More examples on limits and Continuity

Example Let

$$f(x) = \begin{cases} 1/(x+2) & ; \ x < -2 \\ x^2 - 5 & ; \ -2 < x \leq 3 \\ \sqrt{x+13} & ; \ x > 3 \end{cases}$$

Find

(a) $\lim\limits_{x \to -2} f(x)$ (b) $\lim\limits_{x \to 0} f(x)$ (c) $\lim\limits_{x \to 3} f(x)$

Sol^n (a) $\lim\limits_{x \to -2^-} f(x) = \lim\limits_{x \to -2^-} \dfrac{1}{x+2} = -\infty \quad (x < -2)$

$\lim\limits_{x \to -2^+} f(x) = \lim\limits_{x \to -2^+} (x^2 - 5) = -1$

$\therefore \lim\limits_{x \to -2} f(x)$ does <u>not</u> exist #

(b) $\lim\limits_{x \to 0} f(x) = \lim\limits_{x \to 0} x^2 - 5 = -5$ #

(c) $\lim\limits_{x \to 3^-} f(x) = \lim\limits_{x \to 5^-} x^2 - 5 = 4$

$\lim\limits_{x \to 3^+} f(x) = \lim\limits_{x \to 3^+} \sqrt{x+13} = 4$

$\therefore \lim\limits_{x \to 3} f(x) = 4$ #

Example Find $\lim_{x \to \infty} \sqrt[3]{\dfrac{3x+5}{6x-8}}$

Sol^n
$$\lim_{x \to \infty} \sqrt[3]{\frac{3x+5}{6x-8}} = \sqrt[3]{\lim_{x \to \infty} \frac{3x+5}{6x-8}} = \sqrt[3]{\frac{1}{2}} \quad \text{Ans}$$

Example Find (a) $\lim_{x \to \infty} \dfrac{\sqrt{x^2+2}}{3x-6}$ (b) $\lim_{x \to -\infty} \dfrac{\sqrt{x^2+2}}{3x-6}$

Sol^n (a)
$$\lim_{x \to \infty} \frac{\sqrt{x^2+2}}{3x-6} = \lim_{x \to \infty} \frac{\sqrt{x^2+2}\,/\,|x|}{(3x-6)/|x|} = \lim_{x \to \infty} \frac{\sqrt{x^2+2}\,/\sqrt{x^2}}{(3x-6)/|x|}$$
$$= \lim_{x \to \infty} \frac{\sqrt{1+2/x^2}}{3-6/x} = \frac{\lim_{x \to \infty} \sqrt{1+2x^2}}{\lim_{x \to \infty} 3-6/x}$$
$$= \frac{1}{3} \quad \#$$

(b)
$$\lim_{x \to -\infty} \frac{\sqrt{x^2+2}}{3x-6} = \lim_{x \to -\infty} \frac{\sqrt{x^2+2}\,/\sqrt{x^2}}{(3x-6)/|x|} \quad ; \ |x| = -x \text{ if } x < 0$$
$$= \lim_{x \to -\infty} \frac{\sqrt{1+2/x^2}}{(3x-6)/(-x)} = \lim_{x \to -\infty} \frac{\sqrt{1+2/x^2}}{\frac{6}{x}-3} = -\frac{1}{3}$$

$\#$

<u>Example</u> Find

③

(a) $\lim\limits_{x\to\infty} \left(\sqrt{x^6+5} - x^3\right)$ (b) $\lim\limits_{x\to\infty}\left(\sqrt{x^6+5x^3} - x^3\right)$

<u>Soln</u>

(a) limit อยู่ในรูป $\infty - \infty$ จะติด

$$\lim_{x\to\infty} \sqrt{x^6+5} - x^3 = \lim_{x\to\infty} \sqrt{x^6+5} - x^3 \left(\frac{\sqrt{x^6+5}+x^3}{\sqrt{x^6+5}+x^3}\right)$$

$$= \lim_{x\to\infty} \frac{(x^6+5)-x^6}{\sqrt{x^6+5}+x^3} = \lim_{x\to\infty} \frac{5/x^3}{\sqrt{1+5/x^3}+1}$$

$$= \frac{0}{2} = 0 \quad \#$$

(b) $$\lim_{x\to\infty} \frac{\sqrt{x^6+5x^3} - x^3\left(\sqrt{x^6+5x^3}+x^3\right)}{\sqrt{x^6+5x^3}+x^3}$$

$$= \lim_{x\to\infty} \frac{5x^3}{\sqrt{x^6+5x^3}+x^3} = \lim_{x\to\infty} \frac{5}{\sqrt{1+5/x^3}+1}$$

$$= \frac{5}{\sqrt{1+0}+1} = \frac{5}{2}$$ <u>Ans</u>