

Chapter 4

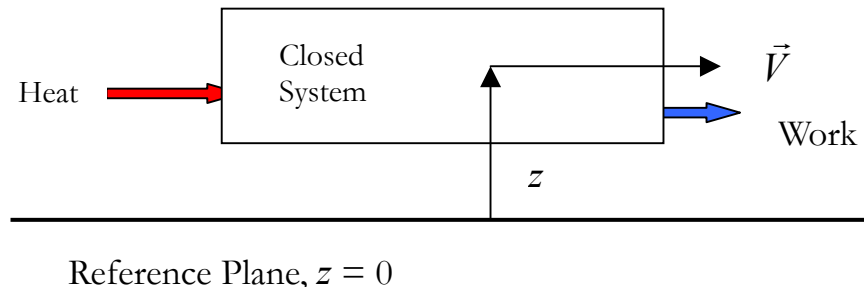
The First Law of Thermodynamics

The first law of thermodynamics is an expression of the conservation of energy principle. Energy can cross the boundaries of a closed system in the form of heat or work. Energy transfer across a system boundary due solely to the temperature difference between a system and its surroundings is called heat.

Work energy can be thought of as the energy expended to lift a weight.

Closed System First Law

A closed system moving relative to a reference plane is shown below where z is the elevation of the center of mass above the reference plane and \vec{V} is the velocity of the center of mass.



For the closed system shown above, the **conservation of energy principle** or **the first law of thermodynamics** is expressed as

$$\left(\begin{array}{c} \text{Total energy} \\ \text{entering the system} \end{array} \right) - \left(\begin{array}{c} \text{Total energy} \\ \text{leaving the system} \end{array} \right) = \left(\begin{array}{c} \text{The change in total} \\ \text{energy of the system} \end{array} \right)$$

or

$$E_{in} - E_{out} = \Delta E_{system}$$

According to classical thermodynamics, we consider the energy added to be net heat transfer to the closed system and the energy leaving the closed system to be net work done by the closed system. So

$$Q_{net} - W_{net} = \Delta E_{system}$$

Normally the stored energy, or total energy, of a system is expressed as the sum of three separate energies. The **total energy of the system**, E_{system} , is given as

$$E = \text{Internal energy} + \text{Kinetic energy} + \text{Potential energy}$$

$$E = U + KE + PE$$

Recall that U is the sum of the energy contained within the molecules of the system other than the kinetic and potential energies of the system as a whole and is called the internal energy. The internal energy U is dependent on the state of the system and the mass of the system.

For a system moving relative to a reference plane, the kinetic energy KE and the potential energy PE are given by

$$KE = \int_{\vec{V}=0}^{\vec{V}} m \vec{V} d\vec{V} = \frac{m \vec{V}^2}{2}$$

$$PE = \int_{z=0}^z mg dz = mgz$$

The change in stored energy for the system is

$$\Delta E = \Delta U + \Delta KE + \Delta PE$$

Now the **conservation of energy principle**, or the **first law of thermodynamics for closed systems**, is written as

$$Q_{net} - W_{net} = \Delta U + \Delta KE + \Delta PE$$

If the system does not move with a velocity and has no change in elevation, the conservation of energy equation reduces to

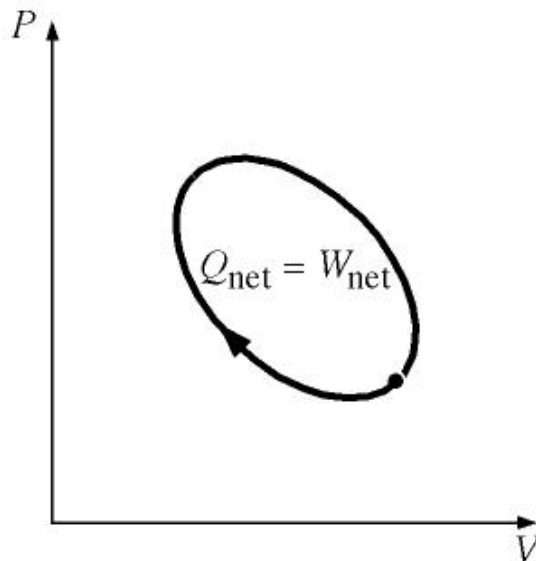
$$Q_{net} - W_{net} = \Delta U$$

We will find that this is the most commonly used form of the first law.
Closed System First Law for a Cycle

Since a thermodynamic cycle is composed of processes that cause the working fluid to undergo a series of state changes through a series of processes such that the final and initial states are identical, the change in internal energy of the working fluid is zero for whole numbers of cycles. The first law for a closed system operating in a thermodynamic cycle becomes

$$Q_{net} - W_{net} = \Delta U_{cycle}^0$$

$$Q_{net} = W_{net}$$



Example 4-1

Complete the table given below for a closed system under going a cycle.

Process	Q_{net} kJ	W_{net} kJ	$U_2 - U_1$ kJ
1-2	+5		-5
2-3	+20	+10	
3-1		-5	
Cycle			

(Answer to above problem) Row 1: +10, Row 2: +10, Row 3: -10, -5
Row 4: +15, +15, 0

Example 4-2

Find the required heat transfer to the water in Example 3-5.

Review the solution procedure of Example 3-5 and then apply the first law to the process.

Conservation of Energy:

$$E_{in} - E_{out} = \Delta E$$

$$Q_{\text{net},14} - W_{\text{net},14} = \Delta U_{14}$$

In Example 3-5 we found that

$$W_{net,14} = 1.2 \text{ kJ}$$

The heat transfer is obtained from the first law as

$$Q_{net,14} = W_{net,14} + \Delta U_{14}$$

where

$$\Delta U_{14} = U_4 - U_1 = m(u_4 - u_1)$$

At state 1, $T_1 = 100^\circ\text{C}$, $v_1 = 0.835 \text{ m}^3/\text{kg}$ and $v_f < v_1 < v_g$ at T_1 . The quality at state 1 is

$$v_1 = v_f + x_1 v_{fg}$$

$$x_1 = \frac{v_1 - v_f}{v_{fg}} = \frac{0.835 - 0.001044}{1.6729 - 0.001044} = 0.499$$

$$u_1 = u_f + x_1 u_{fg}$$

$$= 418.94 + (0.499)(2087.6)$$

$$= 1460.26 \frac{\text{kJ}}{\text{kg}}$$

Because state 4 is a saturated vapor state and $v_4 = 0.841 \text{ m}^3/\text{kg}$, interpolating in either the saturation pressure table or saturation temperature table at $v_4 = v_g$ gives

$$u_4 = 2531.48 \frac{kJ}{kg}$$

So

$$\begin{aligned}\Delta U_{14} &= m(u_4 - u_1) \\ &= (1 \text{ kg})(2531.48 - 1460.26) \frac{kJ}{kg} \\ &= 1071.22 \text{ kJ}\end{aligned}$$

The heat transfer is

$$\begin{aligned}Q_{net,14} &= W_{net,14} + \Delta U_{14} \\ &= 1.2 \text{ kJ} + 1071.22 \text{ kJ} \\ &= 1072.42 \text{ kJ}\end{aligned}$$

Heat in the amount of 1072.42 kJ is added to the water.

Review: Specific Heats and Changes in Internal Energy and Enthalpy for Ideal Gases

Before the first law of thermodynamics can be applied to systems, let's review the ways to calculate the change in internal energy and enthalpy of the substance enclosed by the system boundary. For real substances like water, the property tables are used to find the internal energy and enthalpy change. For ideal gases we learned in Chapter 2 that the internal energy and enthalpy are found by knowing the specific heats.

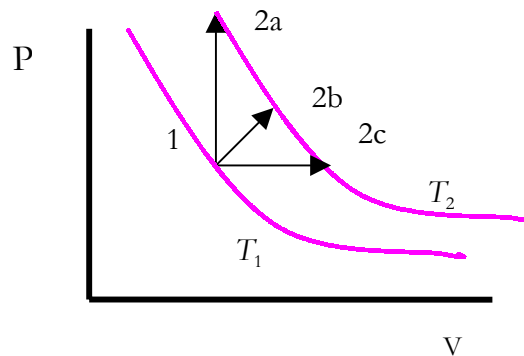
$$C_V = \left(\frac{\partial u}{\partial T} \right)_V = \frac{du}{dT} \Big|_{\text{ideal gas}} \quad \text{and} \quad C_P = \left(\frac{\partial h}{\partial T} \right)_P = \frac{dh}{dT} \Big|_{\text{ideal gas}}$$

The change in internal energy and enthalpy of ideal gases can be expressed as

$$\Delta u = u_2 - u_1 = \int_1^2 C_V(T) dT = C_{V,ave} (T_2 - T_1)$$

$$\Delta h = h_2 - h_1 = \int_1^2 C_P(T) dT = C_{P,ave} (T_2 - T_1)$$

where $C_{V,ave}$ and $C_{P,ave}$ are average or constant values of the specific heats over the temperature range. We will drop the **ave** subscript shortly.



P-V DIAGRAM FOR SEVERAL PROCESSES FOR AN IDEAL GAS.

In the above figure an ideal gas undergoes three different process between the same two temperatures.

Process 1-2a: Constant volume

Process 1-2b: $P = a + bV$, a linear relationship

Process 1-2c: Constant pressure

These ideal gas processes have the same change in internal energy and enthalpy because the processes occur between the same temperature limits.

$$\Delta u_a = \Delta u_b = \Delta u_c = \int_1^2 C_V(T) dT$$

$$\Delta h_a = \Delta h_b = \Delta h_c = \int_1^2 C_P(T) dT$$

To find Δu and Δh we often use average, or constant, values of the specific heats. Some ways to determine these values are as follows:

1. The best average value (the one that gives the exact results)

See Table A-2(c) for variable specific data.

$$C_{v,ave} = \frac{\int_1^2 C_V(T) dT}{T_2 - T_1} \quad \text{and} \quad C_{P,ave} = \frac{\int_1^2 C_P(T) dT}{T_2 - T_1}$$

2. Good average values are

$$C_{v,ave} = \frac{C_V(T_2) + C_V(T_1)}{2} \quad \text{and} \quad C_{P,ave} = \frac{C_P(T_2) + C_P(T_1)}{2}$$

and

$$C_{v,ave} = C_V(T_{ave}) \quad \text{and} \quad C_{P,ave} = C_P(T_{ave})$$

where

$$T_{ave} = \frac{T_2 + T_1}{2}$$

3. Sometimes adequate (and most often used) values are the ones evaluated at 300 K and are given in Table A-2(a).

$$C_{v,ave} = C_V(300K) \quad \text{and} \quad C_{P,ave} = C_P(300K)$$

Let's take a second look at the definition of Δu and Δh for ideal gases. Just consider the enthalpy for now.

$$\Delta h = h_2 - h_1 = \int_1^2 C_P(T) dT$$

Let's perform the integral relative to a reference state where $h = h_{\text{ref}}$ at $T = T_{\text{ref}}$.

$$\Delta h = h_2 - h_1 = \int_{T_1}^{T_{\text{ref}}} C_P(T') dT' + \int_{T_{\text{ref}}}^{T_2} C_P(T') dT'$$

or

$$\begin{aligned} \Delta h = h_2 - h_1 &= \int_{T_{\text{ref}}}^{T_2} C_P(T') dT' - \int_{T_{\text{ref}}}^{T_1} C_P(T') dT' \\ &= (h_2 - h_{\text{ref}}) - (h_1 - h_{\text{ref}}) \end{aligned}$$

At any temperature, we can calculate the enthalpy relative to the reference state as

$$h - h_{\text{ref}} = \int_{T_{\text{ref}}}^T C_P(T') dT'$$

or

$$h = h_{\text{ref}} + \int_{T_{\text{ref}}}^T C_P(T') dT'$$

This result is the basis of Table A-17 for air and Tables A-18 through A-25 for other gases. A similar result is found for the change in internal energy.

The Systematic Thermodynamics Solution Procedure

When we apply a methodical solution procedure, thermodynamics problems are relatively easy to solve. Each thermodynamics problem is approached the same way as shown in the following, which is a modification of the procedure given in the text:

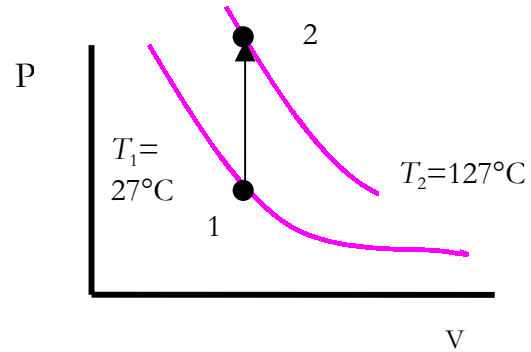
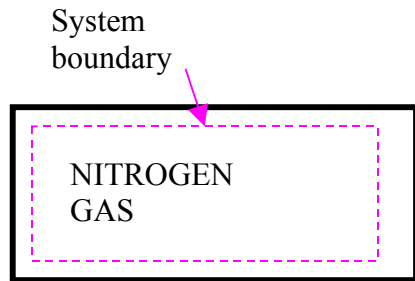
Thermodynamics Solution Method

1. Sketch the system and show energy interactions across the boundaries.
2. Determine the property relation. Is the working substance an ideal gas or a real substance? Begin to set up and fill in a property table.
3. Determine the process and sketch the process diagram. Continue to fill in the property table.
4. Apply conservation of mass and conservation of energy principles.
5. Bring in other information from the problem statement, called physical constraints, such as the volume doubles or the pressure is halved during the process.
6. Develop enough equations for the unknowns and solve.

Example 4-3

A tank contains nitrogen at 27°C. The temperature rises to 127°C by heat transfer to the system. Find the heat transfer and the ratio of the final pressure to the initial pressure.

System: Nitrogen in the tank.



***P-V* diagram for a constant volume process**

Property Relation: Nitrogen is an ideal gas. The ideal gas property relations apply. Let's assume constant specific heats. (You are encouraged to rework this problem using variable specific heat data.)

Process: Tanks are rigid vessels; therefore, the process is constant volume.

Conservation of Mass:

$$m_2 = m_1$$

Using the combined ideal gas equation of state,

$$\frac{P_2 V_2}{T_2} = \frac{P_1 V_1}{T_1}$$

Since R is the particular gas constant, and the process is constant volume,

$$V_2 = V_1$$

$$\frac{P_2}{P_1} = \frac{T_2}{T_1} = \frac{(127 + 273)K}{(27 + 273)K} = 1.333$$

Conservation of Energy:

The first law closed system is

$$E_{in} - E_{out} = \Delta E$$

$$Q_{net} - W_{net} = \Delta U$$

For nitrogen undergoing a constant volume process ($dV = 0$), the net work is ($W_{other} = 0$)

$$W_{net,12} = 0 + W_{b,12} = \int_1^2 P dV = 0$$

Using the ideal gas relations with $W_{net} = 0$, the first law becomes (constant specific heats)

$$Q_{net} - 0 = \Delta U = m \int_1^2 C_V dT = m C_V (T_2 - T_1)$$

The heat transfer per unit mass is

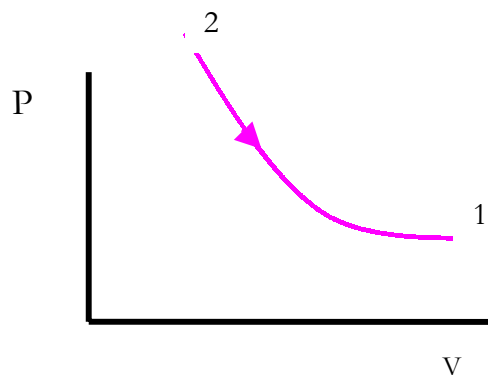
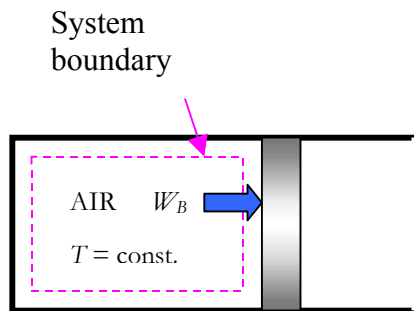
$$\begin{aligned}
 q_{net} &= \frac{Q_{net}}{m} = C_V (T_2 - T_1) \\
 &= 0.743 \frac{kJ}{kg \cdot K} (127 - 27) K \\
 &= 74.3 \frac{kJ}{kg}
 \end{aligned}$$

Example 4-4

Air is expanded isothermally at 100°C from 0.4 MPa to 0.1 MPa. Find the ratio of the final to the initial volume, the heat transfer, and work.

System: Air contained in a piston-cylinder device, a closed system

Process: Constant temperature



P-V diagram for $T = \text{constant}$

Property Relation: Assume air is an ideal gas and use the ideal gas property relations with constant specific heats.

$$PV = mRT$$

$$\Delta u = C_V (T_2 - T_1)$$

Conservation of Energy:

$$E_{in} - E_{out} = \Delta E$$

$$Q_{net} - W_{net} = \Delta U$$

The system mass is constant but is not given and cannot be calculated; therefore, let's find the work and heat transfer per unit mass.

Work Calculation:

$$W_{net,12} = (W_{net})_{other,12} + W_{b,12}$$

$$\begin{aligned} W_{b,12} &= \int_1^2 P dV = \int_1^2 \frac{mRT}{V} dV \\ &= mRT \ln \left(\frac{V_2}{V_1} \right) \end{aligned}$$

Conservation of Mass: For an ideal gas in a closed system (mass = constant), we have

$$\begin{aligned} m_1 &= m_2 \\ \frac{P_1 V_1}{RT_1} &= \frac{P_2 V_2}{RT_2} \end{aligned}$$

Since the R 's cancel and $T_2 = T_1$

$$\frac{V_2}{V_1} = \frac{P_1}{P_2} = \frac{0.4 \text{ MPa}}{0.1 \text{ MPa}} = 4$$

Then the work expression per unit mass becomes

$$\begin{aligned} w_{b,12} &= \frac{W_{b,12}}{m} = RT \ln \left(\frac{V_2}{V_1} \right) \\ w_{b,12} &= \left(0.287 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} \right) (100 + 273) \text{ K} \ln(4) \\ &= 148.4 \frac{\text{kJ}}{\text{kg}} \end{aligned}$$

The net work per unit mass is

$$w_{net,12} = 0 + w_{b,12} = 148.4 \frac{\text{kJ}}{\text{kg}}$$

Now to continue with the conservation of energy to find the heat transfer.
Since $T_2 = T_1 = \text{constant}$,

$$\Delta U_{12} = m \Delta u_{12} = m C_V (T_2 - T_1) = 0$$

So the heat transfer per unit mass is

$$\begin{aligned}
 q_{net} &= \frac{Q_{net}}{m} \\
 q_{net} - w_{net} &= \Delta u = 0 \\
 q_{net} &= w_{net} \\
 &= 148.4 \frac{kJ}{kg}
 \end{aligned}$$

The heat transferred **to** the air during an isothermal expansion process equals the work **done**.

Examples Using Variable Specific Heats

Review the solutions in Chapter 4 to the ideal gas examples where the variable specific heat data are used to determine the changes in internal energy and enthalpy.

Extra Problem for You to Try:

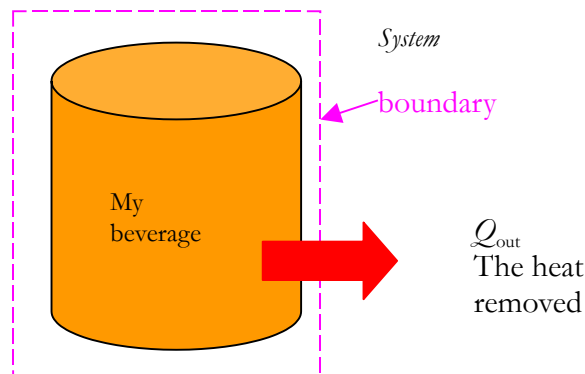
An ideal gas, contained in a piston-cylinder device, undergoes a polytropic process in which the polytropic exponent n is equal to k , the ratio of specific heats. Show that this process is adiabatic. When we get to Chapter 6 you will find that this is an important ideal gas process.

Example 4-5 Incompressible Liquid

A two-liter bottle of your favorite beverage has just been removed from the trunk of your car. The temperature of the beverage is 35°C, and you always drink your beverage at 10°C.

- How much heat energy must be removed from your two liters of beverage?
- You are having a party and need to cool 10 of these two-liter bottles in one-half hour. What rate of heat removal, in kW, is required?
Assuming that your refrigerator can accomplish this and that electricity costs 8.5 cents per kW-hr, how much will it cost to cool these 10 bottles?

System: The liquid in the constant volume, closed system container



Property Relation: Incompressible liquid relations, let's assume that the beverage is mostly water and takes on the properties of liquid water. The specific volume is 0.001 m³/kg, $C = 4.18$ kJ/kg·K.

Process: Constant volume

$$V_2 = V_1$$

Conservation of Mass:

$$m_2 = m_1 = m$$

$$m = \frac{V}{v} = \frac{2 \text{ L}}{0.001 \frac{\text{m}^3}{\text{kg}}} \left(\frac{\text{m}^3}{1000 \text{ L}} \right) = 2 \text{ kg}$$

Conservation of Energy:

The first law closed system is

$$E_{in} - E_{out} = \Delta E$$

Since the container is constant volume and there is no “other” work done on the container during the cooling process, we have

$$W_{net} = (W_{net})_{other} + W_b = 0$$

The only energy crossing the boundary is the heat transfer leaving the container. Assuming the container to be stationary, the conservation of energy becomes

$$-E_{out} = \Delta E$$

$$-Q_{out} = \Delta U = mC\Delta T$$

$$-Q_{out} = (2 \text{ kg})(4.18 \frac{\text{kJ}}{\text{kg} \cdot \text{K}})(10 - 35) \text{ K}$$

$$-Q_{out} = -209.2 \text{ kJ}$$

$$Q_{out} = 209.2 \text{ kJ}$$

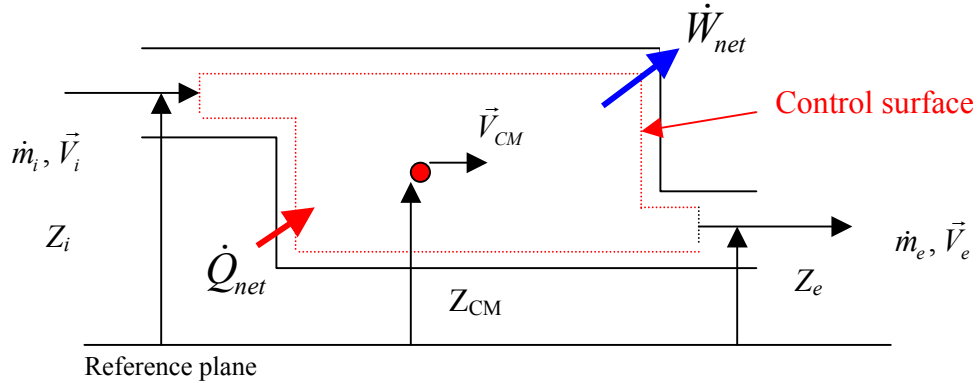
The heat transfer rate to cool the 10 bottles in one-half hour is

$$\begin{aligned} \dot{Q}_{out} &= \frac{(10 \text{ bottles})(209.2 \frac{\text{kJ}}{\text{bottle}})}{0.5 \text{ hr}} \left(\frac{1 \text{ hr}}{3600 \text{ s}} \right) \left(\frac{\text{kW}}{\frac{\text{kJ}}{\text{s}}} \right) \\ &= 1.162 \text{ kW} \end{aligned}$$

$$\begin{aligned} \text{Cost} &= (1.162 \text{ kW})(0.5 \text{ hr}) \frac{\$0.085}{\text{kW} \cdot \text{hr}} \\ &= \$0.05 \end{aligned}$$

Conservation of Energy for Control volumes

The conservation of mass and the conservation of energy principles for open systems or control volumes apply to systems having mass crossing the system boundary or control surface. In addition to the heat transfer and work crossing the system boundaries, mass carries energy with it as it crosses the system boundaries. Thus, the mass and energy content of the open system may change when mass enters or leaves the control volume.



Typical control volume or open system

Thermodynamic processes involving control volumes can be considered in two groups: steady-flow processes and unsteady-flow processes. During a steady-flow process, the fluid flows through the control volume steadily, experiencing no change with time at a fixed position.

Let's review the concepts of mass flow rate and energy transport by mass.

Mass Flow Rate

Mass flow through a cross-sectional area per unit time is called the mass flow rate \dot{m} . If the fluid density and velocity are constant over the flow cross-sectional area, the mass flow rate is

$$\dot{m} = \rho \vec{V}_{av} A = \frac{\vec{V}_{av} A}{\nu}$$

where ρ is the density, kg/m^3 ($= 1/\nu$), A is the cross-sectional area, m^2 ; and \vec{V}_{av} is the average fluid velocity normal to the area, m/s .

The fluid volume flowing through a cross-section per unit time is called the volume flow rate \dot{V} .

$$\dot{V} = \vec{V}A \quad (m^3 / s)$$

The mass and volume flow rate are related by

$$\dot{m} = \rho \dot{V} = \frac{\dot{V}}{v} \quad (kg / s)$$

Conservation of Mass for General Control Volume

The conservation of mass principle for the open system or control volume is expressed as

$$\left[\begin{array}{l} \text{Sum of rate} \\ \text{of mass flowing} \\ \text{into control volume} \end{array} \right] - \left[\begin{array}{l} \text{Sum of rate} \\ \text{of mass flowing} \\ \text{from control volume} \end{array} \right] = \left[\begin{array}{l} \text{Time rate change} \\ \text{of mass inside} \\ \text{control volume} \end{array} \right]$$

or

$$\sum \dot{m}_{in} - \sum \dot{m}_{out} = \Delta \dot{m}_{system} \quad (kg / s)$$

Conservation of Energy for General Control Volume

The conservation of energy principle for the control volume or open system has the same word definition as the first law for the closed system.

Expressing the energy transfers on a rate basis, the control volume first law is

$$\left[\begin{array}{c} \text{Sum of rate} \\ \text{of energy flowing} \\ \text{into control volume} \end{array} \right] - \left[\begin{array}{c} \text{Sum of rate} \\ \text{of energy flowing} \\ \text{from control volume} \end{array} \right] = \left[\begin{array}{c} \text{Time rate change} \\ \text{of energy inside} \\ \text{control volume} \end{array} \right]$$

or

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{system}}_{\text{Rate change in internal, kinetic, potential, etc., energies}} \quad (kW)$$

Considering that energy flows into and from the control volume with the mass, energy enters because net heat is transferred to the control volume, and energy leaves because the control volume does net work on its surroundings, the open system, or control volume, the first law becomes

$$\dot{Q}_{net} + \sum_{\text{for each inlet}} \underbrace{\dot{m}_i \theta_i}_{\text{for each inlet}} - \dot{W}_{net} - \sum_{\text{for each exit}} \underbrace{\dot{m}_e \theta_e}_{\text{for each exit}} = \frac{dE_{CV}}{dt} \quad (kW)$$

where θ is the energy per unit mass flowing into or from the control volume. The energy per unit mass, θ , flowing across the control surface that defines the control volume is composed of four terms: the internal energy, the kinetic energy, the potential energy, and the flow work.

The total energy carried by a unit of mass as it crosses the control surface is

$$\begin{aligned} \theta &= u + Pv + \frac{\vec{V}^2}{2} + gz \\ &= h + \frac{\vec{V}^2}{2} + gz \end{aligned}$$

Here we have used the definition of enthalpy, $h = u + Pv$.

$$\dot{E}_{in} - \dot{E}_{out} = \Delta \dot{E}_{CV}$$

$$\dot{Q}_{net} + \underbrace{\sum \dot{m}_i \left(h_i + \frac{\vec{V}_i^2}{2} + gz_i \right)}_{\text{for each inlet}} - \dot{W}_{net} - \underbrace{\sum \dot{m}_e \left(h_e + \frac{\vec{V}_e^2}{2} + gz_e \right)}_{\text{for each exit}} = \Delta \dot{E}_{CV}$$

Where the time rate change of the energy of the control volume has been written as $\Delta \dot{E}_{CV}$.

Steady-State, Steady-Flow Processes

Most energy conversion devices operate steadily over long periods of time. The rates of heat transfer and work crossing the control surface are constant with time. The states of the mass streams crossing the control surface or boundary are constant with time. Under these conditions the mass and energy content of the control volume are constant with time.

$$\frac{dm_{CV}}{dt} = \Delta \dot{m}_{CV} = 0$$

$$\frac{dE_{CV}}{dt} = \Delta \dot{E}_{CV} = 0$$

Steady-state, Steady-Flow Conservation of Mass:

Since the mass of the control volume is constant with time during the steady-state, steady-flow process, the conservation of mass principle becomes

$$\left[\begin{array}{l} \text{Sum of rate} \\ \text{of mass flowing} \\ \text{into control volume} \end{array} \right] = \left[\begin{array}{l} \text{Sum of rate} \\ \text{of mass flowing} \\ \text{from control volume} \end{array} \right]$$

or

$$\sum \dot{m}_{in} = \sum \dot{m}_{out} \quad (kg / s)$$

Steady-state, steady-flow conservation of energy

Since the energy of the control volume is constant with time during the steady-state, steady-flow process, the conservation of energy principle becomes

$$\left[\begin{array}{c} \text{Sum of rate} \\ \text{of energy flowing} \\ \text{into control volume} \end{array} \right] = \left[\begin{array}{c} \text{Sum of rate} \\ \text{of energy flowing} \\ \text{from control volume} \end{array} \right]$$

or

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{system}^0}_{\text{Rate change in internal, kinetic, potential, etc., energies}} \quad (kW)$$

or

$$\underbrace{\dot{E}_{in}}_{\text{Rate of net energy transfer by heat, work, and mass into the system}} = \underbrace{\dot{E}_{out}}_{\text{Rate of energy transfer by heat, work, and mass from the system}}$$

Considering that energy flows into and from the control volume with the mass, energy enters because heat is transferred to the control volume, and energy leaves because the control volume does work on its surroundings, the steady-state, steady-flow first law becomes

$$\dot{Q}_{in} + \dot{W}_{in} + \underbrace{\sum \dot{m}_i \left(h_i + \frac{\vec{V}_i^2}{2} + gz_i \right)}_{\text{for each inlet}} = \dot{Q}_{out} + \dot{W}_{out} + \underbrace{\sum \dot{m}_e \left(h_e + \frac{\vec{V}_e^2}{2} + gz_e \right)}_{\text{for each exit}}$$

Most often we write this result as

$$\dot{Q}_{net} - \dot{W}_{net} = \underbrace{\sum \dot{m}_e \left(h_e + \frac{\vec{V}_e^2}{2} + gz_e \right)}_{\text{for each exit}} - \underbrace{\sum \dot{m}_i \left(h_i + \frac{\vec{V}_i^2}{2} + gz_i \right)}_{\text{for each inlet}}$$

where

$$\begin{aligned}\dot{Q}_{net} &= \sum \dot{Q}_{in} - \sum \dot{Q}_{out} \\ \dot{W}_{net} &= \sum \dot{W}_{out} - \sum \dot{W}_{in}\end{aligned}$$

Steady-state, steady-flow for one entrance and one exit

A number of thermodynamic devices such as pumps, fans, compressors, turbines, nozzles, diffusers, and heaters operate with one entrance and one exit. The steady-state, steady-flow conservation of mass and first law of thermodynamics for these systems reduce to

$$\dot{m}_1 = \dot{m}_2 \quad (kg / s)$$

$$\frac{1}{v_1} \vec{V}_1 A_1 = \frac{1}{v_2} \vec{V}_2 A_2$$

$$\dot{Q} - \dot{W} = \dot{m} \left[h_2 - h_1 + \frac{\vec{V}_2^2 - \vec{V}_1^2}{2} + g(z_2 - z_1) \right] \quad (kW)$$

where the entrance is state 1 and the exit is state 2 and \dot{m} is the mass flow rate through the device.

When can we neglect the kinetic and potential energy terms in the first law? Consider the kinetic and potential energies per unit mass.

$$ke = \frac{\vec{V}^2}{2}$$

$$\text{For } \vec{V} = 45 \frac{\text{m}}{\text{s}} \quad ke = \frac{(45 \text{ m/s})^2}{2} \frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2} = 1 \frac{\text{kJ}}{\text{kg}}$$

$$\vec{V} = 140 \frac{\text{m}}{\text{s}} \quad ke = \frac{(140 \text{ m/s})^2}{2} \frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2} = 10 \frac{\text{kJ}}{\text{kg}}$$

$$pe = gz$$

$$\text{For } z = 100 \text{ m} \quad pe = 9.8 \frac{\text{m}}{\text{s}^2} 100 \text{ m} \frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2} = 0.98 \frac{\text{kJ}}{\text{kg}}$$

$$z = 1000 \text{ m} \quad pe = 9.8 \frac{\text{m}}{\text{s}^2} 1000 \text{ m} \frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2} = 9.8 \frac{\text{kJ}}{\text{kg}}$$

When compared to the enthalpy of steam ($h \cong 2000$ to 3000 kJ/kg) and the enthalpy of air ($h \cong 200$ to 6000 kJ/kg), the kinetic and potential energies are often neglected.

When the kinetic and potential energies can be neglected, the conservation of energy equation becomes

$$\dot{Q} - \dot{W} = \dot{m}(h_2 - h_1) \quad (\text{kW})$$

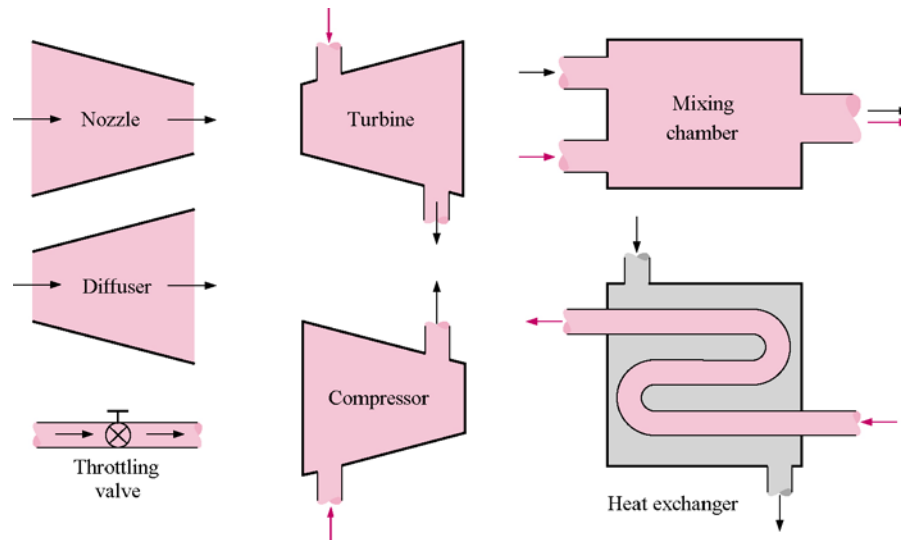
We often write this last result per unit mass flow as

$$q - w = (h_2 - h_1) \quad (\text{kJ/kg})$$

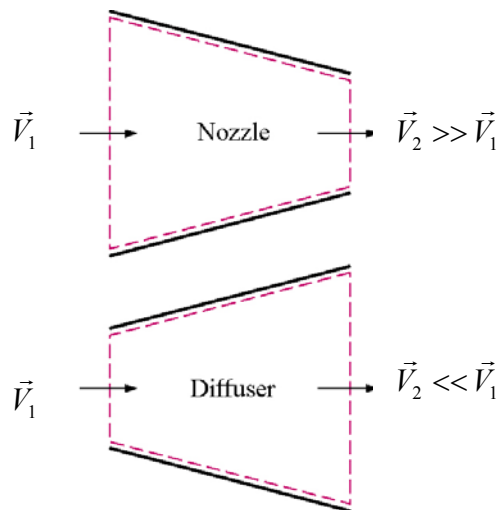
where $q = \frac{\dot{Q}}{\dot{m}}$ and $w = \frac{\dot{W}}{\dot{m}}$.

Some Steady-Flow Engineering Devices

Below are some engineering devices that operate essentially as steady-state, steady-flow control volumes.



Nozzles and Diffusers



For flow through nozzles, the heat transfer, work, and potential energy are normally neglected, and nozzles have one entrance and one exit. The conservation of energy becomes

$$\begin{aligned}
\dot{m}_{in} &= \dot{m}_{out} \\
\dot{m}_1 &= \dot{m}_2 = \dot{m} \\
\dot{E}_{in} &= \dot{E}_{out} \\
\dot{Q}_{net} + \underbrace{\sum \dot{m}_i \left(h_i + \frac{\vec{V}_i^2}{2} + gz_i \right)}_{\text{for each inlet}} &= \dot{W}_{net} + \underbrace{\sum \dot{m}_e \left(h_e + \frac{\vec{V}_e^2}{2} + gz_e \right)}_{\text{for each exit}} \\
\dot{m} \left(h_1 + \frac{\vec{V}_1^2}{2} \right) &= \dot{m} \left(h_2 + \frac{\vec{V}_2^2}{2} \right)
\end{aligned}$$

Solving for \vec{V}_2

$$\vec{V}_2 = \sqrt{2(h_1 - h_2) + \vec{V}_1^2}$$

Example 4-6

Steam at 0.4 MPa, 300°C, enters an adiabatic nozzle with a low velocity and leaves at 0.2 MPa with a quality of 90%. Find the exit velocity, in m/s.

Control Volume: The nozzle

Property Relation: Steam tables

Process: Assume adiabatic, steady-flow

Conservation Principles:

Conservation of mass:

For one entrance, one exit, the conservation of mass becomes

$$\sum \dot{m}_{in} = \sum \dot{m}_{out}$$

$$\dot{m}_1 = \dot{m}_2 = \dot{m}$$

Conservation of energy:

According to the sketched control volume, mass crosses the control surface, but no work or heat transfer crosses the control surface. Neglecting the potential energies, we have

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{m} \left(h_1 + \frac{\vec{V}_1^2}{2} \right) = \dot{m} \left(h_2 + \frac{\vec{V}_2^2}{2} \right)$$

Neglecting the inlet kinetic energy, the exit velocity is

$$\vec{V}_2 = \sqrt{2(h_1 - h_2)}$$

Now, we need to find the enthalpies from the steam tables.

$$\left. \begin{array}{l} T_1 = 300^\circ C \\ P_1 = 0.4 \text{ MPa} \end{array} \right\} h_1 = 3066.8 \frac{\text{kJ}}{\text{kg}} \quad \left. \begin{array}{l} P_2 = 0.2 \text{ MPa} \\ x_2 = 0.90 \end{array} \right\} h_2$$

At 0.2 MPa $h_f = 504.7 \text{ kJ/kg}$ and $h_{fg} = 2201.9 \text{ kJ/kg}$.

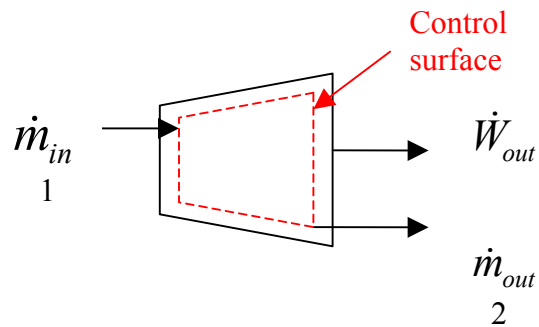
$$h_2 = h_f + x_2 h_{fg}$$

$$= 504.7 + (0.90)(2201.9) = 2486.4 \frac{\text{kJ}}{\text{kg}}$$

$$\vec{V}_2 = \sqrt{2(3066.8 - 2486.4) \frac{kJ}{kg} \frac{1000 m^2 / s^2}{kJ / kg}}$$

$$= 1077.4 \frac{m}{s}$$

Turbines



Turbine control volume

If we neglect the changes in kinetic and potential energies as fluid flows through an adiabatic turbine having one entrance and one exit, the conservation of mass and the steady-state, steady-flow first law becomes

$$\dot{m}_{in} = \dot{m}_{out}$$

$$\dot{m}_1 = \dot{m}_2 = \dot{m}$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{Q}_{net} + \underbrace{\sum \dot{m}_i \left(h_i + \frac{\vec{V}_i^2}{2} + gz_i \right)}_{\text{for each inlet}} = \dot{W}_{net} + \underbrace{\sum \dot{m}_e \left(h_e + \frac{\vec{V}_e^2}{2} + gz_e \right)}_{\text{for each exit}}$$

$$\dot{m}_1 h_1 = \dot{m}_2 h_2 + \dot{W}_{out}$$

$$\dot{W}_{out} = \dot{m}(h_1 - h_2)$$

Example 4-7

High pressure air at 1300 K flows into an aircraft gas turbine and undergoes a steady-state, steady-flow, adiabatic process to the turbine exit at 660 K. Calculate the work done per unit mass of air flowing through the turbine when

- (a) Temperature-dependent data are used.

- (b) $C_{p,ave}$ at the average temperature is used.
- (c) C_p at 300 K is used.

Control Volume: The turbine

Property Relation: Assume air is an ideal gas and use ideal gas relations

Process: Steady-state, steady-flow, adiabatic process

Conservation Principles:

Conservation of mass:

$$\sum \dot{m}_{in} = \sum \dot{m}_{out}$$

$$\dot{m}_1 = \dot{m}_2 = \dot{m}$$

Conservation of energy:

$$\dot{Q}_{in} + \underbrace{\sum \dot{m}_i \left(h_i + \frac{\vec{V}_i^2}{2} + gz_i \right)}_{\text{for each inlet}} = \dot{W}_{out} + \underbrace{\sum \dot{m}_e \left(h_e + \frac{\vec{V}_e^2}{2} + gz_e \right)}_{\text{for each exit}}$$

According to the sketched control volume, mass and work cross the control surface. Neglecting kinetic and potential energies and noting the process is adiabatic, we have

$$0 + \dot{m}_1 h_1 = \dot{W}_{out} + \dot{m}_2 h_2$$

$$\dot{W}_{out} = \dot{m}(h_1 - h_2)$$

The work done by the air per unit mass flow is

$$w_{out} = \frac{\dot{W}_{out}}{\dot{m}} = h_1 - h_2$$

Notice that the work done by a fluid flowing through a turbine is equal to the enthalpy decrease of the fluid.

(a) Using the air tables, Table A-17

$$\text{at } T_1 = 1300 \text{ K, } h_1 = 1395.97 \text{ kJ/kg}$$

$$\text{at } T_2 = 660 \text{ K, } h_2 = 670.47 \text{ kJ/kg}$$

$$\begin{aligned} w_{out} &= h_1 - h_2 \\ &= (1395.97 - 670.47) \frac{\text{kJ}}{\text{kg}} \\ &= 725.5 \frac{\text{kJ}}{\text{kg}} \end{aligned}$$

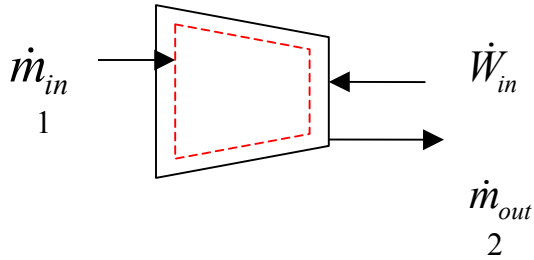
(b) Using Table A-2(c) at $T_{ave} = 980 \text{ K}$, $C_{p, ave} = 1.138 \text{ kJ/kg} \cdot \text{K}$

$$\begin{aligned} w_{out} &= h_1 - h_2 = C_{p, ave} (T_1 - T_2) \\ &= 1.138 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} (1300 - 660) \text{ K} \\ &= 728.3 \frac{\text{kJ}}{\text{kg}} \end{aligned}$$

c. Using Table A-2(a) at $T = 300 \text{ K}$, $C_p = 1.005 \text{ kJ/kg} \cdot \text{K}$

$$\begin{aligned} w_{out} &= h_1 - h_2 = C_p (T_1 - T_2) \\ &= 1.005 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} (1300 - 660) \text{ K} \\ &= 643.2 \frac{\text{kJ}}{\text{kg}} \end{aligned}$$

Compressors and fans



Steady-Flow Compressor

Compressors and fans are essentially the same devices. However, compressors operate over larger pressure ratios than fans. If we neglect the changes in kinetic and potential energies as fluid flows through an adiabatic compressor having one entrance and one exit, the steady-state, steady-flow first law or the conservation of energy equation becomes

$$\dot{Q}_{net} + \underbrace{\sum \dot{m}_i \left(h_i + \frac{\vec{V}_i^2}{2} + gz_i \right)}_{\text{for each inlet}} = \dot{W}_{net} + \underbrace{\sum \dot{m}_e \left(h_e + \frac{\vec{V}_e^2}{2} + gz_e \right)}_{\text{for each exit}}$$

$$-\dot{W}_{net} = \dot{m}(h_2 - h_1)$$

$$-(-\dot{W}_{in}) = \dot{m}(h_2 - h_1)$$

$$\dot{W}_{in} = \dot{m}(h_2 - h_1)$$

Example 4-8

Nitrogen gas is compressed in a steady-state, steady-flow, adiabatic process from 0.1 MPa, 25°C. During the compression process the temperature becomes 125°C. If the mass flow rate is 0.2 kg/s, determine the work done on the nitrogen, in kW.

Control Volume: The compressor (see the compressor sketched above)

Property Relation: Assume nitrogen is an ideal gas and use ideal gas relations

Process: Adiabatic, steady-flow

Conservation Principles:

Conservation of mass:

$$\sum \dot{m}_{in} = \sum \dot{m}_{out}$$
$$\dot{m}_1 = \dot{m}_2 = \dot{m}$$

Conservation of energy:

$$\dot{Q}_{net} + \underbrace{\sum \dot{m}_i \left(h_i + \frac{\vec{V}_i^2}{2} + gz_i \right)}_{\text{for each inlet}} = \dot{W}_{net} + \underbrace{\sum \dot{m}_e \left(h_e + \frac{\vec{V}_e^2}{2} + gz_e \right)}_{\text{for each exit}}$$

According to the sketched control volume, mass and work cross the control surface. Neglecting kinetic and potential energies and noting the process is adiabatic, we have for one entrance and one exit

$$0 + \dot{m}_1(h_1 + 0 + 0) = (-\dot{W}_{in}) + \dot{m}_2(h_2 + 0 + 0)$$
$$\dot{W}_{in} = \dot{m}(h_2 - h_1)$$

The work done **on** the nitrogen is related to the enthalpy rise of the nitrogen as it flows through the compressor. The work done on the nitrogen per unit mass flow is

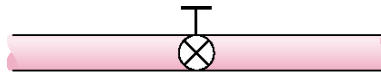
$$w_{in} = \frac{\dot{W}_{in}}{\dot{m}} = h_2 - h_1$$

Assuming constant specific heats at 300 K from Table A-2(a), we write the work as

$$\begin{aligned}
 w_{in} &= C_p (T_2 - T_1) \\
 &= 1.039 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} (125 - 25) \text{K} \\
 &= 103.9 \frac{\text{kJ}}{\text{kg}}
 \end{aligned}$$

$$\begin{aligned}
 \dot{W}_{in} &= \dot{m} w_{in} = 0.2 \frac{\text{kg}}{\text{s}} \left(103.9 \frac{\text{kJ}}{\text{kg}} \right) \\
 &= 20.78 \frac{\text{kJ}}{\text{s}} = 20.78 \text{ kW}
 \end{aligned}$$

Throttling devices



(a) An adjustable valve



(b) A porous plug



(c) A capillary tube

Consider fluid flowing through a one-entrance, one-exit porous plug. The fluid experiences a pressure drop as it flows through the plug. No net work is done by the fluid. Assume the process is adiabatic and that the kinetic and potential energies are neglected; then the conservation of mass and energy equations become

$$\dot{m}_i = \dot{m}_e$$

$$\dot{Q}_{net} + \underbrace{\sum \dot{m}_i \left(h_i + \frac{\vec{V}_i^2}{2} + gz_i \right)}_{\text{for each inlet}} = \dot{W}_{net} + \underbrace{\sum \dot{m}_e \left(h_e + \frac{\vec{V}_e^2}{2} + gz_e \right)}_{\text{for each exit}}$$

$$\dot{m}_i h_i = \dot{m}_e h_e$$

$$h_i = h_e$$

This process is called a throttling process. What happens when an ideal gas is throttled?

$$h_i = h_e$$

$$h_e - h_i = 0$$

$$\int_i^e C_p(T) dT = 0$$

or

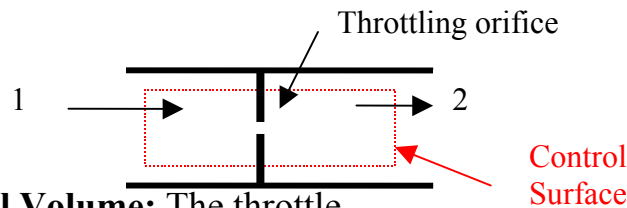
$$T_e = T_i$$

When throttling an ideal gas, the temperature does not change. We will see later in Chapter 10 that the throttling process is an important process in the refrigeration cycle.

A throttling device may be used to determine the enthalpy of saturated steam. The steam is throttled from the pressure in the pipe to ambient pressure in the calorimeter. The pressure drop is sufficient to superheat the steam in the calorimeter. Thus, the temperature and pressure in the calorimeter will specify the enthalpy of the steam in the pipe.

Example 4-9

One way to determine the quality of saturated steam is to throttle the steam to a low enough pressure that it exists as a superheated vapor. Saturated steam at 0.4 MPa is throttled to 0.1 MPa, 100°C. Determine the quality of the steam at 0.4 MPa.



Control Volume: The throttle

Property Relation: The steam tables

Process: Steady-state, steady-flow, no work, no heat transfer, neglect kinetic and potential energies, one entrance, one exit

Conservation Principles:

Conservation of mass:

$$\sum \dot{m}_{in} = \sum \dot{m}_{out}$$

$$\dot{m}_1 = \dot{m}_2 = \dot{m}$$

Conservation of energy:

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{Q}_{net} + \underbrace{\sum \dot{m}_i \left(h_i + \frac{\vec{V}_i^2}{2} + gz_i \right)}_{\text{for each inlet}} = \dot{W}_{net} + \underbrace{\sum \dot{m}_e \left(h_e + \frac{\vec{V}_e^2}{2} + gz_e \right)}_{\text{for each exit}}$$

According to the sketched control volume, mass crosses the control surface. Neglecting kinetic and potential energies and noting the process is adiabatic with no work, we have for one entrance and one exit

$$0 + \dot{m}_1(h_1 + 0 + 0) = 0 + \dot{m}_2(h_2 + 0 + 0)$$

$$\dot{m}_1 h_1 = \dot{m}_2 h_2$$

$$h_1 = h_2$$

$$\left. \begin{array}{l} T_2 = 100^\circ C \\ P_2 = 0.1 \text{ MPa} \end{array} \right\} h_2 = 2676.2 \frac{\text{kJ}}{\text{kg}}$$

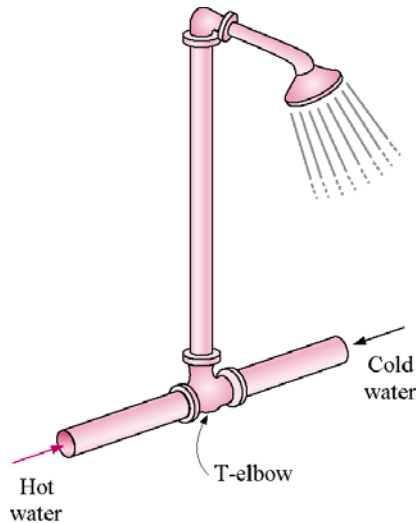
Therefore,

$$\begin{aligned} h_1 = h_2 &= 2676.2 \frac{\text{kJ}}{\text{kg}} \\ &= (h_f + x_1 h_{fg})_{@P_1=0.4 \text{ MPa}} \end{aligned}$$

$$\begin{aligned} x_1 &= \frac{h_1 - h_f}{h_{fg}} \\ &= \frac{2676.2 - 604.74}{2133.8} \\ &= 0.971 \end{aligned}$$

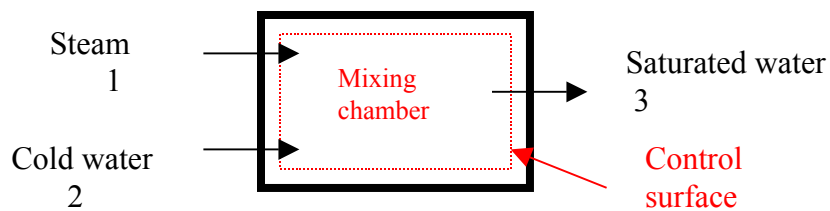
Mixing chambers

The mixing of two fluids occurs frequently in engineering applications. The section where the mixing process takes place is called a mixing chamber. The ordinary shower is an example of a mixing chamber.



Example 4-10

Steam at 0.2 MPa, 300°C, enters a mixing chamber and is mixed with cold water at 20°C, 0.2 MPa, to produce 20 kg/s of saturated liquid water at 0.2 MPa. What are the required steam and cold water flow rates?



Control Volume: The mixing chamber

Property Relation: Steam tables

Process: Assume steady-flow, adiabatic mixing, with no work

Conservation Principles:

Conservation of mass:

$$\sum \dot{m}_{in} = \sum \dot{m}_{out}$$

$$\dot{m}_1 + \dot{m}_2 = \dot{m}_3$$

$$\dot{m}_2 = \dot{m}_3 - \dot{m}_1$$

Conservation of energy:

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{Q}_{net} + \underbrace{\sum \dot{m}_i \left(h_i + \frac{\vec{V}_i^2}{2} + gz_i \right)}_{\text{for each inlet}} = \dot{W}_{net} + \underbrace{\sum \dot{m}_e \left(h_e + \frac{\vec{V}_e^2}{2} + gz_e \right)}_{\text{for each exit}}$$

According to the sketched control volume, mass crosses the control surface. Neglecting kinetic and potential energies and noting the process is adiabatic with no work, we have for two entrances and one exit

$$\begin{aligned}\dot{m}_1 h_1 + \dot{m}_2 h_2 &= \dot{m}_3 h_3 \\ \dot{m}_1 h_1 + (\dot{m}_3 - \dot{m}_1) h_2 &= \dot{m}_3 h_3 \\ \dot{m}_1 (h_1 - h_2) &= \dot{m}_3 (h_3 - h_2) \\ \dot{m}_1 &= \dot{m}_3 \frac{(h_3 - h_2)}{(h_1 - h_2)}\end{aligned}$$

Now, we use the steam tables to find the enthalpies:

$$\left. \begin{array}{l} T_1 = 300^\circ C \\ P_1 = 0.2 \text{ MPa} \end{array} \right\} h_1 = 3071.8 \frac{kJ}{kg}$$

$$\left. \begin{array}{l} T_2 = 20^\circ C \\ P_2 = 0.2 \text{ MPa} \end{array} \right\} h_2 \approx h_{f@20^\circ C} = 83.69 \frac{kJ}{kg}$$

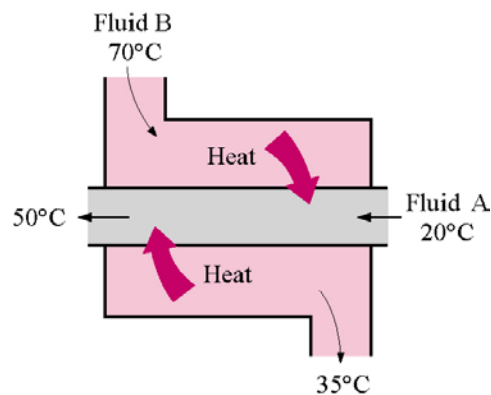
$$\left. \begin{array}{l} P_3 = 0.2 \text{ MPa} \\ \text{Sat. liquid} \end{array} \right\} h_3 = h_{f@0.2 \text{ MPa}} = 504.7 \frac{kJ}{kg}$$

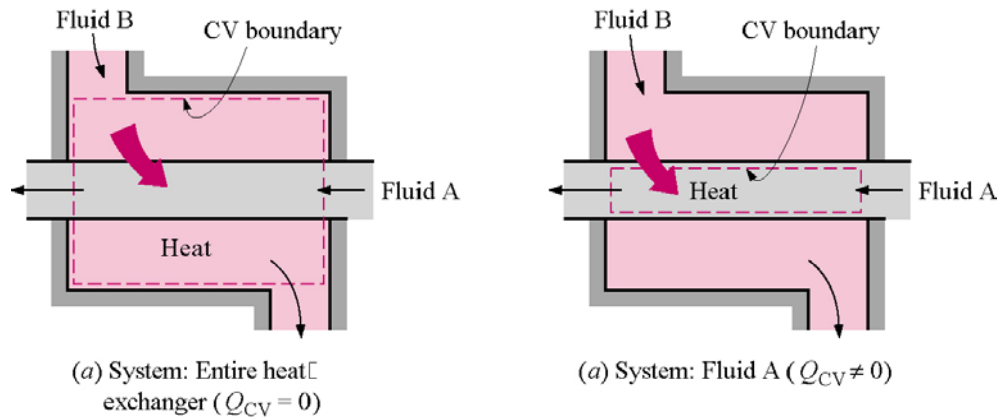
$$\begin{aligned}
 \dot{m}_1 &= \dot{m}_3 \frac{(h_3 - h_2)}{(h_1 - h_2)} \\
 &= 20 \frac{kg}{s} \frac{(504.7 - 83.69) kJ / kg}{(3071.8 - 83.96) kJ / kg} \\
 &= 2.82 \frac{kg}{s}
 \end{aligned}$$

$$\begin{aligned}
 \dot{m}_2 &= \dot{m}_3 - \dot{m}_1 \\
 &= (20 - 2.82) \frac{kg}{s} \\
 &= 17.18 \frac{kg}{s}
 \end{aligned}$$

Heat exchangers

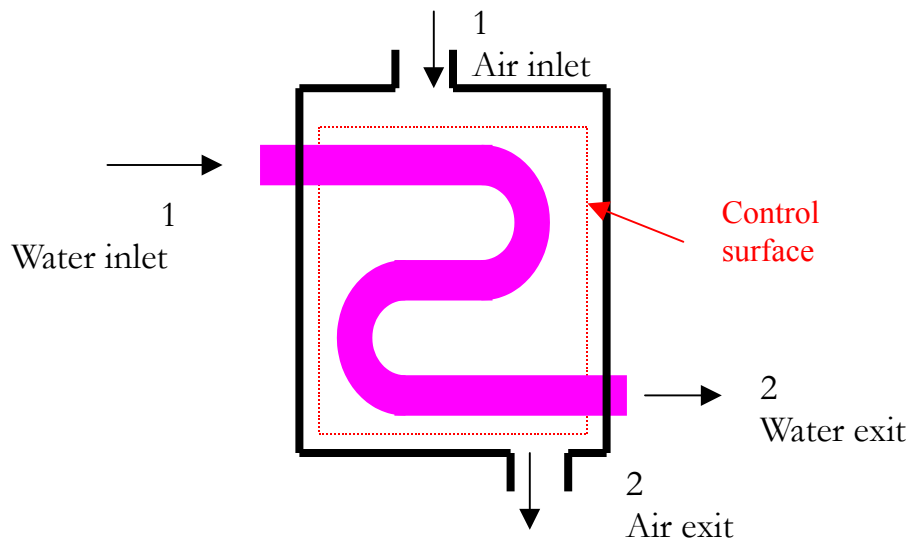
Heat exchangers are normally well-insulated devices that allow energy exchange between hot and cold fluids without mixing the fluids. The pumps, fans, and blowers causing the fluids to flow across the control surface are normally located outside the control surface.





Example 4-11

Air is heated in a heat exchanger by hot water. The water enters the heat exchanger at 45°C and experiences a 20°C drop in temperature. As the air passes through the heat exchanger, its temperature is increased by 25°C . Determine the ratio of mass flow rate of the air to mass flow rate of the water.



Control Volume: The heat exchanger

Property Relation: Air: ideal gas relations

Water: steam tables or incompressible liquid results

Process: Assume adiabatic, steady-flow

Conservation Principles:

Conservation of mass:

$$\dot{m}_{in} - \dot{m}_{out} = \Delta \dot{m}_{system}^{0(\text{steady})} \quad (kg / s)$$

For two entrances, two exits, the conservation of mass becomes

$$\dot{m}_{in} = \dot{m}_{out}$$
$$\dot{m}_{air,1} + \dot{m}_{w,1} = \dot{m}_{air,2} + \dot{m}_{w,2}$$

For two fluid streams that exchange energy but do not mix, it is better to conserve the mass for the fluid streams separately.

$$\dot{m}_{air,1} = \dot{m}_{air,2} = \dot{m}_{air}$$
$$\dot{m}_{w,1} = \dot{m}_{w,2} = \dot{m}_w$$

Conservation of energy:

According to the sketched control volume, mass crosses the control surface, but no work or heat transfer crosses the control surface. Neglecting the kinetic and potential energies, we have for steady-flow

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{system}^{0(\text{steady})}}_{\text{Rate change in internal, kinetic, potential, etc., energies}} \quad (kW)$$

$$\begin{aligned}
\dot{E}_{in} &= \dot{E}_{out} \\
\dot{m}_{air,1}h_{air,1} + \dot{m}_{w,1}h_{w,1} &= \dot{m}_{air,2}h_{air,2} + \dot{m}_{w,2}h_{w,2} \\
\dot{m}_{air}(h_{air,1} - h_{air,2}) &= \dot{m}_w(h_{w,2} - h_{w,1}) \\
\frac{\dot{m}_{air}}{\dot{m}_w} &= \frac{(h_{w,2} - h_{w,1})}{(h_{air,1} - h_{air,2})}
\end{aligned}$$

We assume that the air has constant specific heats at 300 K, Table A-2(a) (we don't know the actual temperatures, just the temperature difference). Because we know the initial and final temperatures for the water, we can use either the incompressible fluid result or the steam tables for its properties.

Using the incompressible fluid approach for the water, Table A-3, $C_{p,w} = 4.18 \text{ kJ/kg}\cdot\text{K}$.

$$\begin{aligned}
\frac{\dot{m}_{air}}{\dot{m}_w} &= \frac{C_{p,w}(T_{w,2} - T_{w,1})}{C_{p,air}(T_{air,1} - T_{air,2})} \\
&= \frac{4.18 \frac{\text{kJ}}{\text{kg}_w \cdot \text{K}} (20 \text{ K})}{1.005 \frac{\text{kJ}}{\text{kg}_{air} \cdot \text{K}} (25 \text{ K})} \\
&= 3.33 \frac{\text{kg}_{air} / \text{s}}{\text{kg}_w / \text{s}}
\end{aligned}$$

A second solution to this problem is obtained by determining the heat transfer rate from the hot water and noting that this is the heat transfer rate to the air. Considering each fluid separately for steady-flow, one entrance, and one exit, and neglecting the kinetic and potential energies, the first law, or conservation of energy, equations become

$$\dot{E}_{in} = \dot{E}_{out}$$

$$air : \quad \dot{m}_{air,1} h_{air,1} + \dot{Q}_{in,air} = \dot{m}_{air,2} h_{air,2}$$

$$water : \quad \dot{m}_{w,1} h_{w,1} = \dot{Q}_{out,w} + \dot{m}_{w,2} h_{w,2}$$

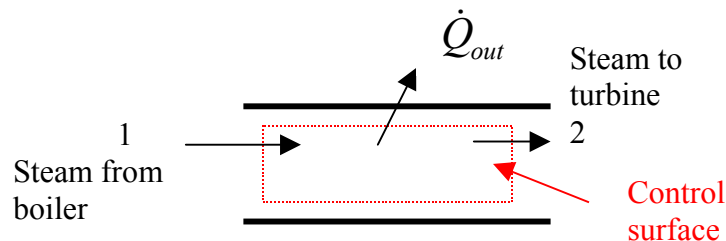
$$\dot{Q}_{in,air} = \dot{Q}_{out,w}$$

Pipe and duct flow

The flow of fluids through pipes and ducts is often a steady-state, steady-flow process. We normally neglect the kinetic and potential energies; however, depending on the flow situation, the work and heat transfer may or may not be zero.

Example 4-12

In a simple steam power plant, steam leaves a boiler at 3 MPa, 600°C, and enters a turbine at 2 MPa, 500°C. Determine the in-line heat transfer from the steam per kilogram mass flowing in the pipe between the boiler and the turbine.



Control Volume: Pipe section in which the heat loss occurs.

Property Relation: Steam tables

Process: Steady-flow

Conservation Principles:

Conservation of mass:

$$\dot{m}_{in} - \dot{m}_{out} = \Delta \dot{m}_{system}^{0(\text{steady})} \quad (kg / s)$$

For one entrance, one exit, the conservation of mass becomes

$$\begin{aligned}\dot{m}_{in} &= \dot{m}_{out} \\ \dot{m}_1 &= \dot{m}_2 = \dot{m}\end{aligned}$$

Conservation of energy:

According to the sketched control volume, heat transfer and mass cross the control surface, but no work crosses the control surface. Neglecting the kinetic and potential energies, we have for steady-flow

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{system}^{0(\text{steady})}}_{\text{Rate change in internal, kinetic, potential, etc., energies}} \quad (kW)$$

We determine the heat transfer rate per unit mass of flowing steam as

$$\dot{m}_1 h_1 = \dot{m}_2 h_2 + \dot{Q}_{out}$$

$$\dot{Q}_{out} = \dot{m}(h_1 - h_2)$$

$$q_{out} = \frac{\dot{Q}_{out}}{\dot{m}} = h_1 - h_2$$

We use the steam tables to determine the enthalpies at the two states as

$$\left. \begin{array}{l} T_1 = 600^\circ C \\ P_1 = 3 MPa \end{array} \right\} h_1 = 3682.3 \frac{kJ}{kg}$$

$$\left. \begin{array}{l} T_2 = 500^\circ C \\ P_2 = 2 MPa \end{array} \right\} h_2 = 3467.6 \frac{kJ}{kg}$$

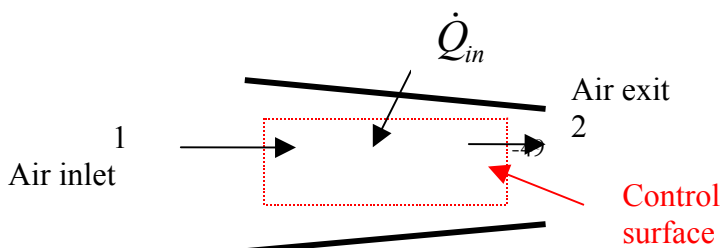
$$q_{out} = h_1 - h_2$$

$$= (3682.3 - 3467.6) \frac{kJ}{kg}$$

$$= 214.7 \frac{kJ}{kg}$$

Example 4-13

Air at 100°C, 0.15 MPa, 40 m/s, flows through a converging duct with a mass flow rate of 0.2 kg/s. The air leaves the duct at 0.1 MPa, 113.6 m/s. The exit-to-inlet duct area ratio is 0.5. Find the required rate of heat transfer to the air when no work is done by the air.



Control Volume: The converging duct

Property Relation: Assume air is an ideal gas and use ideal gas relations

Process: Steady-flow

Conservation Principles:

Conservation of mass:

$$\dot{m}_{in} - \dot{m}_{out} = \Delta \dot{m}_{system} \quad \overset{0(\text{steady})}{\nearrow} \quad (kg / s)$$

For one entrance, one exit, the conservation of mass becomes

$$\dot{m}_{in} = \dot{m}_{out}$$

$$\dot{m}_1 = \dot{m}_2 = \dot{m}$$

Conservation of energy:

According to the sketched control volume, heat transfer and mass cross the control surface, but no work crosses the control surface. Here keep the kinetic energy and still neglect the potential energies, we have for steady-state, steady-flow process

$$\nearrow \quad 0(\text{steady})$$

$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{system}}_{\text{Rate change in internal, kinetic, potential, etc., energies}} \quad (kW)$$

$$\dot{m}_1 \left(h_1 + \frac{\vec{V}_1^2}{2} \right) + \dot{Q}_{in} = \dot{m}_2 \left(h_2 + \frac{\vec{V}_2^2}{2} \right)$$

$$\dot{Q}_{in} = \dot{m} \left((h_2 - h_1) + \frac{\vec{V}_2^2 - \vec{V}_1^2}{2} \right)$$

In the first law equation, the following are known: P_1 , T_1 (and h_1), \vec{V}_1 , \vec{V}_2 , \dot{m} , and A_2/A_1 . The unknowns are \dot{Q}_{in} , and h_2 (or T_2). We use the first law and the conservation of mass equation to solve for the two unknowns.

$$\dot{m}_1 = \dot{m}_2 \quad (kg / s)$$

$$\frac{1}{v_1} \vec{V}_1 A_1 = \frac{1}{v_2} \vec{V}_2 A_2$$

$$\vec{V}_1 A_1 \frac{P_1}{RT_1} = \vec{V}_2 A_2 \frac{P_2}{RT_2}$$

Solving for T_2

$$\begin{aligned}
 T_2 &= T_1 \frac{P_2}{P_1} \frac{A_2}{A_1} \frac{\vec{V}_2}{\vec{V}_1} \\
 &= (100 + 273) K \left(\frac{0.1 \text{ MPa}}{0.15 \text{ MPa}} \right) (0.5) \left(\frac{113.6 \text{ m/s}}{40 \text{ m/s}} \right) \\
 &= 353.1 \text{ K}
 \end{aligned}$$

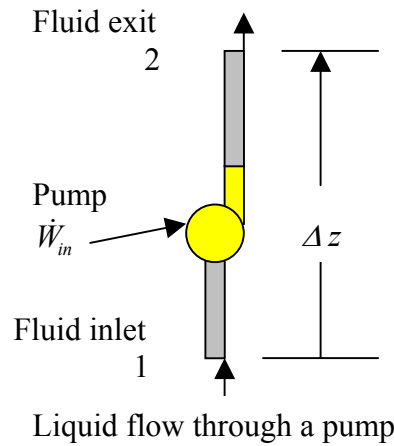
Assuming $C_p = \text{constant}$, $h_2 - h_1 = C_p(T_2 - T_1)$

$$\begin{aligned}
 \dot{Q}_{in} &= \dot{m} \left(C_p (T_2 - T_1) + \frac{\vec{V}_2^2 - \vec{V}_1^2}{2} \right) \\
 &= 0.2 \frac{\text{kg}}{\text{s}} \left(1.005 \frac{\text{kJ}}{\text{kg} \cdot \text{K}} (353.1 - 373) \text{ K} \right. \\
 &\quad \left. + \frac{(113.6^2 - 40^2) \text{ m}^2 / \text{s}^2}{2} \frac{\text{kJ} / \text{kg}}{1000 \text{ m}^2 / \text{s}^2} \right) \\
 &= -2.87 \frac{\text{kJ}}{\text{s}} = -2.87 \text{ kW}
 \end{aligned}$$

Looks like we made the wrong assumption for the direction of the heat transfer. The heat is really leaving the flow duct. (What type of device is this anyway?)

$$\dot{Q}_{out} = -\dot{Q}_{in} = 2.87 \text{ kW}$$

Liquid pumps



The work required when pumping an incompressible liquid in an adiabatic steady-state, steady-flow process is given by

$$\dot{Q} - \dot{W} = \dot{m} \left[h_2 - h_1 + \frac{\vec{V}_2^2 - \vec{V}_1^2}{2} + g(z_2 - z_1) \right] \quad (kW)$$

The enthalpy difference can be written as

$$h_2 - h_1 = (u_2 - u_1) + [(Pv)_2 - (Pv)_1]$$

For incompressible liquids we assume that the density and specific volume are constant. The pumping process for an incompressible liquid is essentially isothermal, and the internal energy change is approximately zero (we will see this more clearly after introducing the second law). Thus, the enthalpy difference reduces to the difference in the pressure-specific volume products. Since $v_2 = v_1 = v$ the work input to the pump becomes

$$-\dot{W} = \dot{m} \left[v(P_2 - P_1) + \frac{\vec{V}_2^2 - \vec{V}_1^2}{2} + g(z_2 - z_1) \right] \quad (kW)$$

\dot{W} is the net work done by the control volume, and it is noted that work is input to the pump; so, $\dot{W} = -\dot{W}_{in, pump}$.

If we neglect the changes in kinetic and potential energies, the pump work becomes

$$-(-\dot{W}_{in, pump}) = \dot{m}[v(P_2 - P_1)] \quad (kW)$$

$$\dot{W}_{in, pump} = \dot{m}[v(P_2 - P_1)]$$

We use this result to calculate the work supplied to boiler feedwater pumps in steam power plants.

Uniform-State, Uniform-Flow Problems

During unsteady energy transfer to or from open systems or control volumes, the system may have a change in the stored energy and mass. Several unsteady thermodynamic problems may be treated as uniform-state, uniform-flow problems. The assumptions for uniform-state, uniform-flow are

- The process takes place over a specified time period.
- The state of the mass within the control volume is uniform at any instant of time but may vary with time.
- The state of mass crossing the control surface is uniform and steady. The mass flow may be different at different control surface locations.

To find the amount of mass crossing the control surface at a given location, we integrate the mass flow rate over the time period.

$$\text{Inlets: } m_i = \int_0^t \dot{m}_i dt \quad \text{Exits: } m_e = \int_0^t \dot{m}_e dt$$

The change in mass of the control volume in the time period is

$$(m_2 - m_1)_{CV} = \int_0^t \left. \frac{dm}{dt} \right|_{CV} dt$$

The uniform-state, uniform-flow conservation of mass becomes

$$\sum m_i - \sum m_e = (m_2 - m_1)_{CV}$$

The change in internal energy for the control volume during the time period is

$$(m_2 u_2 - m_1 u_1)_{CV} = \int_0^t \left. \frac{dU}{dt} \right|_{CV} dt$$

The heat transferred and work done in the time period are

$$Q = \int_0^t \dot{Q} dt \quad \text{and} \quad W = \int_0^t \dot{W} dt$$

The energy crossing the control surface with the mass in the time period is

$$\sum m_j \theta_j = \int_0^t \dot{m}_j \left(h_j + \frac{\vec{V}_j^2}{2} + g z_j \right) dt$$

where

$$j = \begin{array}{l} \text{i, for inlets} \\ \text{e, for exits} \end{array}$$

The first law for uniform-state, uniform-flow becomes

$$E_{in} - E_{out} = \Delta E_{CV}$$

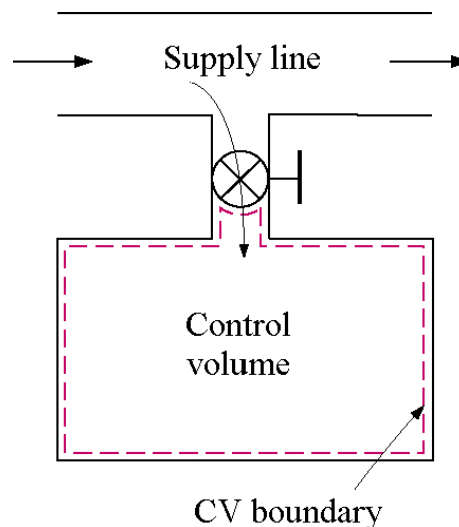
$$Q - W = \sum m_e \left(h_e + \frac{\vec{V}_e^2}{2} + gz_e \right) - \sum m_i \left(h_i + \frac{\vec{V}_i^2}{2} + gz_i \right) + (m_2 e_2 - m_1 e_1)_{CV}$$

When the kinetic and potential energy changes associated with the control volume and the fluid streams are negligible, it simplifies to

$$Q - W = \sum m_e h_e - \sum m_i h_i + (m_2 u_2 - m_1 u_1)_{CV} \quad (kJ)$$

Example 4-14

Consider an evacuated, insulated, rigid tank connected through a closed valve to a high-pressure line. The valve is opened and the tank is filled with the fluid in the line. If the fluid is an ideal gas, determine the final temperature in the tank when the tank pressure equals that of the line.



Control Volume: The tank

Property Relation: Ideal gas relations

Process: Assume uniform-state, uniform-flow

Conservation Principles:

Conservation of mass:

$$\sum m_i - \sum m_e = (m_2 - m_1)_{CV}$$

Or, for one entrance, no exit, and initial mass of zero, this becomes

$$m_i = (m_2)_{CV}$$

Conservation of energy:

For an insulated tank Q is zero and for a rigid tank with no shaft work W is zero. For a one-inlet mass stream and no-exit mass stream and neglecting changes in kinetic and potential energies, the uniform-state, uniform-flow conservation of energy reduces to

$$\begin{aligned} Q - W &= \sum m_e h_e - \sum m_i h_i + (m_2 u_2 - m_1 u_1)_{CV} \quad (kJ) \\ 0 &= -m_i h_i + (m_2 u_2)_{CV} \end{aligned}$$

or

$$\begin{aligned} m_i h_i &= (m_2 u_2)_{CV} \\ h_i &= u_2 \\ u_i + P_i v_i &= u_2 \\ u_2 - u_i &= P_i v_i \\ C_v (T_2 - T_i) &= P_i v_i \end{aligned}$$

$$\begin{aligned}
 C_v(T_2 - T_i) &= RT_i \\
 T_2 &= \frac{C_v + R}{C_v} T_i = \frac{C_p}{C_v} T_i \\
 &= kT_i
 \end{aligned}$$

If the fluid is air, $k = 1.4$ and the absolute temperature in the tank at the final state is 40 percent higher than the fluid absolute temperature in the supply line. The internal energy in the full tank differs from the internal energy of the supply line by the amount of flow work done to push the fluid from the line into the tank.

Extra Assignment

Rework the above problem for a 10 m^3 tank initially open to the atmosphere at 25°C and being filled from an air supply line at 90 psig, 25°C , until the pressure inside the tank is 70 psig.