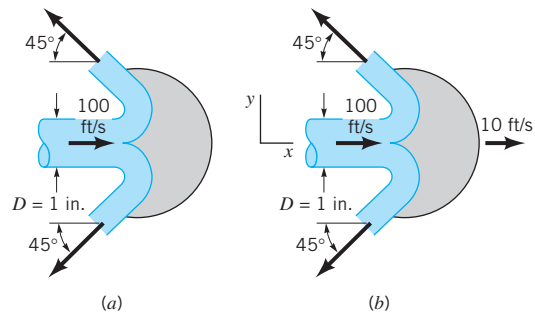
† **5.64** A truck carrying chickens is too heavy for a bridge that it needs to cross. The empty truck is within the weight limits; with the chickens it is overweight. It is suggested that if one could get the chickens to fly around the truck (i.e., by banging on the truck side) it would be safe to cross the bridge. Do you agree? Explain.

**5.65** A 3-in.-diameter horizontal jet of water strikes a flat plate as indicated in Fig. P5.65. Determine the jet velocity if a 10-lb horizontal force is required to (a) hold the plate stationary, (b) allow the plate to move at a constant speed of 10 ft/s to the right.



■ FIGURE P5.65

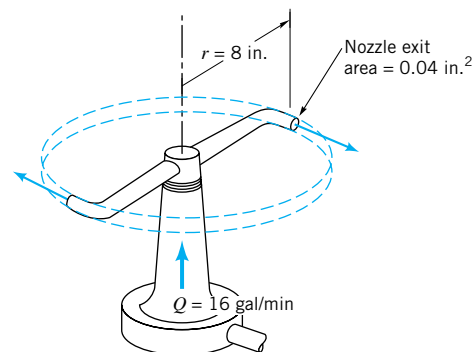
**5.66** A Pelton wheel vane directs a horizontal, circular cross-sectional jet of water symmetrically as indicated in Fig. P5.66 and Video V5.4. The jet leaves the nozzle with a velocity of 100 ft/s. Determine the  $x$  direction component of anchoring force required to (a) hold the vane stationary, (b) confine the speed of the vane to a value of 10 ft/s to the right. The fluid speed magnitude remains constant along the vane surface.



■ FIGURE P5.66

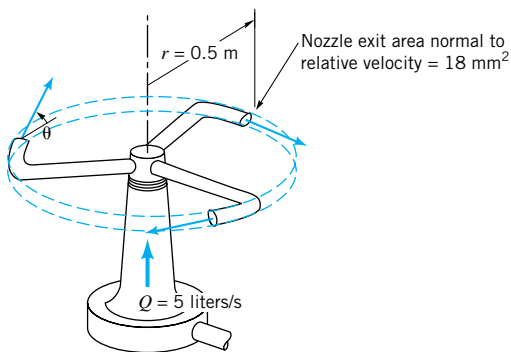
**5.67** How much power is transferred to the moving vane of Problem 5.66?

**5.68** Water enters a rotating lawn sprinkler through its base at the steady rate of 16 gal/min as shown in Fig. P5.68. The exit cross-sectional area of each of the two nozzles is  $0.04 \text{ in.}^2$ , and the flow leaving each nozzle is tangential. The radius from the axis of rotation to the centerline of each nozzle is 8 in. (a) Determine the resisting torque required to hold the sprinkler head stationary. (b) Determine the resisting torque associated with the sprinkler rotating with a constant speed of 500 rev/min. (c) Determine the angular velocity of the sprinkler if no resisting torque is applied.



■ FIGURE P5.68

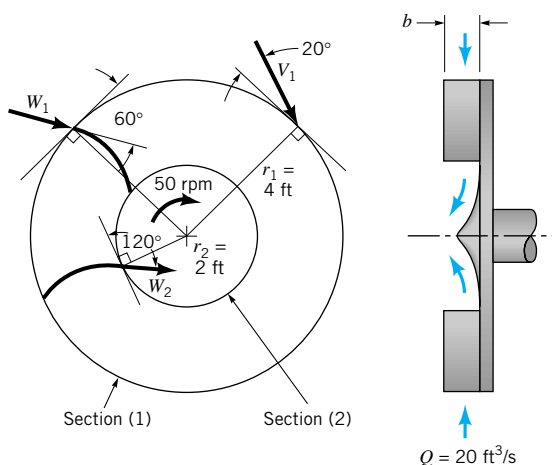
**5.69** Five liters/s of water enter the rotor shown in Video V5.5 and Fig. P5.69 along the axis of rotation. The cross-sectional area of each of the three nozzle exits normal to the relative velocity is  $18 \text{ mm}^2$ . How large is the resisting torque required to hold the rotor stationary? How fast will the rotor spin steadily if the resisting torque is reduced to zero and (a)  $\theta = 0^\circ$ , (b)  $\theta = 30^\circ$ , (c)  $\theta = 60^\circ$ ?



■ FIGURE P5.69

† 5.70 After examining an impulse-type lawn sprinkler like the one shown in Video V5.6, explain how it works.

5.71 A water turbine wheel rotates at the rate of 50 rpm in the direction shown in Fig. P5.71. The inner radius,  $r_2$ , of the blade row is 2 ft, and the outer radius,  $r_1$ , is 4 ft. The absolute velocity vector at the turbine rotor entrance makes an angle of  $20^\circ$  with the tangential direction. The inlet blade angle is  $60^\circ$  relative to the tangential direction. The blade outlet angle is  $120^\circ$ . The flowrate is  $20 \text{ ft}^3/\text{s}$ . For the flow tangent to the rotor blade surface at inlet and outlet, determine an appropriate constant blade height,  $b$ , and the corresponding power available at the rotor shaft.



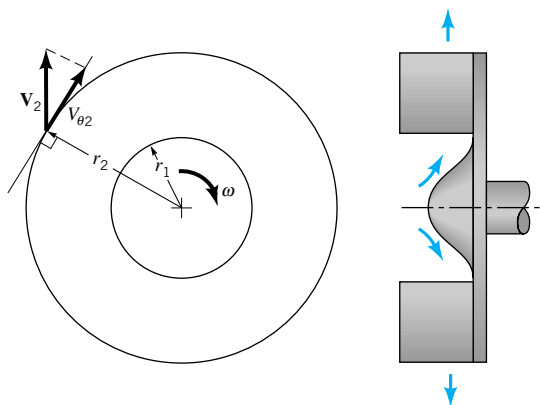
■ FIGURE P5.71

5.72 An incompressible fluid flows outward through a blower as indicated in Fig. P5.72. The shaft torque involved,  $T_{\text{shaft}}$ , is estimated with the following relationship:

$$T_{\text{shaft}} = \dot{m} r_2 V_{\theta 2}$$

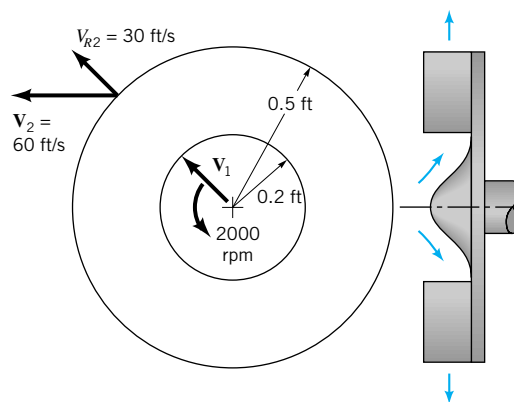
where  $\dot{m}$  = mass flowrate through the blower,  $r_2$  = outer radius of blower, and  $V_{\theta 2}$  = tangential component of absolute

fluid velocity leaving the blower. State the flow conditions that make this formula valid.



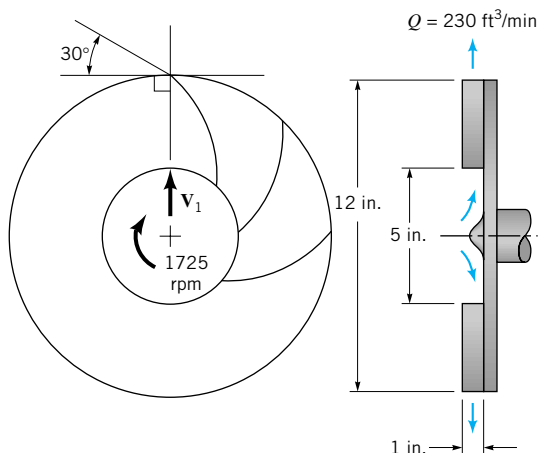
■ FIGURE P5.72

5.73 The radial component of velocity of water leaving the centrifugal pump sketched in Fig. P5.73 is  $30 \text{ ft/s}$ . The magnitude of the absolute velocity at the pump exit is  $60 \text{ ft/s}$ . The fluid enters the pump rotor radially. Calculate the shaft work required per unit mass flowing through the pump.



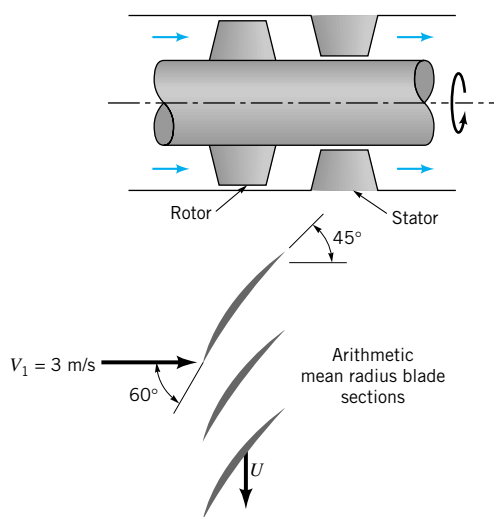
■ FIGURE P5.73

5.74 A fan (see Fig. P5.74) has a bladed rotor of 12-in. outside diameter and 5-in. inside diameter and runs at 1725 rpm. The width of each rotor blade is 1 in. from blade inlet to outlet. The volume flowrate is steady at  $230 \text{ ft}^3/\text{min}$ , and the absolute velocity of the air at blade inlet,  $V_1$ , is purely radial. The blade discharge angle is  $30^\circ$  measured with respect to the tangential direction at the outside diameter of the rotor. (a) What would be a reasonable blade inlet angle (measured with respect to the tangential direction at the inside diameter of the rotor)? (b) Find the power required to run the fan.



■ FIGURE P5.74

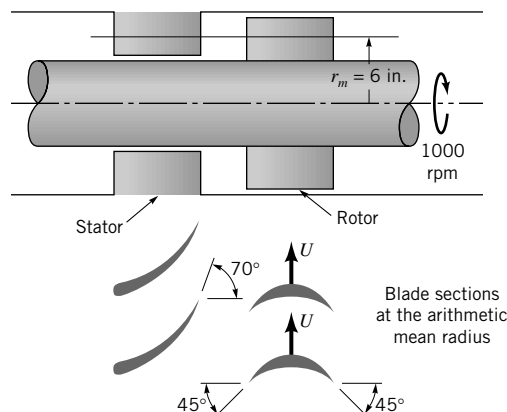
**5.75** An axial flow gasoline pump (see Fig. P5.75) consists of a rotating row of blades (rotor) followed downstream by a stationary row of blades (stator). The gasoline enters the rotor axially (without any angular momentum) with an absolute velocity of 3 m/s. The rotor blade inlet and exit angles are  $60^\circ$  and  $45^\circ$  from the axial direction. The pump annulus passage cross-sectional area is constant. Consider the flow as being tangent to the blades involved. Sketch velocity triangles for flow just upstream and downstream of the rotor and just downstream of the stator where the flow is axial. How much energy is added to each kilogram of gasoline?



■ FIGURE P5.75

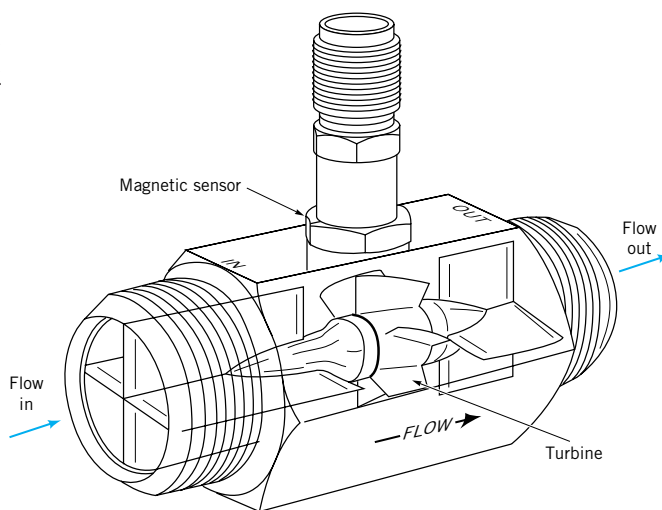
**5.76** A sketch of the arithmetic mean radius blade sections of an axial-flow water turbine stage is shown in Fig. P5.76. The rotor speed is 1000 rpm. (a) Sketch and label velocity triangles for the flow entering and leaving the rotor row. Use  $\mathbf{V}$  for absolute velocity,  $\mathbf{W}$  for relative velocity, and  $\mathbf{U}$  for blade velocity. Assume flow enters and leaves each blade row at the blade

angles shown. (b) Calculate the work per unit mass delivered at the shaft.



■ FIGURE P5.76

**5.77** Sketch the velocity triangles for the flows entering and leaving the rotor of the turbine-type flow meter shown in Fig. P5.77. Show how rotor angular velocity is proportional to average fluid velocity.



■ FIGURE P5.77 (Courtesy of EG&amp;G Flow Technology, Inc.)

**5.78** By using velocity triangles for flow upstream (1) and downstream (2) of a turbomachine rotor, prove that the shaft work in per unit mass flowing through the rotor is

$$w_{\text{shaft net in}} = \frac{V_2^2 - V_1^2 - U_2^2 - U_1^2 + W_1^2 - W_2^2}{2}$$

where  $V$  = absolute flow velocity magnitude,  $W$  = relative flow velocity magnitude, and  $U$  = blade speed.

**\*5.79** Summarized next are air flow data for flow across a low-speed axial-flow fan. Calculate the change in rate of flow of axial direction angular momentum across this rotor and evaluate the shaft power input involved. The inner and outer radii

of the fan annulus are 142 and 203 mm, respectively. The rotor speed is 2400 rpm.

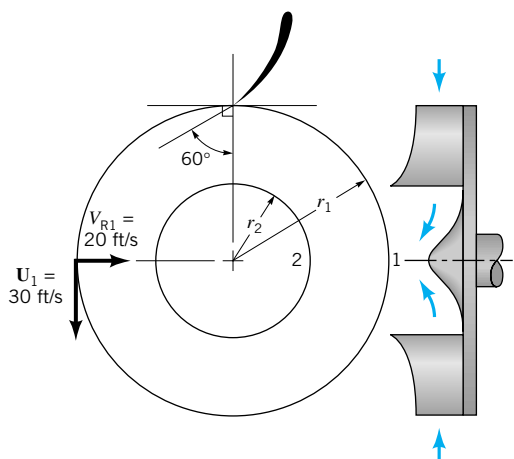
Radius (mm)	Upstream of Rotor		Downstream of Rotor	
	Axial Velocity (m/s)	Absolute Tangential Velocity (m/s)	Axial Velocity (m/s)	Absolute Tangential Velocity (m/s)
142	0	0	0	0
148	32.03	0	32.28	12.64
169	32.03	0	32.37	12.24
173	32.04	0	31.78	11.91
185	32.03	0	31.50	11.35
197	31.09	0	29.64	11.66
203	0	0	0	0

**5.80** Air enters a radial blower with zero angular momentum. It leaves with an absolute tangential velocity,  $V_\theta$ , of 200 ft/s. The rotor blade speed at rotor exit is 170 ft/s. If the stagnation pressure rise across the rotor is 0.4 psi, calculate the loss of available energy across the rotor and the rotor efficiency.

**5.81** Water enters a pump impeller radially. It leaves the impeller with a tangential component of absolute velocity of 10 m/s. The impeller exit diameter is 60 mm, and the impeller speed is 1800 rpm. If the stagnation pressure rise across the impeller is 45 kPa, determine the loss of available energy across the impeller and the hydraulic efficiency of the pump.

**5.82** Water enters an axial-flow turbine rotor with an absolute velocity tangential component,  $V_\theta$ , of 15 ft/s. The corresponding blade velocity,  $U$ , is 50 ft/s. The water leaves the rotor blade row with no angular momentum. If the stagnation pressure drop across the turbine is 12 psi, determine the hydraulic efficiency of the turbine.

**5.83** An inward flow radial turbine (see Fig. P5.83) involves a nozzle angle,  $\alpha_1$ , of  $60^\circ$  and an inlet rotor tip speed,  $U_1$ , of 30 ft/s. The ratio of rotor inlet to outlet diameters is 2.0. The radial component of velocity remains constant at 20 ft/s through the rotor, and the flow leaving the rotor at section (2) is with-



■ FIGURE P5.83

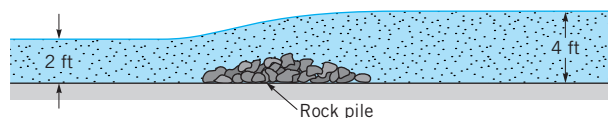
out angular momentum. If the flowing fluid is water and the stagnation pressure drop across the rotor is 16 psi, determine the loss of available energy across the rotor and the hydraulic efficiency involved.

**5.84** An inward flow radial turbine (see Fig. P5.83) involves a nozzle angle,  $\alpha_1$ , of  $60^\circ$  and an inlet rotor tip speed of 30 ft/s. The ratio of rotor inlet to outlet diameters is 2.0. The radial component of velocity remains constant at 20 ft/s through the rotor, and the flow leaving the rotor at section (2) is without angular momentum. If the flowing fluid is air and the static pressure drop across the rotor is 0.01 psi, determine the loss of available energy across the rotor and the rotor aerodynamic efficiency.

**5.85** How much available energy is lost during the process shown in Video V5.7?

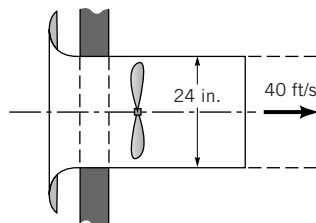
**5.86** What is the size of the head loss that is needed to raise the temperature of water by  $1^\circ\text{F}$ ?

**5.87** A 100-ft-wide river with a flowrate of 2400 ft<sup>3</sup>/s flows over a rock pile as shown in Fig. P5.87. Determine the direction of flow and the head loss associated with the flow across the rock pile.



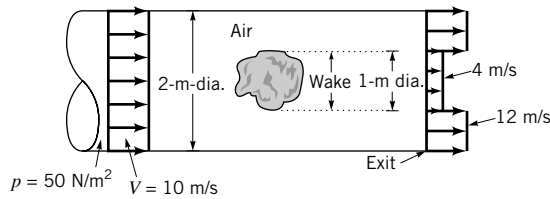
■ FIGURE P5.87

**5.88** If a  $\frac{3}{4}$ -hp motor is required by a ventilating fan to produce a 24-in. stream of air having a velocity of 40 ft/s as shown in Fig. P5.88, estimate (a) the efficiency of the fan and (b) the thrust of the supporting member on the conduit enclosing the fan.



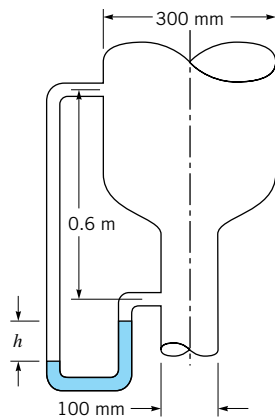
■ FIGURE P5.88

**5.89** Air flows past an object in a pipe of 2-m diameter and exits as a free jet as shown in Fig. P5.89. The velocity and pressure upstream are uniform at 10 m/s and 50 N/m<sup>2</sup>, respectively. At the pipe exit the velocity is nonuniform as indicated. The shear stress along the pipe wall is negligible. (a) Determine the head loss associated with a particle as it flows from the uniform velocity upstream of the object to a location in the wake at the exit plane of the pipe. (b) Determine the force that the air puts on the object.



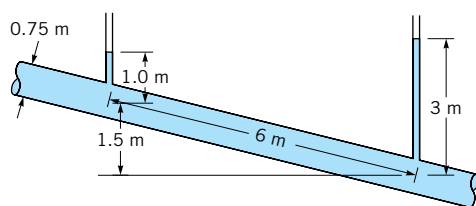
■ FIGURE P5.89

**5.90** Oil ( $SG = 0.9$ ) flows downward through a vertical pipe contraction as shown in Fig. P5.90. If the mercury manometer reading,  $h$ , is 100 mm, determine the volume flowrate for frictionless flow. Is the actual flowrate more or less than the frictionless value? Explain.



■ FIGURE P5.90

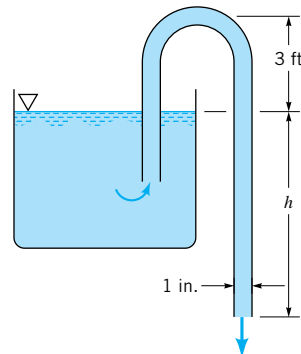
**5.91** An incompressible liquid flows steadily along the pipe shown in Fig. P5.91. Determine the direction of flow and the head loss over the 6-m length of pipe.



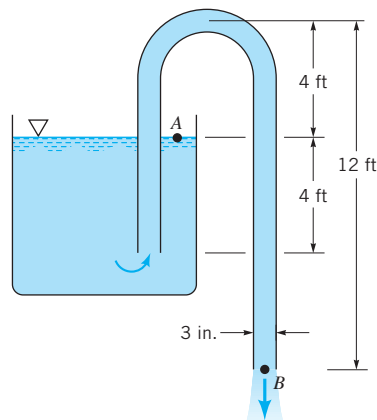
■ FIGURE P5.91

**5.92** A siphon is used to draw water at 70 °F from a large container as indicated in Fig. P5.92. The inside diameter of the siphon line is 1 in., and the pipe centerline rises 3 ft above the essentially constant water level in the tank. Show that by varying the length of the siphon below the water level,  $h$ , the rate of flow through the siphon can be changed. Assuming frictionless flow, determine the maximum flowrate possible through the siphon. The limiting condition is the occurrence of cavitation in the siphon. Will the actual maximum flow be more or less than the frictionless value? Explain.

**5.93** A water siphon having a constant inside diameter of 3 in. is arranged as shown in Fig. P5.93. If the friction loss between A and B is  $0.8V^2/2$ , where  $V$  is the velocity of flow in the siphon, determine the flowrate involved.

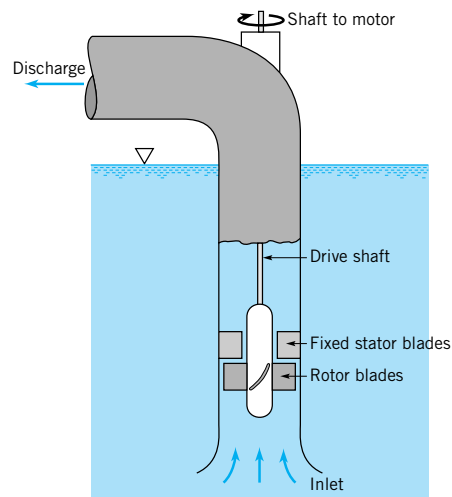


■ FIGURE P5.92



■ FIGURE P5.93

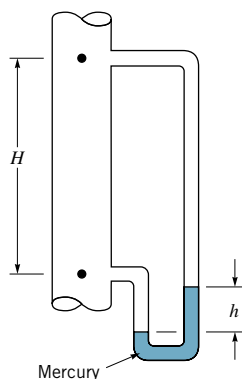
† **5.94** Explain how you would estimate the fluid pressure at the inlet of the axial-flow pump shown in Fig. P5.94. Is this pressure greater or less than atmospheric pressure? Comment on the placement of the pump in the pipe to avoid cavitation in the pump.



■ FIGURE P5.94



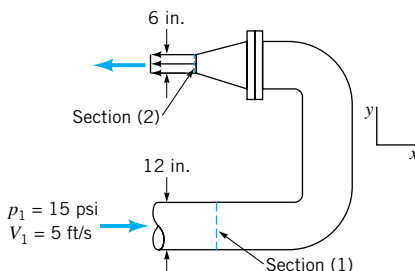
**5.95** Water flows through a vertical pipe, as is indicated in Fig. P5.95. Is the flow up or down in the pipe? Explain.



■ FIGURE P5.95

**5.96** A fire hose nozzle is designed to deliver water that will rise 40 m vertically. Calculate the stagnation pressure required at the nozzle inlet if (a) no loss is assumed, (b) a loss of  $30 \text{ N} \cdot \text{m/kg}$  is assumed.

**5.97** For the  $180^\circ$  elbow and nozzle flow shown in Fig. P5.97, determine the loss in available energy from section (1) to section (2). How much additional available energy is lost from section (2) to where the water comes to rest?



■ FIGURE P5.97

**5.98** An automobile engine will work best when the back pressure at the interface of the exhaust manifold and the engine block is minimized. Show how reduction of losses in the exhaust manifold, piping, and muffler will also reduce the back pressure. How could losses in the exhaust system be reduced? What primarily limits the minimization of exhaust system losses?

**5.99** Water flows vertically upward in a circular cross-sectional pipe. At section (1), the velocity profile over the cross-sectional area is uniform. At section (2), the velocity profile is

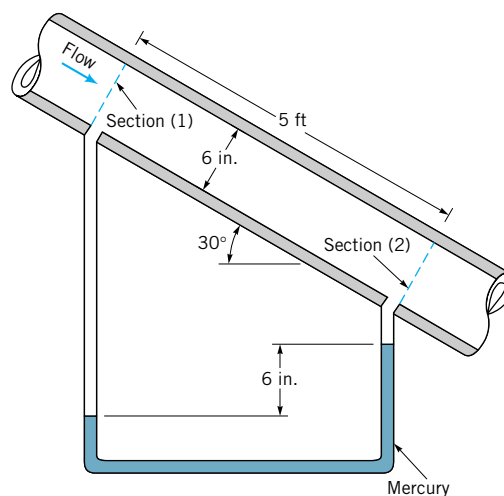
$$\mathbf{V} = w_c \left( \frac{R-r}{R} \right)^{1/7} \hat{\mathbf{k}}$$

where  $\mathbf{V}$  = local velocity vector,  $w_c$  = centerline velocity in the axial direction,  $R$  = pipe inside radius, and,  $r$  = radius from pipe axis. Develop an expression for the loss in available energy between sections (1) and (2).

**5.100** Discuss the causes of loss of available energy in a fluid flow.

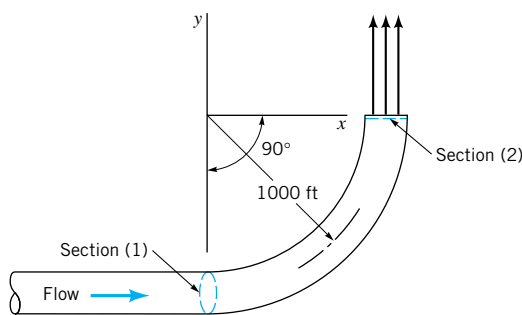
**5.101** Consider the flow shown in Fig. P5.91. If the flowing fluid is water, determine the axial (along the pipe) and normal (perpendicular to the pipe) components of force that the pipe puts on the fluid in the 6-m section shown.

**5.102** Water flows steadily down the inclined pipe as indicated in Fig. P5.102. Determine the following: (a) the difference in pressure  $p_1 - p_2$ , (b) the loss between sections (1) and (2), (c) the net axial force exerted by the pipe wall on the flowing water between sections (1) and (2).



■ FIGURE P5.102

**5.103** Water flows through a 2-ft-diameter pipe arranged horizontally in a circular arc as shown in Fig. P5.103. If the pipe discharges to the atmosphere ( $p = 14.7 \text{ psia}$ ), determine the  $x$  and  $y$  components of the resultant force exerted by the water on the piping between sections (1) and (2). The steady flowrate is  $3000 \text{ ft}^3/\text{min}$ . The loss in pressure due to fluid friction between sections (1) and (2) is 25 psi.

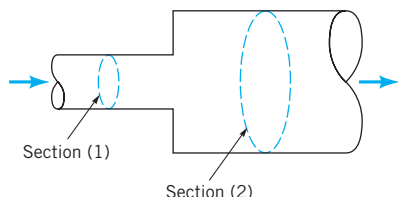


■ FIGURE P5.103

**5.104** When fluid flows through an abrupt expansion as indicated in Fig. P5.104, the loss in available energy across the expansion,  $\text{loss}_{\text{ex}}$ , is often expressed as

$$\text{loss}_{\text{ex}} = \left(1 - \frac{A_1}{A_2}\right)^2 \frac{V_1^2}{2}$$

where  $A_1$  = cross-sectional area upstream of expansion,  $A_2$  = cross-sectional area downstream of expansion, and  $V_1$  = velocity of flow upstream of expansion. Derive this relationship.



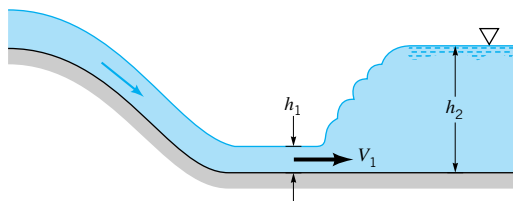
■ FIGURE P5.104

**5.105** Near the downstream end of a river spillway, a hydraulic jump often forms, as illustrated in Fig. P5.105 and [Video V10.5](#). The velocity of the channel flow is reduced abruptly across the jump. Using the conservation of mass and linear momentum principles, derive the following expression for  $h_2$ ,

$$h_2 = -\frac{h_1}{2} + \sqrt{\left(\frac{h_1}{2}\right)^2 + \frac{2V_1^2 h_1}{g}}$$

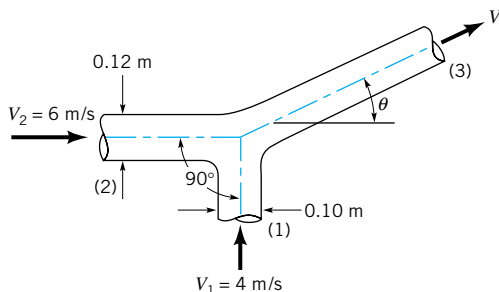
The loss of available energy across the jump can also be determined if energy conservation is considered. Derive the loss expression

$$\text{jump loss} = \frac{g(h_2 - h_1)^3}{4h_1 h_2}$$



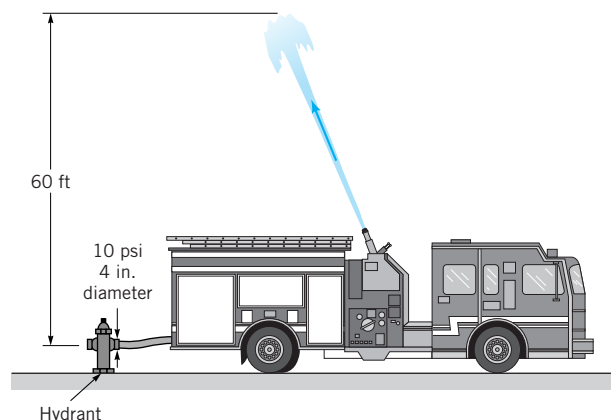
■ FIGURE P5.105

**5.106** Two water jets collide and form one homogeneous jet as shown in Fig. P5.106. (a) Determine the speed,  $V$ , and direction,  $\theta$ , of the combined jet. (b) Determine the loss for a fluid particle flowing from (1) to (3), from (2) to (3). Gravity is negligible.



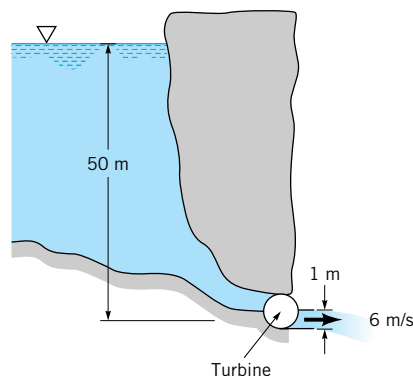
■ FIGURE P5.106

**5.107** The pumper truck shown in Fig. P5.107 is to deliver  $1.5 \text{ ft}^3/\text{s}$  to a maximum elevation of 60 ft above the hydrant. The pressure at the 4-in. diameter outlet of the hydrant is 10 psi. If head losses are negligibly small, determine the power that the pump must add to the water.



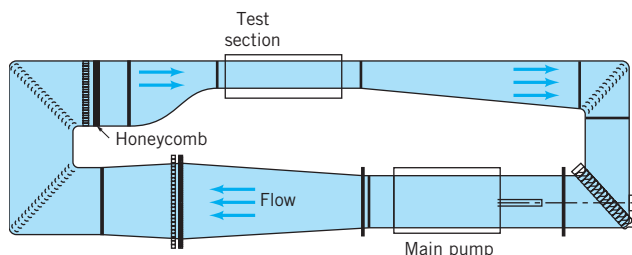
■ FIGURE P5.107

**5.108** What is the maximum possible power output of the hydroelectric turbine shown in Fig. P5.108?



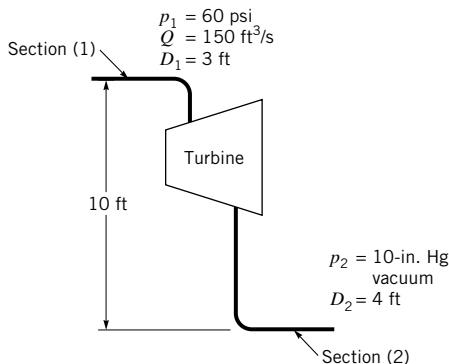
■ FIGURE P5.108

**5.109** Estimate the power in hp needed to drive the main pump of the large-scale water tunnel shown in Fig. P5.109. The design condition head loss is specified as 14 ft of water for a flowrate of  $4900 \text{ ft}^3/\text{s}$ .



■ FIGURE P5.109

**5.110** Water is supplied at  $150 \text{ ft}^3/\text{s}$  and 60 psi to a hydraulic turbine through a 3-ft inside-diameter inlet pipe as indicated in Fig. P5.110. The turbine discharge pipe has a 4-ft inside diameter. The static pressure at section (2), 10 ft below the turbine inlet, is 10-in. Hg vacuum. If the turbine develops 2500 hp, determine the power lost between sections (1) and (2).



■ FIGURE P5.110

**5.111** A steam turbine receives steam having a static pressure,  $p_1$ , of 400 psia, an enthalpy,  $\hat{h}_1$ , of 1407 Btu/lbm, and a velocity,  $V_1$ , of 100 ft/s. The steam leaves the turbine as a mixture of vapor and liquid having an enthalpy,  $\hat{h}_2$ , of 1098 Btu/lbm, a pressure,  $p_2$ , of 2 psia, and a velocity,  $V_2$ , of 200 ft/s. If the flow through the turbine is essentially adiabatic and the change in elevation of the steam is negligible, calculate (a) the actual work output per unit mass of steam, (b) the efficiency of the turbine if the ideal work output is 467 Btu/lbm.

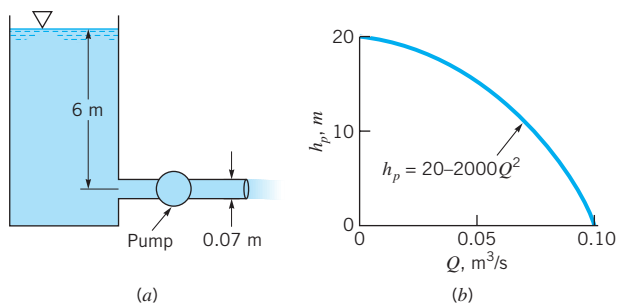
**5.112** A centrifugal air compressor stage operates between an inlet stagnation pressure of 14.7 psia and an exit stagnation pressure of 60 psia. The inlet stagnation temperature is 80 °F. If the loss of total pressure through the compressor stage associated with irreversible flow phenomena is 10 psi, estimate the actual and ideal stagnation temperature rise through the compressor. Estimate the ratio of ideal to actual temperature rise to obtain an approximate value of the efficiency.

† **5.113** Explain how, in terms of the loss of available energy involved, a home sink water faucet valve works to vary the flow from the shutoff condition to maximum flow. Explain how you would estimate the size of the overflow drain holes needed in the sink of Video V5.1 (Video V3.5 may be helpful).

\***5.114** Total head-rise values measured for air flowing across a fan are listed below as a function of volume flowrate. Determine the flowrate that will result when this fan is connected to a piping system whose loss in total head is described by loss =  $K_L Q^2$  when: (a)  $K_L = 49 \text{ mm H}_2\text{O}/(\text{m}^3/\text{s})^2$ ; (b)  $K_L = 91 \text{ mm H}_2\text{O}/(\text{m}^3/\text{s})^2$ ; (c)  $K_L = 140 \text{ mm H}_2\text{O}/(\text{m}^3/\text{s})^2$ .

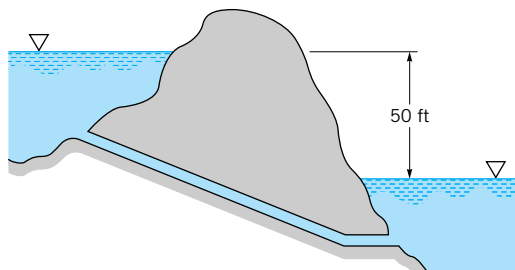
$Q \text{ (m}^3/\text{s)}$	Total Head Rise (mm H <sub>2</sub> O)
0	79
0.14	79
0.28	76
0.42	67
0.57	65
0.71	70
0.85	76
0.99	79
1.13	75
1.27	64

**5.115** Water is pumped from the tank shown in Fig. P5.115a. The head loss is known to be  $1.2 V^2/2g$ , where  $V$  is the average velocity in the pipe. According to the pump manufacturer, the relationship between the pump head and the flowrate is as shown in Fig. P5.115b:  $h_p = 20 - 2000 Q^2$ , where  $h_p$  is in meters and  $Q$  is in  $\text{m}^3/\text{s}$ . Determine the flowrate,  $Q$ .



■ FIGURE P5.115

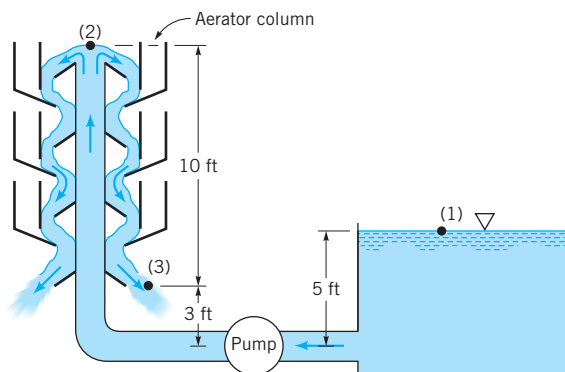
**5.116** Water flows by gravity from one lake to another as sketched in Fig. P5.116 at the steady rate of 80 gpm. What is the loss in available energy associated with this flow? If this same amount of loss is associated with pumping the fluid from the lower lake to the higher one at the same flowrate, estimate the amount of pumping power required.



■ FIGURE P5.116

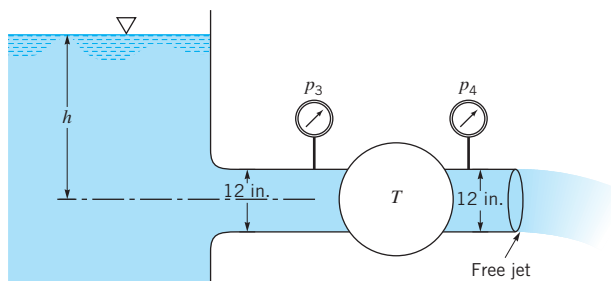
**5.117** A  $\frac{3}{4}$ -hp motor is required by an air ventilating fan to produce a 24-in.-diameter stream of air having a uniform speed of 40 ft/s. Determine the aerodynamic efficiency of the fan.

**5.118** Water is pumped from a tank, point (1), to the top of a water plant aerator, point (2), as shown in [Video V5.8](#) and Fig. P5.118 at a rate of  $3.0 \text{ ft}^3/\text{s}$ . (a) Determine the power that the pump adds to the water if the head loss from (1) to (2) where  $V_2 = 0$  is 4 ft. (b) Determine the head loss from (2) to the bottom of the aerator column, point (3), if the average velocity at (3) is  $V_3 = 2 \text{ ft/s}$ .



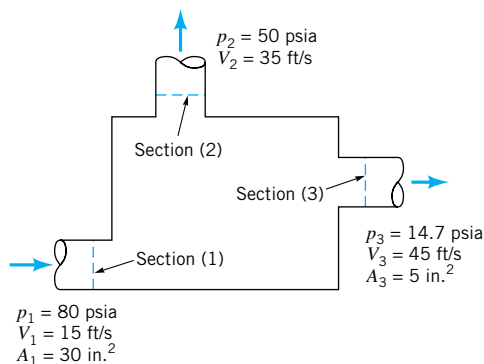
■ FIGURE P5.118

**5.119** The turbine shown in Fig. P5.119 develops 100 hp when the flowrate of water is  $20 \text{ ft}^3/\text{s}$ . If all losses are negligible, determine (a) the elevation  $h$ , (b) the pressure difference across the turbine, and (c) the flowrate expected if the turbine were removed.



■ FIGURE P5.119

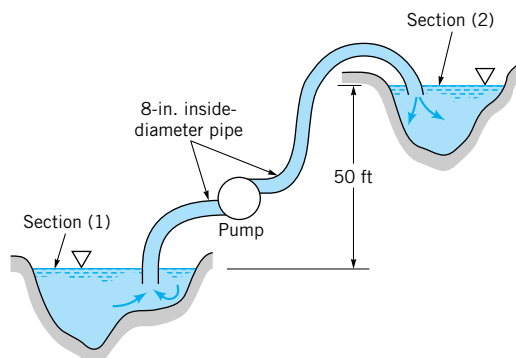
**5.120** A liquid enters a fluid machine at section (1) and leaves at sections (2) and (3) as shown in Fig. P5.120. The den-



■ FIGURE P5.120

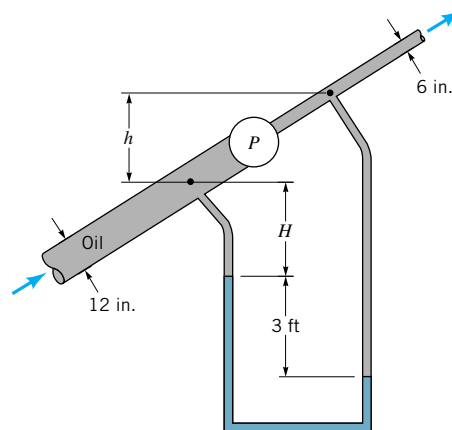
sity of the fluid is constant at  $2 \text{ slugs/ft}^3$ . All of the flow occurs in a horizontal plane and is frictionless and adiabatic. For the above-mentioned and additional conditions indicated in Fig. P5.120, determine the amount of shaft power involved.

**5.121** Water is to be moved from one large reservoir to another at a higher elevation as indicated in Fig. P5.121. The loss of available energy associated with  $2.5 \text{ ft}^3/\text{s}$  being pumped from sections (1) to (2) is  $61\bar{V}^2/2 \text{ ft}^2/\text{s}^2$ , where  $\bar{V}$  is the average velocity of water in the 8-in. inside-diameter piping involved. Determine the amount of shaft power required.



■ FIGURE P5.121

**5.122** Oil ( $SG = 0.88$ ) flows in an inclined pipe at a rate of  $5 \text{ ft}^3/\text{s}$  as shown in Fig. P5.122. If the differential reading in the mercury manometer is 3 ft, calculate the power that the pump supplies to the oil if head losses are negligible.



■ FIGURE P5.122

† **5.123** Explain how you would estimate the vacuum needed in a shop vac like the one shown in [Video V5.2](#).

**5.124** The velocity profile in a turbulent pipe flow may be approximated with the expression

$$\frac{u}{u_c} = \left( \frac{R - r}{R} \right)^{1/n}$$

where  $u$  = local velocity in the axial direction,  $u_c$  = centerline velocity in the axial direction,  $R$  = pipe inner radius from pipe axis,  $r$  = local radius from pipe axis, and  $n$  = constant. Determine the kinetic energy coefficient,  $\alpha$ , for (a)  $n = 5$ , (b)  $n = 6$ , (c)  $n = 7$ , (d)  $n = 8$ , (e)  $n = 9$ , (f)  $n = 10$ .

**5.125** A small fan moves air at a mass flowrate of 0.004 lbm/s. Upstream of the fan, the pipe diameter is 2.5 in., the flow is laminar, the velocity distribution is parabolic, and the kinetic energy coefficient,  $\alpha_1$ , is equal to 2.0. Downstream of the fan, the pipe diameter is 1 in., the flow is turbulent, the velocity profile is quite flat, and the kinetic energy coefficient,  $\alpha_2$ , is equal to 1.08. If the rise in static pressure across the fan is 0.015 psi and the fan shaft draws 0.00024 hp, compare the value of loss calculated: (a) assuming uniform velocity distributions, (b) considering actual velocity distributions.

**5.126** This problem involves the force that a jet of air exerts on a flat plate as the air is deflected by the plate. To proceed with this problem, [click here](#) in the E-book.

**5.127** This problem involves the pressure distribution produced on a flat plate that deflects a jet of air. To proceed with this problem, [click here](#) in the E-book.

**5.128** This problem involves the force that a jet of water exerts on a vane when the vane turns the jet through a given angle. To proceed with this problem, [click here](#) in the E-book.

**5.129** This problem involves the force needed to hold a pipe elbow stationary. To proceed with this problem, [click here](#) in the E-book.