Chapter 3: Energy Transport by Heat, Work, and Mass

We will soon learn how to apply the first law of thermodynamics as the expression of the conservation of energy principle. But, first we study the ways in which energy may be transported across the boundary of a general thermodynamic system. For closed systems (fixed mass systems) energy can cross the boundaries of a closed system only in the form of heat or work. For open systems or control volumes energy can cross the control surface in the form of heat, work, and energy transported by the mass streams crossing the control surface. We now consider each of these modes of energy transport across the boundaries of the general thermodynamic system.

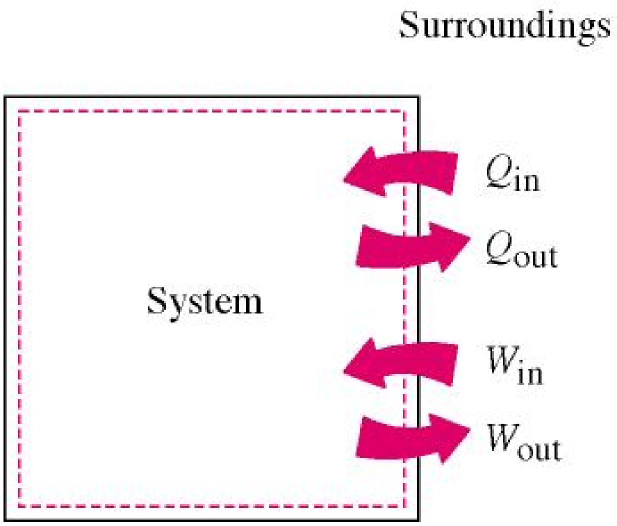
Classical Sign Convention for Energy Transport by Heat and Work

Energy transfer across a system boundary due solely to the temperature difference between a system and its surroundings is called heat.

Work energy can be thought of as the energy expended to lift a weight.

A sign convention is required for heat and work energy transfers, and the classical sign convention is selected for these notes. According to the classical sign convention, heat transfer to a system and work done by a system are positive; heat transfer from a system and work done on a system are negative. The system shown below has heat supplied to it and work done by it.

In these study guide we will use the concept of net heat and net work.



Energy Transport by Heat

Recall that heat is energy in transition across the system boundary solely due to the temperature difference between the system and its surroundings. The net heat transferred to a system is defined as

$$Q_{net} = \sum Q_{in} - \sum Q_{out}$$

Here, Q_{in} and Q_{out} are the magnitudes of the heat transfer values. In most thermodynamics texts, the quantity Q is meant to be the net heat transferred to the system, Q_{net}. Since heat transfer is process dependent, the differential of heat transfer δQ is called inexact. We often think about the heat transfer per unit mass of the system, Q.

$$q = \frac{Q}{m}$$

Heat transfer has the units of energy joules (we will use kilojoules, kJ) or the units of energy per unit mass, kJ/kg.

Since heat transfer is energy in transition across the system boundary due to a temperature difference, there are three modes of heat transfer at the boundary that depend on the temperature difference between the boundary surface and the surroundings. These are conduction, convection, and radiation. However, when solving problems in thermodynamics involving heat transfer to a system, the heat transfer is usually given or is calculated by applying the first law, or the conservation of energy, to the system.

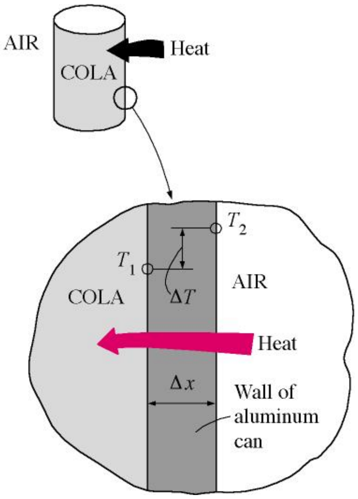
An **adiabatic** process is one in which the system is perfectly insulated and the heat transfer is zero.

For those of us who do not have the opportunity to have a complete course in heat transfer theory and applications, the following is a short introduction to the basic mechanisms of heat transfer. Those of us who have a complete course in heat transfer theory may elect to omit this material at this time.

Heat transfer is energy in transition due to a temperature difference. The three modes of heat transfer are conduction, convection, and radiation.

Conduction through Plane Walls

Conduction heat transfer is a progressive exchange of energy between the molecules of a substance.



Fourier's law of heat conduction is

$$\dot{Q}_{cond} = -A\,k_t\,\frac{dT}{dx}$$

here

$\dot{Q}_{cond}$ = heat flow per unit time (W)
k_t = thermal conductivity (W/m·K)
A = area normal to heat flow (m^2)
$\frac{dT}{dx}$ = temperature gradient in the direction of heat flow (°C/m)

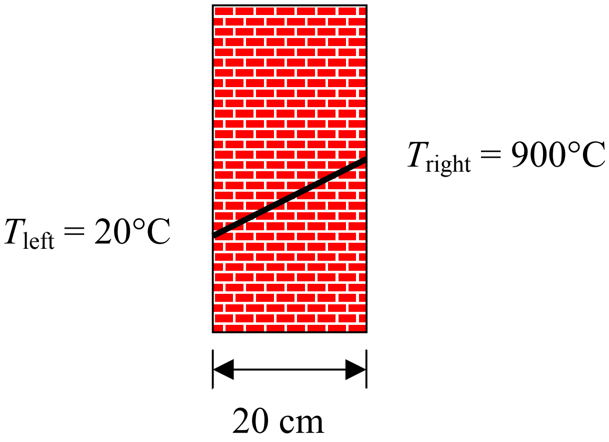
Integrating Fourier's law

$$\dot{Q}_{cond} = kA\frac{\Delta T}{\Delta x}$$

Since $T_2 > T_1$, the heat flows from right to left in the above figure.

Example 3-1

A flat wall is composed of 20 cm of brick ($k_t = 0.72$ W/m·K, see Table 3-1). The right face temperature of the brick is 900°C, and the left face temperature of the brick is 20°C. Determine the rate of heat conduction through the wall per unit area of wall.



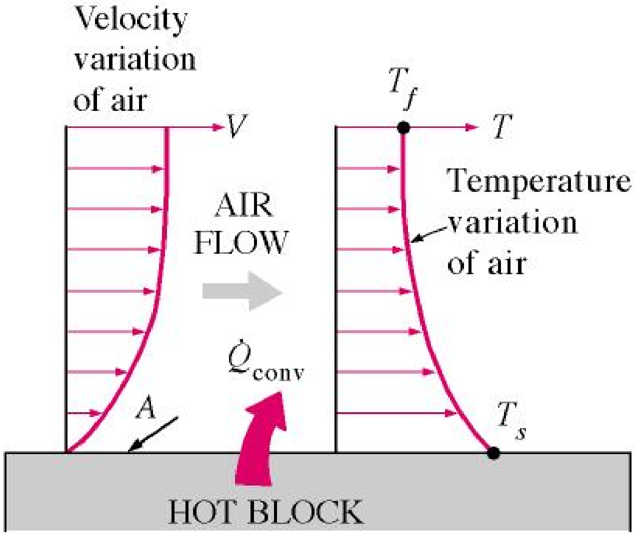
$$\dot{Q}_{cond} = k_t A \frac{\Delta T}{\Delta x}$$

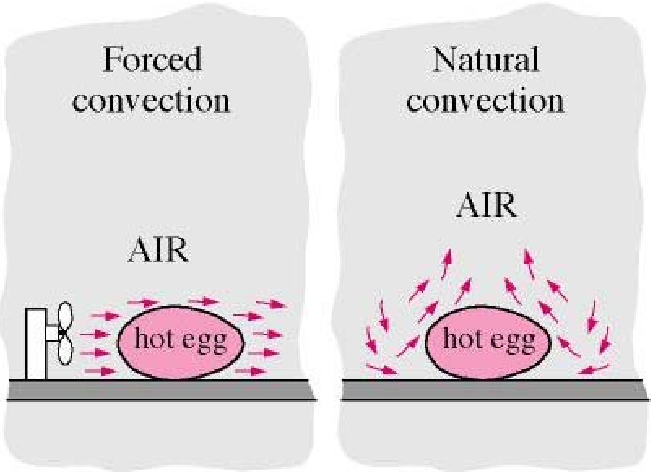
$$\frac{\dot{Q}_{cond}}{A} = k_t \frac{\Delta T}{\Delta x} = 0.72 \frac{W}{m \cdot K} \left(\frac{(900 - 20)K}{0.2m} \right)$$

$$= 3168 \frac{W}{m^2}$$

Convection Heat Transfer

Convection heat transfer is the mode of energy transfer between a solid surface and the adjacent liquid or gas that is in motion, and it involves the combined effects of conduction and fluid motion.





The rate of heat transfer by convection $\dot{Q}_{conv}$ is determined from Newton's law of cooling, expressed as

$$\dot{Q}_{conv} = h\,A\,(T_s - T_f)$$

here

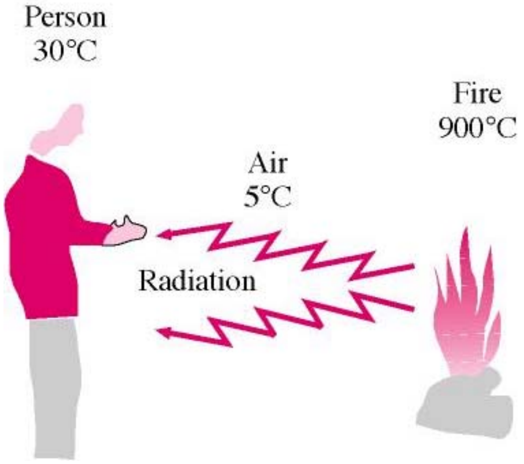
$\dot{Q}_{conv}$ = heat transfer rate (W)
A = heat transfer area (m^2)
h = convective heat transfer coefficient ($W/m^2 \cdot K$)
T_s = surface temperature (K)
T_f = bulk fluid temperature away from the surface (K)

The convective heat transfer coefficient is an experimentally determined parameter that depends upon the surface geometry, the nature of the fluid motion, the properties of the fluid, and the bulk fluid velocity. Ranges of the convective heat transfer coefficient are given below.

	h $W/m^2 \cdot K$
free convection of gases	2-25
free convection of liquids	50-100
forced convection of gases	25-250
forced convection of liquids	50-20,000
convection in boiling and condensation	2500-100,000

Radiative Heat Transfer

Radiative heat transfer is energy in transition from the surface of one body to the surface of another due to electromagnetic radiation. The radiative energy transferred is proportional to the difference in the fourth power of the absolute temperatures of the bodies exchanging energy.



The net exchange of radiative heat transfer between a body surface and its surroundings is given by

$$\dot{Q}_{rad} = \varepsilon \, \sigma \, A \left(T_s^4 - T_{surr}^4 \right)$$

here

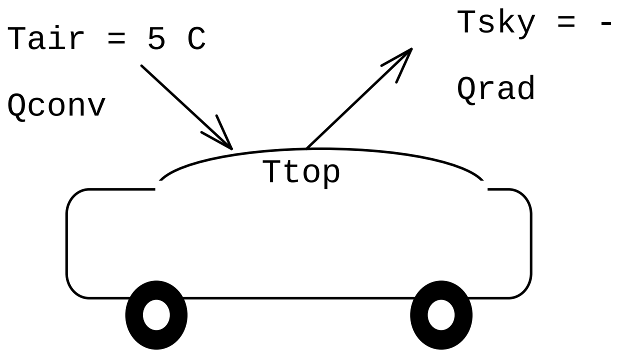
$\dot{Q}_{rad}$ = heat transfer per unit time (W)
A = surface area for heat transfer (m^2)
σ = Stefan-Boltzmann constant, $5.67\text{x}10^{-8}$ W/m^2K^4 and $0.1713\text{x}10^{-8}$ BTU/h ft^2 R^4
ε = emissivity
T_s = absolute temperature of surface (K)
T_{surr} = absolute temperature of surroundings (K)

Example 3-2

A vehicle is to be parked overnight in the open away from large surrounding objects. It is desired to know if dew or frost may form on the vehicle top. Assume the following:

- Convection coefficient *h* from ambient air to vehicle top is 6.0 W/m$^2\cdot$°C.
- Equivalent sky temperature is -18°C.
- Emissivity of vehicle top is 0.84.
- Negligible conduction from inside vehicle to top of vehicle.

Determine the temperature of the vehicle top when the air temperature is 5 ^{o}F. State which formation (dew or frost) occurs.



Under steady-state conditions, the energy convected to the vehicle top is equal to the energy radiated to the sky.

$$\dot{Q}_{conv} = \dot{Q}_{rad}$$

The energy convected from the ambient air to the vehicle top is

$$\dot{Q}_{conv} = A_{top}\, h\, (T_{air} - T_{top})$$

The energy radiated from the top to the night sky is

$$\dot{Q}_{rad} = \varepsilon\,\sigma\,A_{top}\left(T_{top}^4 - T_{sky}^4\right)$$

Setting these two heat transfers equal gives

$$A_{top}\,h\,(T_{air} - T_{top}) = \varepsilon\,\sigma\,A_{top}\left(T_{top}^4 - T_{sky}^4\right)$$

$$h\,(T_{air} - T_{top}) = \varepsilon\,\sigma\left(T_{top}^4 - T_{sky}^4\right)$$

$$6.0\frac{W}{m^2\ K}\left[(5+273) - T_{top}\right]K$$

$$= (0.84)\left(5.67x10^{-8}\ \frac{W}{m^2\ K^4}\right)\left[T_{top}^4 - (-18+273)^4\right]K^4$$

Write the equation for T_{top} in °C (T K = T°C + 273)

$$\left(5 - T_{top}\right) = \frac{(0.84)(5.67)}{6.0}\left[\left(\frac{T_{top}+273}{100}\right)^4 - (2.55)^4\right]$$

Using the EES software package

$$T_{top} = -3.38\,°\mathrm{C}$$

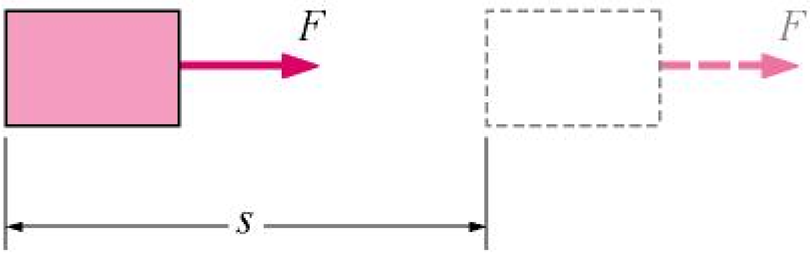
Since T_{top} is below the triple point of water, 0.01°C, the water vapor in the air will form frost on the car top (see Chapter 13).

Extra Problem

Explore what happens to T_{top} as you vary the convective heat transfer coefficient. On a night when the atmosphere is particularly still and cold and has a clear sky, why do fruit growers use fans to increase the air velocity in their fruit groves?

Energy Transfer by Work

Work is energy expended by a force acting through a distance. Thermodynamic work is defined as energy in transition across the system boundary and is done by a system if the sole effect external to the boundaries could have been the raising of a weight.



Mathematically, the differential of work is expressed as

$$\delta W = \vec{F} \cdot d\vec{s} = F\,ds\cos\Theta$$

here Θ is the angle between the force vector and the displacement vector. As with the heat transfer, the Greek symbol δ means that work is a path-dependent function and has an inexact differential. If the angle between the force and the displacement is zero, the work done between two states is

$$W_{12} = \int_1^2 \delta W = \int_1^2 F\,ds$$

Work has the units of energy force times displacement or newton times meter or joule (we will use kilojoules). Work per unit mass of a system is measured in kJ/kg.

Common Types of Work Energy

The net work done by the system may be in two forms. First, there can be work crossing the system boundary in the form of a rotating shaft or electrical work. We will call shaft work and electrical work "other" work, that is, work not associated with a moving boundary. In thermodynamics electrical energy is normally considered to be work energy rather than heat energy, but the placement of the system boundary dictates whether to include electrical energy as work or heat. Second, the system may do work on its surroundings because of moving boundaries.

The net work done by a closed system is defined by

$$W_{net} = \left(\sum W_{out} - \sum W_{in}\right)_{other} + W_b$$

Here, W_{out} and W_{in} are the magnitudes of the other work forms crossing the boundary. W_b is the work due to the moving boundary and will be positive or negative depending upon the process.

Or

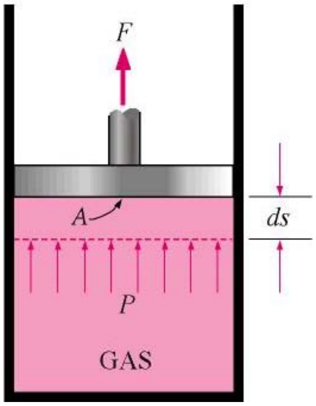
$$W_{net} = \left(W_{net}\right)_{other} + W_b$$

Several types of "other" work (shaft work, electrical work, etc.) are discussed in the text.

In the next section we will look at boundary work in detail. Review the text material on other types of work such as shaft work, spring work, electrical work.

Boundary Work

Work is energy expended when a force acts through a displacement. Boundary work occurs because the mass of the substance contained within the system boundary causes a force, the pressure times the surface area, to act on the boundary surface and make it move. This is what happens when steam, the "gas" in the figure below, contained in a piston-cylinder device expands against the piston and forces the piston to move; thus, boundary work is done by the steam on the piston. Boundary work is then calculated from



$$W_b = \int_1^2 \delta W_b = \int_1^2 F\,ds = \int_1^2 \frac{F}{A} A\,ds$$
$$= \int_1^2 P\,dV$$

Since the work is process dependent, the differential of boundary work δW_b

$$\delta W_b = P\,dV$$

is called inexact. The above equation for W_b is valid for a quasi-equilibrium process and gives the maximum work done during expansion and the minimum work input during compression. In an expansion process the boundary work must overcome friction, push the atmospheric air out of the way, and rotate a crankshaft.

$$W_{\mathrm{b}} = W_{\mathrm{friction}} + W_{\mathrm{atm}} + W_{\mathrm{crank}}$$

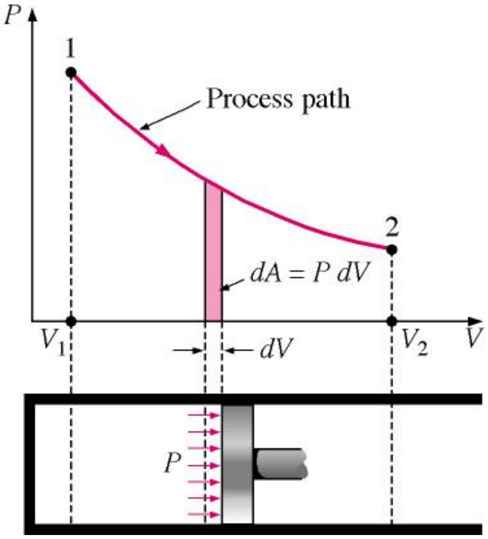
$$= \int_1^2 (F_{\mathrm{friction}} + P_{\mathrm{atm}} A + F_{\mathrm{crank}})ds$$

To calculate the boundary work, the process by which the system changed states must be known. Once the process is determined, the pressure-volume relationship for the process can be obtained and the integral in the boundary work equation can be performed. For each process we need to determine

$$P = f(V)$$

So as we work problems, we will be asking, “What is the pressure-volume relationship for the process?” Remember that this relation is really the force-displacement function for the process.

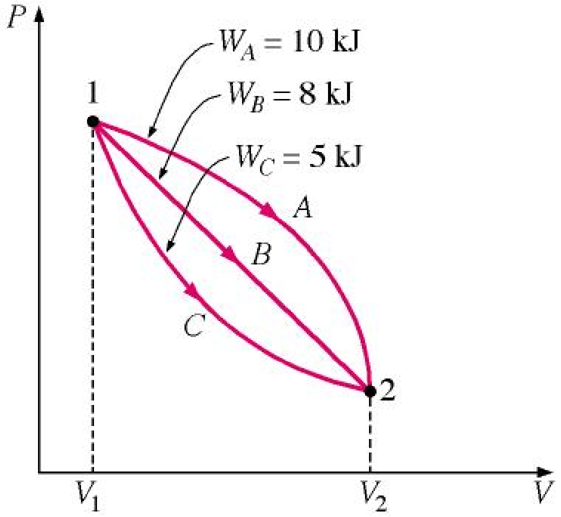
The boundary work is equal to the area under the process curve plotted on the pressure-volume diagram.



Note form the above figure:

P is the absolute pressure and is always positive.
When dV is positive, W_b is positive.
When dV is negative, W_b is negative.

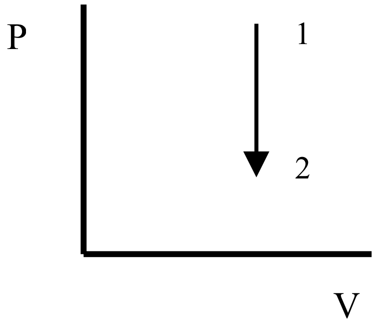
Since the areas under different process curves on a P-V diagram are different, the boundary work for each process will be different. The next figure shows that each process gives a different value for the boundary work.



Some Typical Processes

Constant volume

If the volume is held constant, $dV = 0$, and the boundary work equation becomes

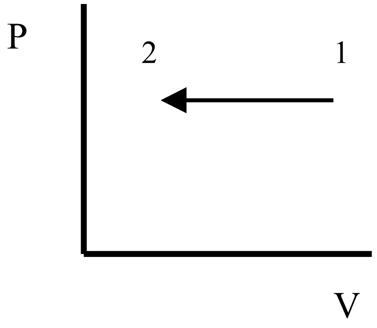


P-V diagram for V = constant

$$W_b = \int_1^2 P dV = 0$$

If the working fluid is an ideal gas, what will happen to the temperature of the gas during this constant volume process?

Constant pressure



P-V diagram for *P* = constant

If the pressure is held constant, the boundary work equation becomes

$$W_b = \int_1^2 P dV = P\int_1^2 dV = P(V_2 - V_1)$$

For the constant pressure process shown above, is the boundary work positive or negative and why?

Constant temperature, ideal gas

If the temperature of an ideal gas system is held constant, then the equation of state provides the pressure-volume relation

$$P = \frac{mRT}{V}$$

Then, the boundary work is

$$W_b = \int_1^2 P dV = \int_1^2 \frac{mRT}{V} dV$$

$$= mRT \ln\left(\frac{V_2}{V_1}\right)$$

Note: The above equation is the result of applying the ideal gas assumption for the equation of state. For real gases undergoing an isothermal (constant temperature) process, the integral in the boundary work equation would be done numerically.

The polytropic process

The polytropic process is one in which the pressure-volume relation is given as

$$PV^n = \text{constant}$$

The exponent *n* may have any value from minus infinity to plus infinity depending on the process. Some of the more common values are given below.

Process	**Exponent *n***
Constant pressure	0
Constant volume	∞
Isothermal & ideal gas	1
Adiabatic & ideal gas	$k = C_P/C_V$

Here, *k* is the ratio of the specific heat at constant pressure C_P to specific heat at constant volume C_V. The specific heats will be discussed later.

The boundary work done during the polytropic process is found by substituting the pressure-volume relation into the boundary work equation. The result is

$$W_b = \int_1^2 P dV = \int_1^2 \frac{Const}{V^n} dV$$

$$= \frac{P_2 V_2 - P_1 V_1}{1 - n}, \ \mathrm{n} \neq 1$$

$$= PV \ln\left(\frac{V_2}{V_1}\right), \ \mathrm{n} = 1$$

For an ideal gas under going a polytropic process, the boundary work is

$$W_b = \int_1^2 P dV = \int_1^2 \frac{Const}{V^n} dV$$

$$= \frac{mR(T_2 - T_1)}{1 - n}, \ \mathrm{n} \neq 1$$

$$= mRT \ln\left(\frac{V_2}{V_1}\right), \ \mathrm{n} = 1$$

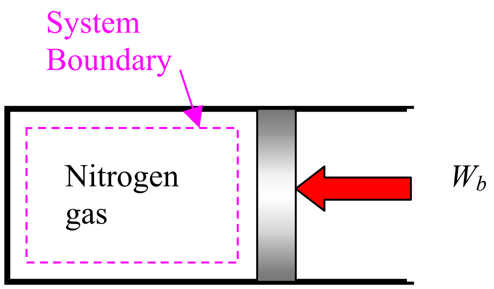
Notice that the results we obtained for an ideal gas undergoing a polytropic process when n = 1 are identical to those for an ideal gas undergoing the isothermal process.

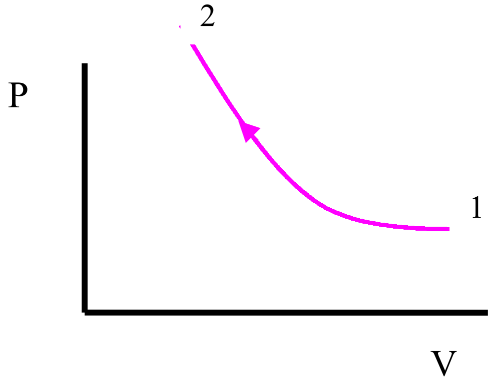
Example 3-3

Three kilograms of nitrogen gas at 27°C and 0.15 MPa are compressed isothermally to 0.3 MPa in a piston-cylinder device. Determine the minimum work of compression, in kJ.

<u>System</u>: Nitrogen contained in a piston-cylinder device.

<u>Process</u>: Constant temperature





P-*V* diagram for *T* = constant

<u>Property Relation</u>: Check the reduced temperature and pressure for nitrogen. The critical state properties are found in Table A-1.

$$T_{R1} = \frac{T_1}{T_{cr}} = \frac{(27 + 273)K}{126.2K} = 2.38 = T_{R2}$$

$$P_{R1} = \frac{P_1}{P_{cr}} = \frac{0.15\,MPa}{3.39\,MPa} = 0.044$$

$$P_{R2} = 2P_{R1} = 0.088$$

Since P_R<<1 and T>2T_{cr}, nitrogen is an ideal gas, and we use the ideal gas equation of state as the property relation.

$$PV = mRT$$

<u>Work Calculation</u>:

$$W_{net,12} = \left(W_{net}\right)_{other,12} + W_{b,12}$$

$$W_{b,12} = \int_1^2 P\,dV = \int_1^2 \frac{mRT}{V}\,dV$$

$$= mRT\ln\left(\frac{V_2}{V_1}\right)$$

For an ideal gas in a closed system (mass = constant), we have

$$m_1 = m_2$$

$$\frac{P_1V_1}{RT_1} = \frac{P_2V_2}{RT_2}$$

Since the R's cancel, we obtain the combined ideal gas equation. Since $T_2 = T_1$,

$$\frac{V_2}{V_1} = \frac{P_1}{P_2}$$

$$W_{b,12} = mRT\ln\left(\frac{P_1}{P_2}\right)$$

$$= (3\,kg)\left(0.2968\frac{kJ}{kg-K}\right)(300K)\ln\left(\frac{0.15\,MPa}{0.30\,MPa}\right)$$

$$= -184.5\,kJ$$

The net work is

$$W_{net,12} = 0 + W_{b,12} = -184.5\,kJ$$

On a per unit mass basis

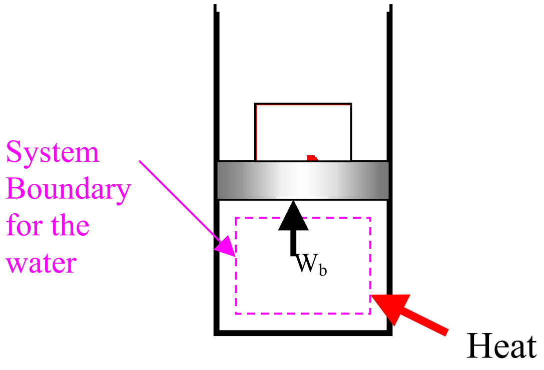
$$w_{net,12} = \frac{W_{net,12}}{m} = -61.5\frac{kJ}{kg}$$

The net work is negative because work is done **on** the system during the compression process. Thus, the work **done on** the system is 184.5 kJ, or 184.5 kJ of work energy is required to compress the nitrogen.

Example 3-4

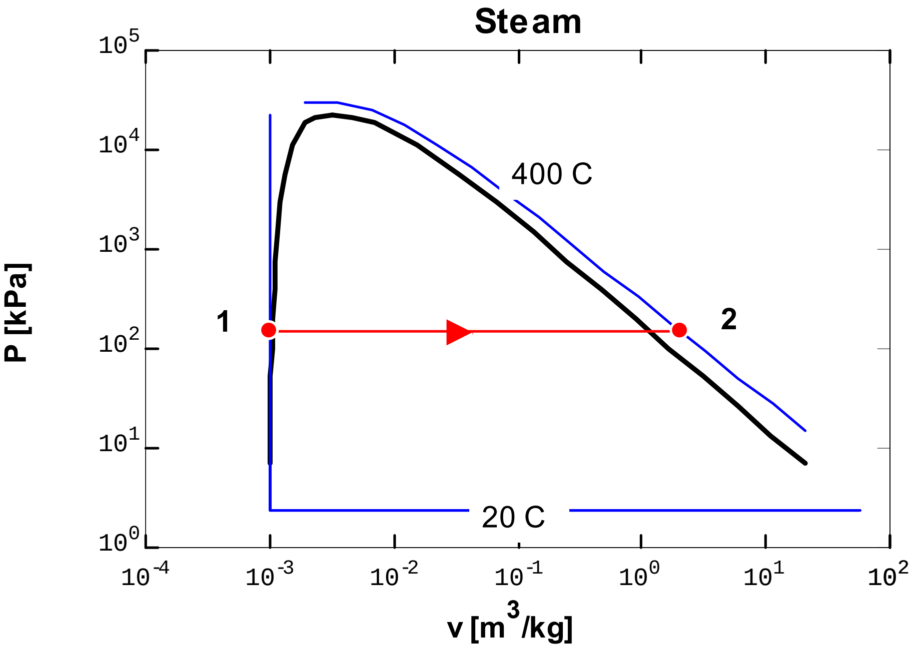
Water is placed in a piston-cylinder device at 20 °C, 0.1 MPa. Weights are placed on the piston to maintain a constant force on the water as it is heated to 400 °C. How much work does the water do on the piston?

System: The water contained in the piston-cylinder device



Property Relation: Steam tables

Process: Constant pressure



Work Calculation:

Since there is no W_{other} mentioned in the problem, the net work is

$$W_{net,12} = W_{b,12} = \int_1^2 P dV = P\int_1^2 dV = P(V_2 - V_1)$$

Since the mass of the water is unknown, we calculate the work per unit mass.

$$w_{b,12} = \frac{W_{b,12}}{m} = \frac{P\left(V_2 - V_1\right)}{m} = P\left(v_2 - v_1\right)$$

At T_1 = 20°C, P_{sat} = 2.339 kPa. Since P_1 > 2.339 kPa, state 1 is compressed liquid. Thus,

$$v_1 \cong v_f \text{ at } 20\ {}^\circ\text{C} = 0.001002\ \text{m}^3/\ \text{kg}$$

At $P_2 = P_1$ = 0.1 MPa, $T_2 > T_{sat}$ at 0.1 MPa = 99.63°C.

So, state 2 is superheated. Using the superheated tables,

$$v_2 = 3.103\ \text{m}^3/\text{kg}$$

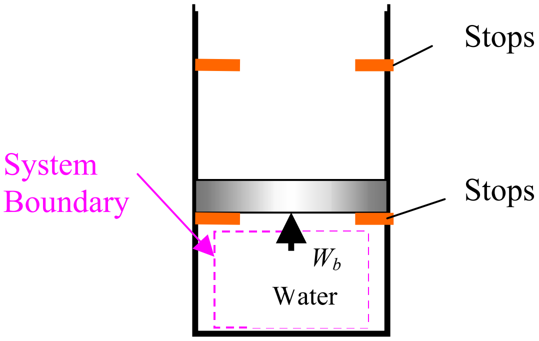
$$\begin{aligned} w_{b,12} &= P\left(v_2 - v_1\right) \\ &= 0.1\ MPa(3.103 - 0.001002)\frac{m^3}{kg}\frac{10^3\,kPa}{MPa}\frac{kJ}{m^3kPa} \\ &= 310.2\,\frac{kJ}{kg} \end{aligned}$$

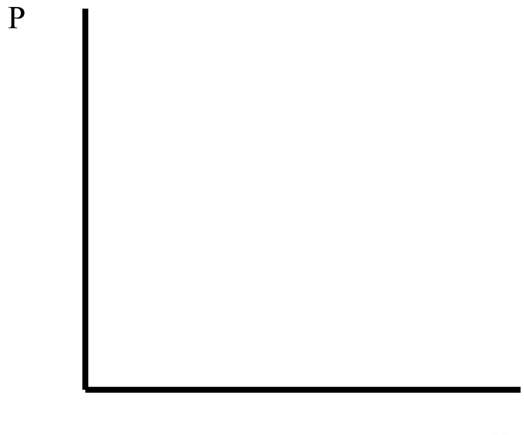
The water does work on the piston in the amount of 310.2 kJ/kg.

Example 3-5

One kilogram of water is contained in a piston-cylinder device at 100 °C. The piston rests on lower stops such that the volume occupied by the water is 0.835 m^3. The cylinder is fitted with an upper set of stops. When the piston rests against the upper stops, the volume enclosed by the piston-cylinder device is 0.841 m^3. A pressure of 200 kPa is required to support the piston. Heat is added to the water until the water exists as a saturated vapor. How much work does the water do on the piston?

System: The water contained in the piston-cylinder device





Property Relation: Steam tables

Process: Combination of constant volume and constant pressure process to be shown on the *P*-*v* diagram as the problem is solved

Work Calculation:

The specific volume at state 1 is

$$v_1 = \frac{V_1}{m} = \frac{0.835\ \mathrm{m}^3}{1\ \mathrm{kg}} = 0.835\frac{\mathrm{m}^3}{\mathrm{kg}}$$

At $T_1 = 100°\mathrm{C}$,

$$v_f = 0.0010044\frac{m^3}{kg} \qquad v_g = 1.6729\frac{m^3}{kg}$$

Therefore, $v_f < v_1 < v_g$ and state 1 is in the saturation region; so $P_1 = 101.35$ kPa.

Now let's consider the processes for the water.

Process 1-2: The volume stays constant until the pressure increases to 200 kPa. Then the piston will move.

$$v_2 = v_1 = 0.835 \frac{m^3}{kg}$$

Process 2-3: Piston lifts off the bottom stops while the pressure stays constant. Does the piston hit the upper stops before or after reaching the saturated vapor state?

Let's set

$$v_3 = \frac{V_3}{m} = \frac{0.841 m^3}{1 kg} = 0.841 \frac{m^3}{kg}$$

At $P_3 = P_2 = 200$ kPa

$$v_f = 0.0010061 \frac{m^3}{kg} \qquad v_g = 0.8857 \frac{m^3}{kg}$$

Thus, $v_f < v_3 < v_g$. So, the piston hits the upper stops before the water reaches the saturated vapor state. Now we have to consider a third process.

Process 3-4: With the piston against the upper stops, the volume remains constant during the final heating to the saturated vapor state and the pressure increases.

Because the volume is constant in process 3-to-4, $v_4 = v_3 = 0.841$ m^3/kg and v_4 is a saturated vapor state. Interpolating in either the saturation pressure table or saturation temperature table at $v_4 = v_g$ gives

$$\text{State } 4:\quad \left.\begin{array}{l} P_4 = 211.3\ kPa \\ v_4 = v_g \end{array}\right\} \quad T_4 = 122^\circ C$$

The net work for the heating process is (the "other" work is zero)

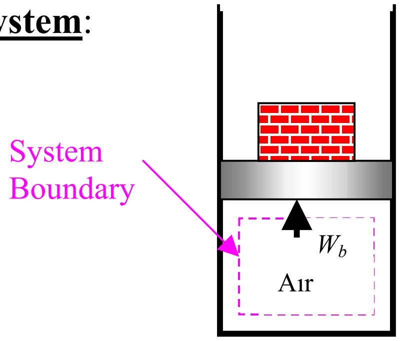
$$\begin{aligned} W_{net,14} &= W_{b,14} = \int_1^4 PdV = \int_1^2 PdV + \int_2^3 PdV + \int_3^4 PdV \\ &= 0 + mP(v_3 - v_2) + 0 \\ &= (1kg)(200kPa)(0.841 - 0.835)\frac{m^3}{kg}\frac{kJ}{m^3 kPa} \\ &= 1.2\ kJ \end{aligned}$$

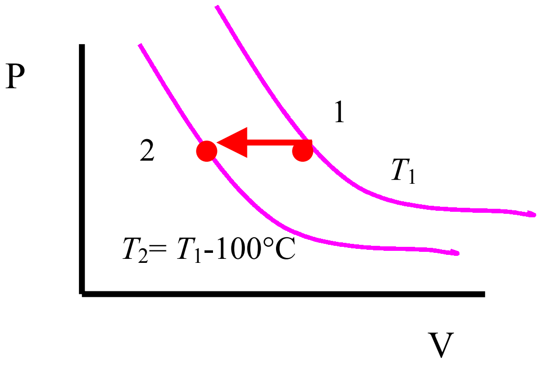
Later in Chapter 4, we will apply the conservation of energy, or the first law of thermodynamics, to this process to determine the amount of heat transfer required.

Example 3-6

Air undergoes a constant pressure cooling process in which the temperature decreases by 100°C. What is the magnitude and direction of the work for this process?

System:





P-V diagram for T = constant

Property Relation: Ideal gas law, $Pv = RT$

Process: Constant pressure

Work Calculation: Neglecting the "other" work

$$W_{net,12} = 0 + W_{b,12} = \int_1^2 PdV = P(V_2 - V_1)$$

$$= mR(T_2 - T_1)$$

The work per unit mass is

$$w_{net,12} = \frac{W_{net,12}}{m} = R(T_2 - T_1)$$

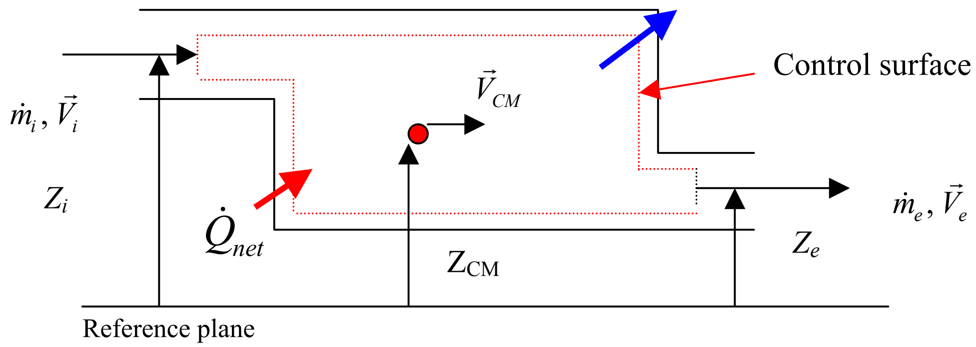
$$= (0.287\frac{kJ}{kg \cdot K})(-100\,K) = -28.7\frac{kJ}{kg}$$

The work **done on** the air is 28.7 kJ/kg.

Energy Transport by Mass

The conservation of mass and the conservation of energy principles for open systems or control volumes apply to systems having mass crossing the system boundary or control surface. In addition to the heat transfer and work crossing the system boundaries, mass carries energy with it as it crosses the system boundaries. Thus, the mass and energy content of the open system may change when mass enters or leaves the control volume.

$\dot{W}_{net}$



Typical control volume or open system

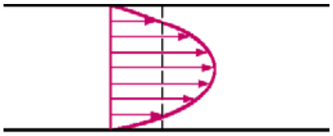
Thermodynamic processes involving control volumes can be considered in two groups: steady-flow processes and unsteady-flow processes. During a steady-flow process, the fluid flows through the control volume steadily, experiencing no change with time at a fixed position**.**

Mass Flow Rate

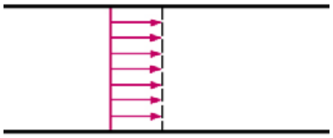
Mass flow through a cross-sectional area per unit time is called the mass flow rate $\dot{m}$. It is expressed as

$$\dot{m} = \int_A \rho \vec{V}_n \, dA$$

where $\vec{V}_n$ is the velocity normal to the cross-sectional flow area.



(*a*) Actual



(*b*) Average

If the fluid density and velocity are constant over the flow cross-sectional area, the mass flow rate is

$$\dot{m} = \rho \vec{V}_{av} A = \frac{\vec{V}_{av} A}{v}$$

where ρ is the density, kg/m^3 (= 1/*v*), A is the cross-sectional area, m^2; and $\vec{V}_{av}$ is the average fluid velocity normal to the area, m/s.

Example 3-7

Refrigerant-134a at 200 kPa, 40% quality, flows through a 1.1-cm inside diameter, *d*, tube with a velocity of 50 m/s. Find the mass flow rate of the refrigerant-134a.

At $P = 200$ kPa, $x = 0.4$

$$\begin{aligned} v &= v_f + x v_{fg} \\ &= 0.0007532 + 0.4(0.0993 - 0.0007532) \\ &= 0.0402 \frac{m^3}{kg} \end{aligned}$$

$$\begin{aligned} \dot{m} &= \frac{\vec{V}_{av} A}{v} = \frac{\vec{V}_{av}}{v} \frac{\pi d^2}{4} \\ &= \frac{50\ m/s}{0.0402\ m^3/kg} \frac{\pi (0.011\ m)^2}{4} \\ &= 0.118 \frac{kg}{s} \end{aligned}$$

The fluid volume flowing through a cross-section per unit time is called the volume flow rate $\dot{V}$. The volume flow rate is given by integrating the product of the velocity normal to the flow area and the differential flow area over the flow area. If the velocity over the flow area is a constant, the volume flow rate is given by (note we are dropping the "av" subscript on the velocity)

$$\dot{V} = \vec{V} A \qquad (m^3/s)$$

The mass and volume flow rate are related by

$$\dot{m} = \rho \dot{V} = \frac{\dot{V}}{v} \qquad (kg/s)$$

Example 3-8

Air at 100 kPa, 50°C, flows through a pipe with a volume flow rate of 40 m^3/min. Find the mass flow rate through the pipe, in kg/s.

Assume air to be an ideal gas, so

$$v = \frac{RT}{P} = 0.287\frac{kJ}{kg \cdot K}\frac{(50+273)K}{100kPa}\frac{m^3 kPa}{kJ}$$

$$= 0.9270\frac{m^3}{kg}$$

$$\dot{m} = \frac{\dot{V}}{v} = \frac{40m^3/\min}{0.9270m^3/kg}\frac{1\min}{60s}$$

$$= 0.719\frac{kg}{s}$$

Conservation of Mass for General Control Volume

The conservation of mass principle for the open system or control volume is expressed as

$$\begin{bmatrix}\text{Sum of rate}\\ \text{of mass flowing}\\ \text{into control volume}\end{bmatrix} - \begin{bmatrix}\text{Sum of rate}\\ \text{of mass flowing}\\ \text{from control volume}\end{bmatrix} = \begin{bmatrix}\text{Time rate change}\\ \text{of mass inside}\\ \text{control volume}\end{bmatrix}$$

or

$$\sum \dot{m}_{in} - \sum \dot{m}_{out} = \Delta \dot{m}_{system} \qquad (kg/s)$$

Steady-State, Steady-Flow Processes

Most energy conversion devices operate steadily over long periods of time. The rates of heat transfer and work crossing the control surface are constant with time. The states of the mass streams crossing the control surface or boundary are constant with time. Under these conditions the mass and energy content of the control volume are constant with time.

$$\frac{dm_{CV}}{dt} = \Delta \dot{m}_{CV} = 0$$

Steady-state, Steady-Flow Conservation of Mass:

Since the mass of the control volume is constant with time during the steady-state, steady-flow process, the conservation of mass principle becomes

$$\begin{bmatrix} \text{Sum of rate} \\ \text{of mass flowing} \\ \text{into control volume} \end{bmatrix} = \begin{bmatrix} \text{Sum of rate} \\ \text{of mass flowing} \\ \text{from control volume} \end{bmatrix}$$

or

$$\sum \dot{m}_{in} = \sum \dot{m}_{out} \qquad (kg/s)$$

Special Case: Steady Flow of an Incompressible Fluid

The mass flow rate is related to volume flow rate and fluid density by

$$\dot{m} = \rho \dot{V}$$

For one entrance, one exit steady flow control volume, the mass flow rates are related by

$$\dot{m}_{\text{in}} = \dot{m}_{\text{out}} \quad \text{(kg/s)}$$

$$\rho_{\text{in}} \dot{V}_{\text{in}} = \rho_{\text{out}} \dot{V}_{\text{out}}$$

$$\rho_{\text{in}} = \rho_{\text{out}} \quad \text{incompressible assumption}$$

$$\dot{V}_{\text{in}} = \dot{V}_{\text{out}}$$

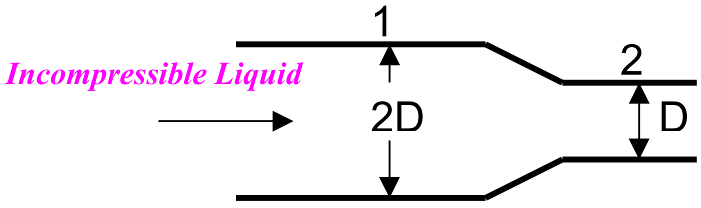
$$\vec{V}_{\text{in}} A_{\text{in}} = \vec{V}_{\text{out}} A_{\text{out}}$$

Word of caution: This result applies only to incompressible fluids. Most thermodynamic systems deal with processes involving compressible fluids such as ideal gases, steam, and the refrigerants for which the above relation **will not apply**.

Example 3-9 Geometry Effects on Fluid Flow

An incompressible liquid flows through the pipe shown in the figure. The velocity at location 2 is

A) $\frac{1}{4}\vec{V}_1$ B) $\frac{1}{2}\vec{V}_1$ C) $2\vec{V}_1$ D) $4\vec{V}_1$



Solution:

$$\dot{m} = \rho\dot{V}$$

$$\sum_{Inlets} \dot{m}_{in} = \sum_{Outlets} \dot{m}_{out}$$

$$\rho\dot{V}_1 = \rho\dot{V}_2$$

$$\dot{V}_1 = \dot{V}_2$$

$$A_1\vec{V}_1 = A_2\vec{V}_2$$

$$\vec{V}_2 = \frac{A_1}{A_2}\vec{V}_1 = \frac{\pi D_1^2 / 4}{\pi D_2^2 / 4}\vec{V}_1$$

$$\vec{V}_2 = \left(\frac{D_1}{D_2}\right)^2 \vec{V}_1 = \left(\frac{2D}{D}\right)^2 \vec{V}_1$$

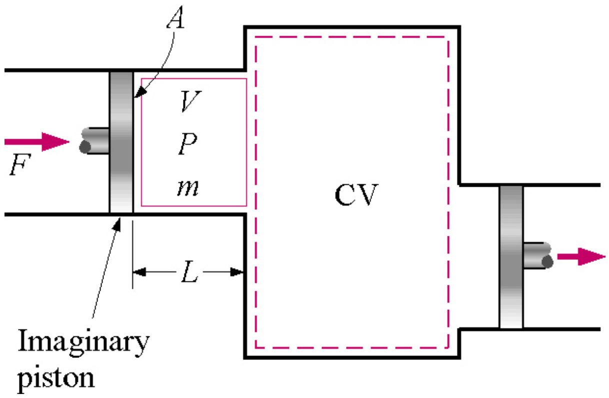
$$\vec{V}_2 = 4\vec{V}_1$$

Answer: D

Flow work and the energy of a flowing fluid

Energy flows into and from the control volume with the mass. The energy required to push the mass into or out of the control volume is known as the flow work or flow energy.

The fluid up steam of the control surface acts as a piston to push a unit of mass into or out of the control volume. Consider the unit of mass entering the control volume shown below.



As the fluid upstream pushes mass across the control surface, work done on that unit of mass is

$$W_{flow} = FL = FL\frac{A}{A} = PV = Pmv$$

$$w_{flow} = \frac{W_{flow}}{m} = Pv$$

The term Pv is called the flow work done on the unit of mass as it crosses the control surface.

The total energy of flowing fluid

The total energy carried by a unit of mass as it crosses the control surface is the sum of the internal energy, flow work, potential energy, and kinetic energy.

$$\begin{aligned}\theta &= u + Pv + \frac{\vec{V}^2}{2} + gz \\ &= h + \frac{\vec{V}^2}{2} + gz\end{aligned}$$

Here we have used the definition of enthalpy, ***h* = *u* + *Pv***.

Energy transport by mass

Amount of energy transport:

$$E_{mass} = m\theta = m\left(h + \frac{\vec{V}^2}{2} + gz\right) \qquad \text{(kJ)}$$

Rate of energy transport:

$$\dot{E}_{mass} = \dot{m}\theta = \dot{m}\left(h + \frac{\vec{V}^2}{2} + gz\right) \qquad (kW)$$