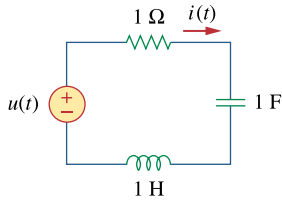


### Chapter 16, Problem 1.

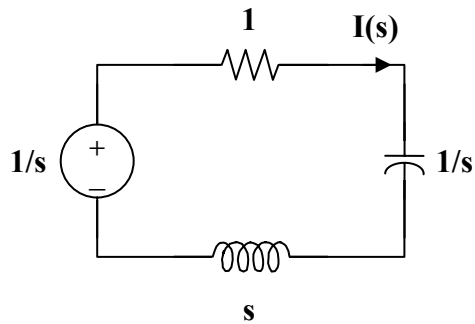
Determine  $i(t)$  in the circuit of Fig. 16.35 by means of the Laplace transform.



**Figure 16.35**  
For Prob. 16.1.

### Chapter 16, Solution 1.

Consider the s-domain form of the circuit which is shown below.



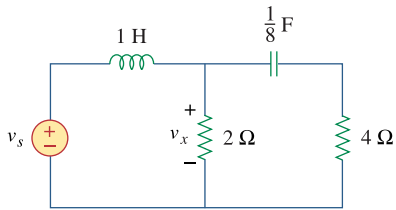
$$I(s) = \frac{1/s}{1 + s + 1/s} = \frac{1}{s^2 + s + 1} = \frac{1}{(s + 1/2)^2 + (\sqrt{3}/2)^2}$$

$$i(t) = \frac{2}{\sqrt{3}} e^{-t/2} \sin\left(\frac{\sqrt{3}}{2} t\right)$$

$$i(t) = \underline{\underline{1.155 e^{-0.5t} \sin(0.866t) \text{ A}}}$$

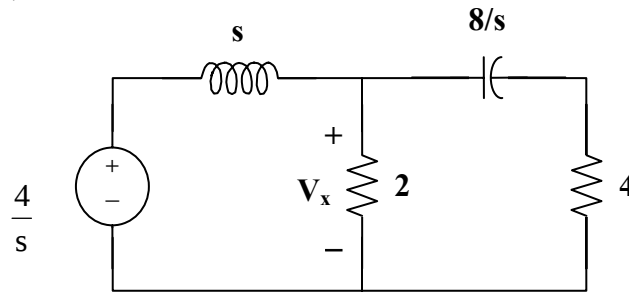
### Chapter 16, Problem 2.

Find  $v_x$  in the circuit shown in Fig. 16.36 given  $v_s = 4u(t)$  V.



**Figure 16.36**  
For Prob. 16.2.

**Chapter 16, Solution 2.**



$$\frac{V_x - \frac{4}{s}}{s} + \frac{V_x - 0}{2} + \frac{V_x - 0}{4 + \frac{8}{s}} = 0$$

$$V_x(4s + 8) - \frac{(16s + 32)}{s} + (2s^2 + 4s)V_x + s^2V_x = 0$$

$$V_x(3s^2 + 8s + 8) = \frac{16s + 32}{s}$$

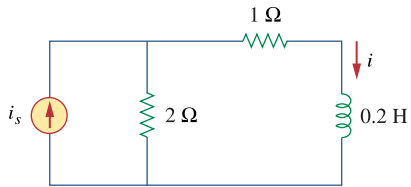
$$V_x = -16 \frac{s + 2}{s(3s^2 + 8s + 8)} = -16 \left( \frac{0.25}{s} + \frac{-0.125}{s + \frac{4}{3} + j\frac{\sqrt{8}}{3}} + \frac{-0.125}{s + \frac{4}{3} - j\frac{\sqrt{8}}{3}} \right)$$

$$v_x = \underline{(-4 + 2e^{-(1.3333 + j0.9428)t} + 2e^{-(1.3333 - j0.9428)t})u(t) \text{ V}}$$

$$\underline{v_x = 4u(t) - e^{-4t/3} \cos\left(\frac{2\sqrt{2}}{3}t\right) - \frac{6}{\sqrt{2}}e^{-4t/3} \sin\left(\frac{2\sqrt{2}}{3}t\right) \text{ V}}$$

### Chapter 16, Problem 3.

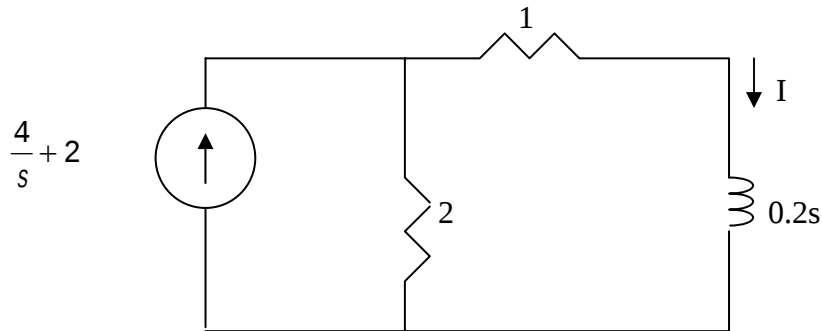
Find  $i(t)$  for  $t > 0$  for the circuit in Fig. 16.37. Assume  $i_s = 4u(t) + 2\delta(t)$  mA. (Hint: Can we use superposition to help solve this problem?)



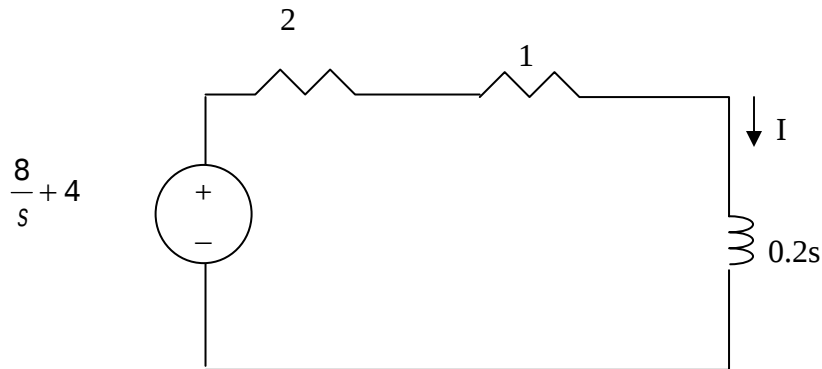
**Figure 16.37**  
For Prob. 16.3.

### Chapter 16, Solution 3.

In the s-domain, the circuit becomes that shown below.



We transform the current source to a voltage source and obtain the circuit shown below.



$$I = \frac{\frac{8}{s} + 4}{3 + 0.2s} = \frac{20s + 40}{s(s + 15)} = \frac{A}{s} + \frac{B}{s + 15}$$

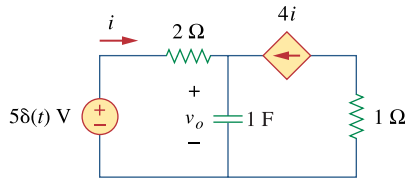
$$A = \frac{40}{15} = \frac{8}{3}, \quad B = \frac{-15 \times 20 + 40}{-15} = \frac{52}{3}$$

$$I = \frac{8/3}{s} + \frac{52/3}{s + 15}$$

$$i(t) = \left[ \frac{8}{3} + \frac{52}{3} e^{-15t} \right] u(t)$$

### Chapter 16, Problem 4.

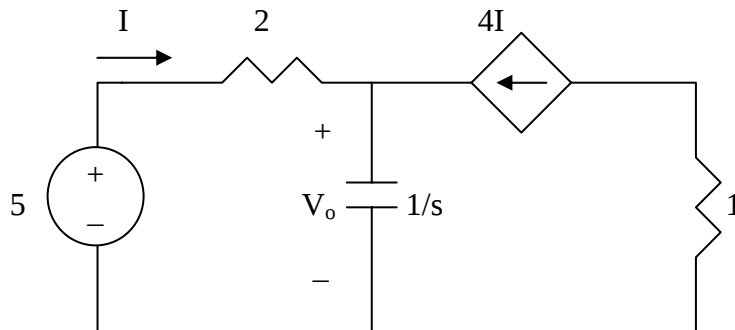
The capacitor in the circuit of Fig. 16.38 is initially uncharged. Find  $v_o(t)$  for  $t > 0$ .



**Figure 16.38**  
For Prob. 16.4.

### Chapter 16, Solution 4.

The circuit in the s-domain is shown below.



$$I + 4I = \frac{V_o}{1/s} \longrightarrow 5I = sV_o$$

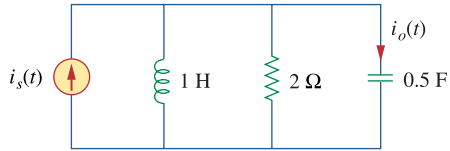
$$\text{But } I = \frac{5 - V_o}{2}$$

$$5\left(\frac{5 - V_o}{2}\right) = sV_o \longrightarrow V_o = \frac{12.5}{s + 5/2}$$

$$v_o(t) = \underline{12.5e^{-2.5t} \text{ V}}$$

### Chapter 16, Problem 5.

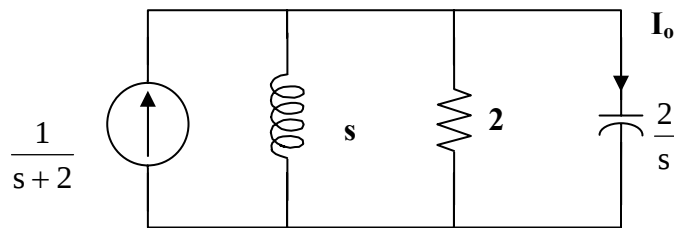
If  $i_s(t) = e^{-2t} u(t)$  A in the circuit shown in Fig. 16.39, find the value of  $i_o(t)$ .



**Figure 16.39**

For Prob. 16.5.

### Chapter 16, Solution 5.



$$V = \frac{1}{s+2} \left( \frac{1}{\frac{1}{s} + \frac{1}{2} + \frac{s}{2}} \right) = \frac{1}{s+2} \left( \frac{2s}{s^2 + s + 2} \right) = \frac{2s}{(s+2)(s+0.5+j1.3229)(s+0.5-j1.3229)}$$

$$I_o = \frac{Vs}{2} = \frac{s^2}{(s+2)(s+0.5+j1.3229)(s+0.5-j1.3229)}$$

$$= \frac{1}{s+2} + \frac{(-0.5-j1.3229)^2}{(1.5-j1.3229)(-j2.646)} + \frac{(-0.5+j1.3229)^2}{(1.5+j1.3229)(+j2.646)}$$

$$i_o(t) = \left( e^{-2t} + 0.3779e^{-90^\circ} e^{-t/2} e^{-j1.3229t} + 0.3779e^{90^\circ} e^{-t/2} e^{j1.3229t} \right) u(t) \text{ A}$$

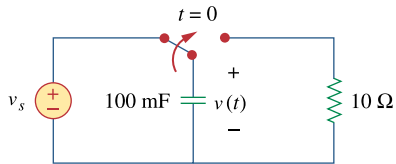
or

$$= \left( e^{-2t} - 0.7559 \sin 1.3229t \right) u(t) \text{ A}$$

$$\text{or } i_o(t) = \left( e^{-2t} - \frac{2}{\sqrt{7}} \sin \left( \frac{\sqrt{7}}{2} t \right) \right) u(t) \text{ A}$$

### Chapter 16, Problem 6.

Find  $v(t)$ ,  $t > 0$  in the circuit of Fig. 16.40. Let  $v_s = 20$  V.



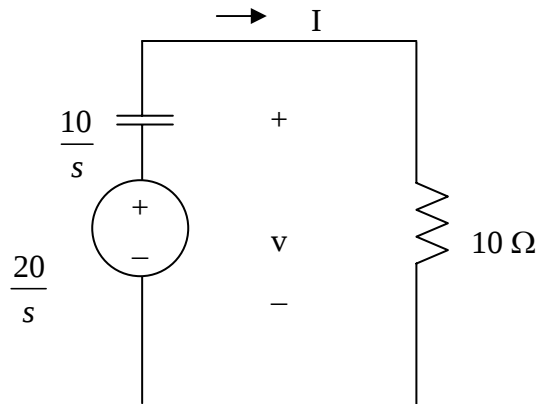
**Figure 16.40**

For Prob. 16.6.

### Chapter 16, Solution 6.

For  $t < 0$ ,  $v(0) = v_s = 20$  V

For  $t > 0$ , the circuit in the s-domain is as shown below.



$$100\text{mF} = 0.1\text{F} \quad \longrightarrow \quad \frac{1}{sC} = \frac{10}{s}$$

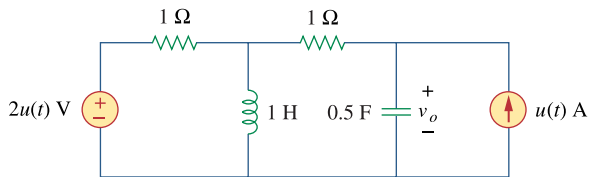
$$I = \frac{20/s}{10 + 10/s} = \frac{2}{s+1}$$

$$V = 10I = \frac{20}{s+1}$$

$$v(t) = \underline{20e^{-t}u(t)}$$

### Chapter 16, Problem 7.

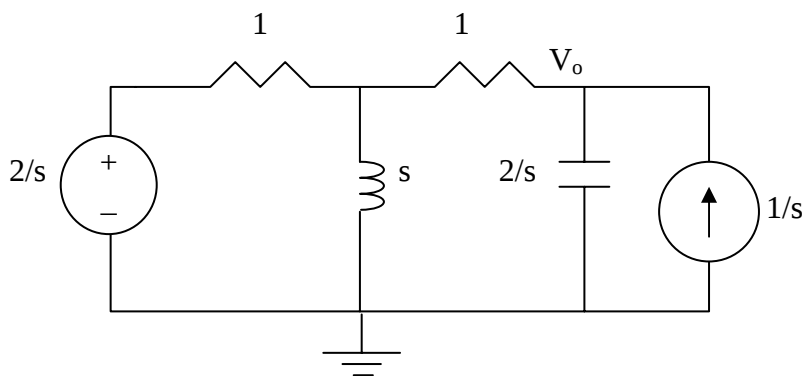
Find  $v_o(t)$ , for all  $t > 0$ , in the circuit of Fig. 16.41.



**Figure 16.41**  
For Prob. 16.7.

### Chapter 16, Solution 7.

The circuit in the s-domain is shown below. Please note,  $i_L(0) = 0$  and  $v_o(0) = 0$  because both sources were equal to zero for all  $t < 0$ .



At node 1

$$\frac{2/s - V_1}{1} = \frac{V_1}{s} + \frac{V_1 - V_o}{1} \longrightarrow \frac{2}{s} = V_1(2 + 1/s) - V_o \quad (1)$$

At node O,

$$\frac{V_1 - V_o}{1} + \frac{1}{s} = \frac{V_o}{2/s} = \frac{s}{2}V_o \longrightarrow V_1 = (1 + s/2)V_o - 1/s \quad (2)$$

Substituting (2) into (1) gives

$$2/s = (2 + 1/s)(1 + s/2)V_o - \frac{1}{s}(2 + \frac{1}{s}) - V_o$$

$$V_o = \frac{(4s+1)}{s(s^2+1.5s+1)} = \frac{A}{s} + \frac{Bs+C}{s^2+1.5s+1}$$

$$4s+1 = A(s^2+1.5s+1) + Bs^2 + Cs$$

We equate coefficients.

$$s^2: \quad 0 = A + B \text{ or } B = -A$$

$$s: \quad 4 = 1.5A + C$$

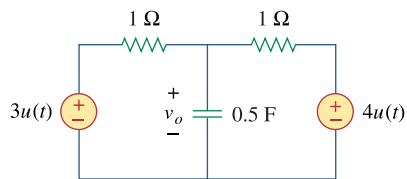
$$\text{constant:} \quad 1 = A, \quad B = -1, \quad C = 4 - 1.5A = 2.5$$

$$V_o = \frac{1}{s} + \frac{-s+2.5}{s^2+1.5s+1} = \frac{1}{s} - \frac{s+3/4}{(s+3/4)^2 + \left(\frac{\sqrt{7}}{4}\right)^2} + \frac{\frac{3.25}{\sqrt{7}} \times \frac{\sqrt{7}}{4}}{(s+3/4)^2 + \left(\frac{\sqrt{7}}{4}\right)^2}$$

$$v(t) = u(t) - e^{-3t/4} \cos \frac{\sqrt{7}}{4} t + 4.9135 e^{-3t/4} \sin \frac{\sqrt{7}}{4} t$$

### Chapter 16, Problem 8.

If  $v_o(0) = -1\text{V}$ , obtain  $v_o(t)$  in the circuit of Fig. 16.42.

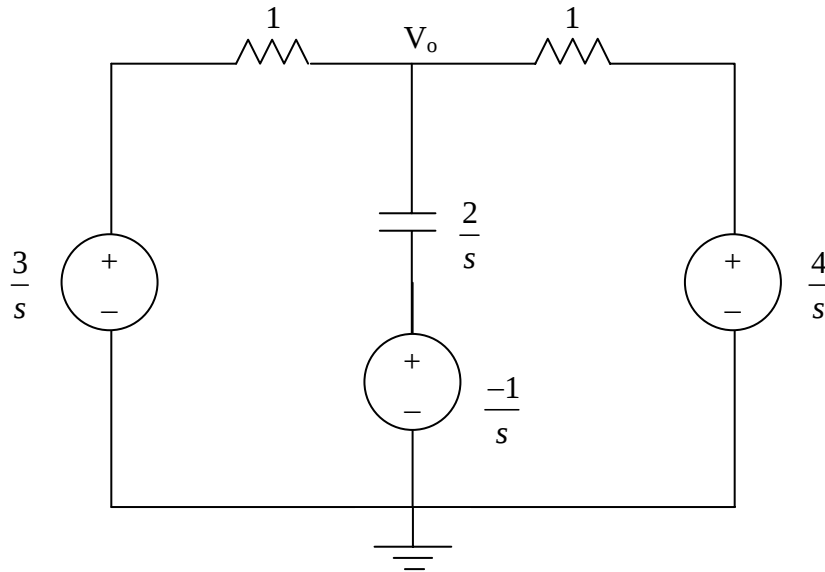


**Figure 16.42**  
For Prob. 16.8.

**Chapter 16, Solution 8.**

$$\frac{1}{2}F \longrightarrow \frac{1}{sC} = \frac{2}{s}$$

We analyze the circuit in the s-domain as shown below. We apply nodal analysis.



$$\frac{\frac{3}{s} - V_o}{1} + \frac{-\frac{1}{s} - V_o}{\frac{2}{s}} + \frac{\frac{4}{s} - V_o}{1} = 0 \longrightarrow V_o = \frac{14 - s}{s(s + 4)}$$

$$V_o = \frac{A}{s} + \frac{B}{s + 4}$$

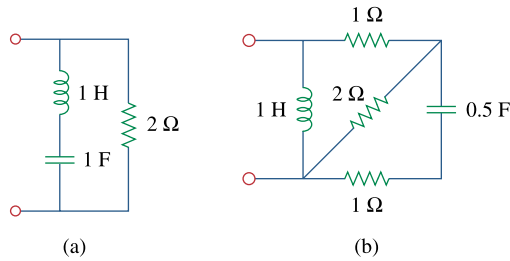
$$A = \frac{14}{4} = 7/2, \quad B = \frac{18}{-4} = -9/2$$

$$V_o = \frac{7/2}{s} - \frac{9/2}{s + 4}$$

$$v_o(t) = \left( \frac{7}{2} - \frac{9}{2}e^{-4t} \right) u(t)$$

### Chapter 16, Problem 9.

Find the input impedance  $Z_{in}(s)$  of each of the circuits in Fig. 16.43.



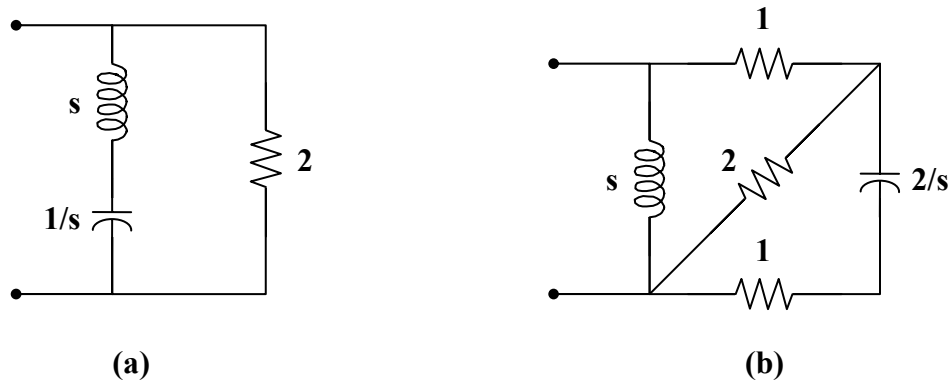
**Figure 16.43**

For Prob. 16.9.

### Chapter 16, Solution 9.

- (a) The  $s$ -domain form of the circuit is shown in Fig. (a).

$$Z_{in} = 2 \parallel (s + 1/s) = \frac{2(s + 1/s)}{2 + s + 1/s} = \underline{\underline{\frac{2(s^2 + 1)}{s^2 + 2s + 1}}}$$



- (b) The  $s$ -domain equivalent circuit is shown in Fig. (b).

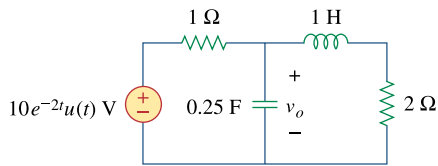
$$2 \parallel (1 + 2/s) = \frac{2(1 + 2/s)}{3 + 2/s} = \frac{2(s + 2)}{3s + 2}$$

$$1 + 2 \parallel (1 + 2/s) = \frac{5s + 6}{3s + 2}$$

$$Z_{in} = s \parallel \left( \frac{5s + 6}{3s + 2} \right) = \frac{s \cdot \left( \frac{5s + 6}{3s + 2} \right)}{s + \left( \frac{5s + 6}{3s + 2} \right)} = \underline{\underline{\frac{s(5s + 6)}{3s^2 + 7s + 6}}}$$

### Chapter 16, Problem 10.

Use Thevenin's theorem to determine  $v_o(t)$ ,  $t > 0$  in the circuit of Fig. 16.44.

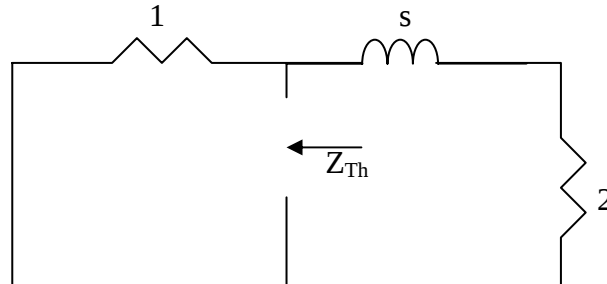


**Figure 16.44**  
For Prob. 16.10.

### Chapter 16, Solution 10.

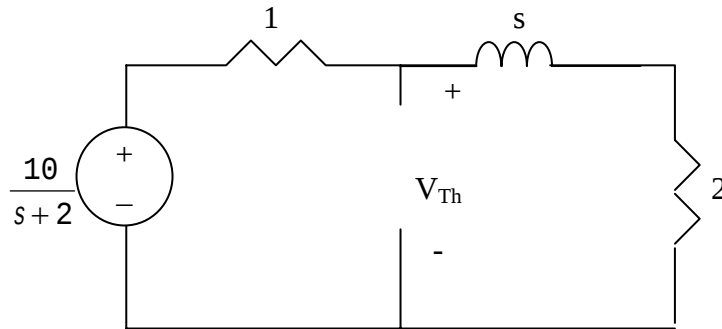
$1H \longrightarrow 1s$  and  $i_L(0) = 0$  (the source is zero for all  $t < 0$ ).  
 $\frac{1}{4}F \longrightarrow \frac{1}{sC} = \frac{4}{s}$  and  $v_C(0) = 0$  (again, there are no source contributions for all  $t < 0$ ).

To find  $Z_{Th}$ , consider the circuit below.



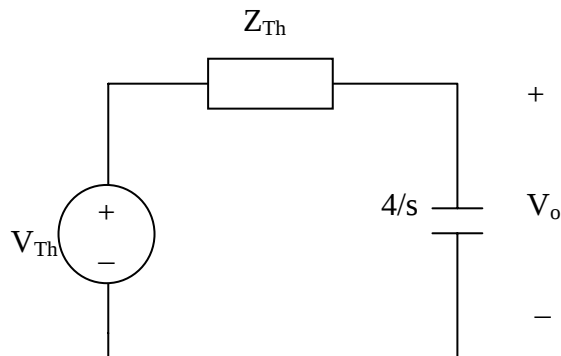
$$Z_{Th} = 1 // (s + 2) = \frac{s + 2}{s + 3}$$

To find  $V_{Th}$ , consider the circuit below.



$$V_{Th} = \frac{s+2}{s+3} \cdot \frac{10}{s+2} = \frac{10}{s+3}$$

The Thevenin equivalent circuit is shown below

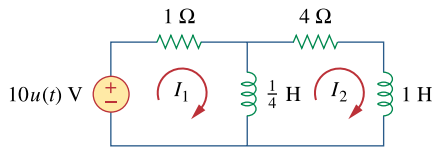


$$V_o = \frac{\frac{4}{s}}{\frac{4}{s} + Z_{Th}} V_{Th} = \frac{\frac{4}{s}}{\frac{4}{s} + \frac{s+2}{s+3}} \cdot \frac{10}{s+3} = \frac{40}{s^2 + 6s + 12} = \frac{\frac{40}{\sqrt{3}} \sqrt{3}}{(s+3)^2 + (\sqrt{3})^2}$$

$$v_o(t) = \underline{23.094e^{-3t} \sin \sqrt{3}t}$$

### Chapter 16, Problem 11.

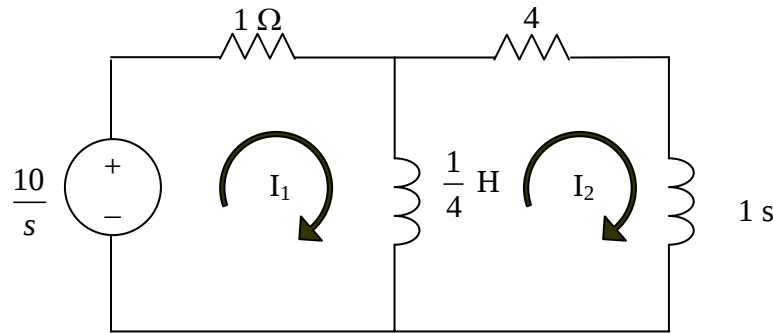
Solve for the mesh currents in the circuit of Fig. 16.45. You may leave your results in the s-domain.



**Figure 16.45**  
For Prob. 16.11.

### Chapter 16, Solution 11.

In the s-domain, the circuit is as shown below.



$$\frac{10}{s} = \left(1 + \frac{s}{4}\right)I_1 - \frac{1}{4}sI_2 \quad (1)$$

$$-\frac{1}{4}sI_1 + I_2\left(4 + \frac{5}{4}s\right) = 0 \quad (2)$$

In matrix form,

$$\begin{bmatrix} \frac{10}{s} \\ 0 \end{bmatrix} = \begin{bmatrix} 1 + \frac{s}{4} & -\frac{1}{4}s \\ -\frac{1}{4}s & 4 + \frac{5}{4}s \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\Delta = \frac{1}{4}s^2 + \frac{9}{4}s + 4$$

$$\Delta_1 = \begin{vmatrix} \frac{10}{s} & -\frac{1}{4}s \\ 0 & 4 + \frac{5}{4}s \end{vmatrix} = \frac{40}{s} + \frac{50}{4}$$

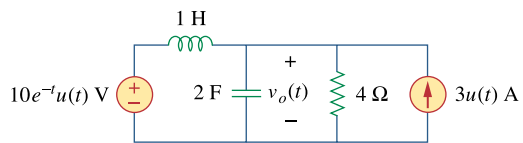
$$\Delta_2 = \begin{vmatrix} 1 + \frac{s}{4} & \frac{10}{s} \\ -\frac{1}{4}s & 0 \end{vmatrix} = \frac{5}{2}$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{\frac{40}{s} + \frac{25}{2}}{0.25s^2 + 2.25s + 4} = \frac{50s + 160}{s(s^2 + 9s + 16)}$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{2.5}{0.25s^2 + 2.25s + 4} = \frac{10}{s^2 + 9s + 16}$$

### Chapter 16, Problem 12.

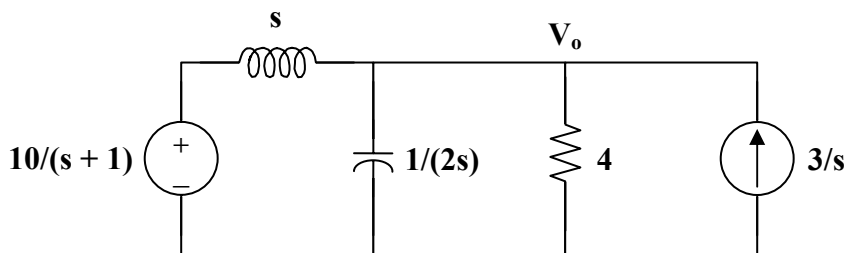
Find  $v_o(t)$  in the circuit of Fig. 16.46.



**Figure 16.46**  
For Prob. 16.12.

### Chapter 16, Solution 12.

We apply nodal analysis to the s-domain form of the circuit below.



$$\frac{10}{s+1} - \frac{V_o}{s} + \frac{3}{s} = \frac{V_o}{4} + 2sV_o$$

$$(1 + 0.25s + s^2)V_o = \frac{10}{s+1} + 15 = \frac{10 + 15s + 15}{s+1}$$

$$V_o = \frac{15s + 25}{(s+1)(s^2 + 0.25s + 1)} = \frac{A}{s+1} + \frac{Bs + C}{s^2 + 0.25s + 1}$$

$$A = (s+1)V_o \Big|_{s=-1} = \frac{40}{7}$$

$$15s + 25 = A(s^2 + 0.25s + 1) + B(s^2 + s) + C(s+1)$$

Equating coefficients :

$$s^2: \quad 0 = A + B \quad \longrightarrow \quad B = -A$$

$$s^1: \quad 15 = 0.25A + B + C = -0.75A + C$$

$$s^0: \quad 25 = A + C$$

$$A = 40/7, \quad B = -40/7, \quad C = 135/7$$

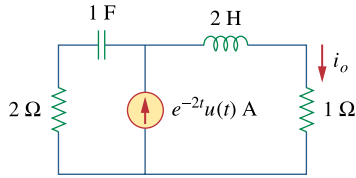
$$V_o = \frac{40}{7} \frac{1}{s+1} + \frac{-40}{7} \frac{s}{\left(s + \frac{1}{2}\right)^2 + \frac{3}{4}} + \frac{135}{7} \frac{1}{\left(s + \frac{1}{2}\right)^2 + \frac{3}{4}} = \frac{40}{7} \frac{1}{s+1} - \frac{40}{7} \frac{s + \frac{1}{2}}{\left(s + \frac{1}{2}\right)^2 + \frac{3}{4}} + \left(\frac{155}{7} \cdot \frac{2}{\sqrt{3}}\right) \frac{\frac{\sqrt{3}}{2}}{\left(s + \frac{1}{2}\right)^2 + \frac{3}{4}}$$

$$v_o(t) = \frac{40}{7} e^{-t} - \frac{40}{7} e^{-t/2} \cos\left(\frac{\sqrt{3}}{2}t\right) + \frac{(155)(2)}{(7)(\sqrt{3})} e^{-t/2} \sin\left(\frac{\sqrt{3}}{2}t\right)$$

$$v_o(t) = \underline{\underline{5.714 e^{-t} - 5.714 e^{-t/2} \cos(0.866t) + 25.57 e^{-t/2} \sin(0.866t) \text{ V}}}$$

### Chapter 16, Problem 13.

Determine  $i_o(t)$  in the circuit of Fig. 16.47.

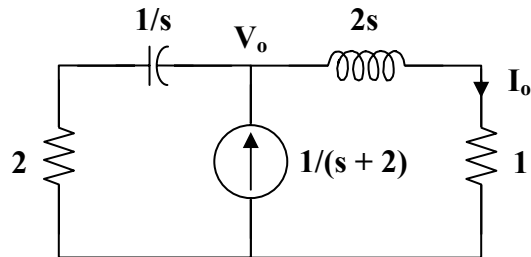


**Figure 16.47**

For Prob. 16.13.

### Chapter 16, Solution 13.

Consider the following circuit.



Applying KCL at node o,

$$\frac{1}{s+2} = \frac{V_o}{2s+1} + \frac{V_o}{2+1/s} = \frac{s+1}{2s+1} V_o$$

$$V_o = \frac{2s+1}{(s+1)(s+2)}$$

$$I_o = \frac{V_o}{2s+1} = \frac{1}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$$

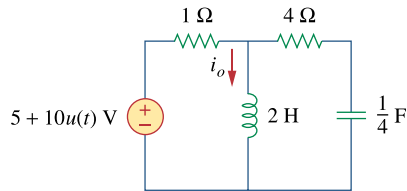
$$A = 1, \quad B = -1$$

$$I_o = \frac{1}{s+1} - \frac{1}{s+2}$$

$$i_o(t) = \underline{\underline{(e^{-t} - e^{-2t})u(t) \text{ A}}}$$

### Chapter 16, Problem 14.

\* Determine  $i_o(t)$  in the network shown in Fig. 16.48.



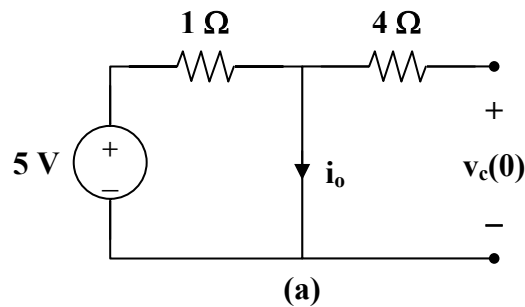
**Figure 16.48**

For Prob. 16.14.

\* An asterisk indicates a challenging problem.

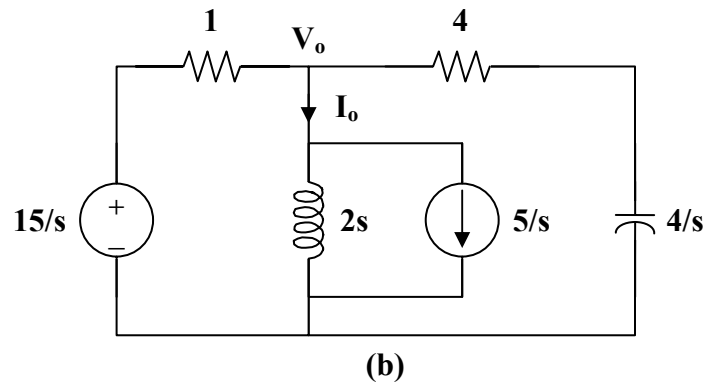
### Chapter 16, Solution 14.

We first find the initial conditions from the circuit in Fig. (a).



$$i_o(0^-) = 5 \text{ A}, \quad v_c(0^-) = 0 \text{ V}$$

We now incorporate these conditions in the s-domain circuit as shown in Fig.(b).



At node o,

$$\frac{V_o - 15/s}{1} + \frac{V_o}{2s} + \frac{5}{s} + \frac{V_o - 0}{4 + 4/s} = 0$$

$$\frac{15}{s} - \frac{5}{s} = \left(1 + \frac{1}{2s} + \frac{s}{4(s+1)}\right) V_o$$

$$\frac{10}{s} = \frac{4s^2 + 4s + 2s + 2 + s^2}{4s(s+1)} V_o = \frac{5s^2 + 6s + 2}{4s(s+1)} V_o$$

$$V_o = \frac{40(s+1)}{5s^2 + 6s + 2}$$

$$I_o = \frac{V_o}{2s} + \frac{5}{s} = \frac{4(s+1)}{s(s^2 + 1.2s + 0.4)} + \frac{5}{s}$$

$$I_o = \frac{5}{s} + \frac{A}{s} + \frac{Bs + C}{s^2 + 1.2s + 0.4}$$

$$4(s+1) = A(s^2 + 1.2s + 0.4) + Bs^2 + Cs$$

Equating coefficients :

$$s^0: \quad 4 = 0.4A \quad \longrightarrow \quad A = 10$$

$$s^1: \quad 4 = 1.2A + C \quad \longrightarrow \quad C = -1.2A + 4 = -8$$

$$s^2: \quad 0 = A + B \quad \longrightarrow \quad B = -A = -10$$

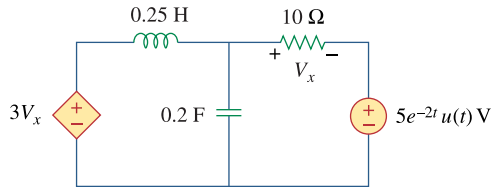
$$I_o = \frac{5}{s} + \frac{10}{s} - \frac{10s + 8}{s^2 + 1.2s + 0.4}$$

$$I_o = \frac{15}{s} - \frac{10(s + 0.6)}{(s + 0.6)^2 + 0.2^2} - \frac{10(0.2)}{(s + 0.6)^2 + 0.2^2}$$

$$i_o(t) = \underline{\underline{[15 - 10e^{-0.6t}(\cos(0.2t) - \sin(0.2t))]} u(t) \text{ A}}$$

### Chapter 16, Problem 15.

Find  $V_x(s)$  in the circuit shown in Fig. 16.49.

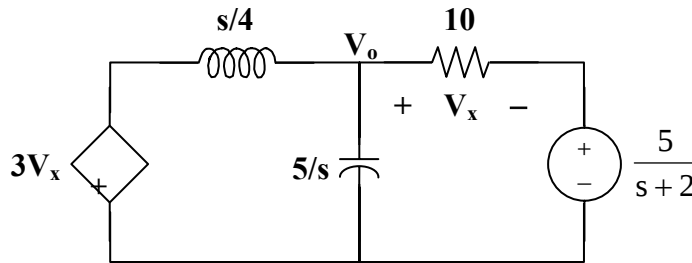


**Figure 16.49**

For Prob. 16.15.

### Chapter 16, Solution 15.

First we need to transform the circuit into the  $s$ -domain.



$$\frac{V_o - 3V_x}{s/4} + \frac{V_o - 0}{5/s} + \frac{V_o - \frac{5}{s+2}}{10} = 0$$

$$40V_o - 120V_x + 2s^2V_o + sV_o - \frac{5s}{s+2} = 0 = (2s^2 + s + 40)V_o - 120V_x - \frac{5s}{s+2}$$

$$\text{But, } V_x = V_o - \frac{5}{s+2} \rightarrow V_o = V_x + \frac{5}{s+2}$$

We can now solve for  $V_x$ .

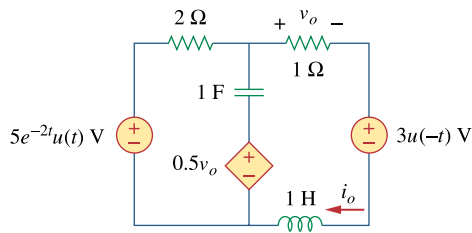
$$(2s^2 + s + 40)\left(V_x + \frac{5}{s+2}\right) - 120V_x - \frac{5s}{s+2} = 0$$

$$2(s^2 + 0.5s - 40)V_x = -10\frac{(s^2 + 20)}{s+2}$$

$$V_x = -5\frac{(s^2 + 20)}{(s+2)(s^2 + 0.5s - 40)}$$

### Chapter 16, Problem 16.

\* Find  $i_o(t)$  for  $t > 0$  in the circuit of Fig. 16.50.



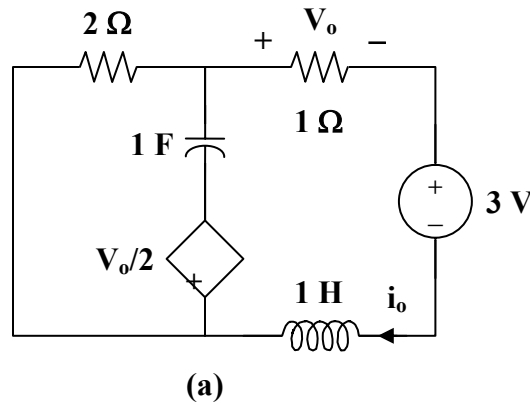
**Figure 16.50**

For Prob. 16.16.

\* An asterisk indicates a challenging problem.

## Chapter 16, Solution 16.

We first need to find the initial conditions. For  $t < 0$ , the circuit is shown in Fig. (a).

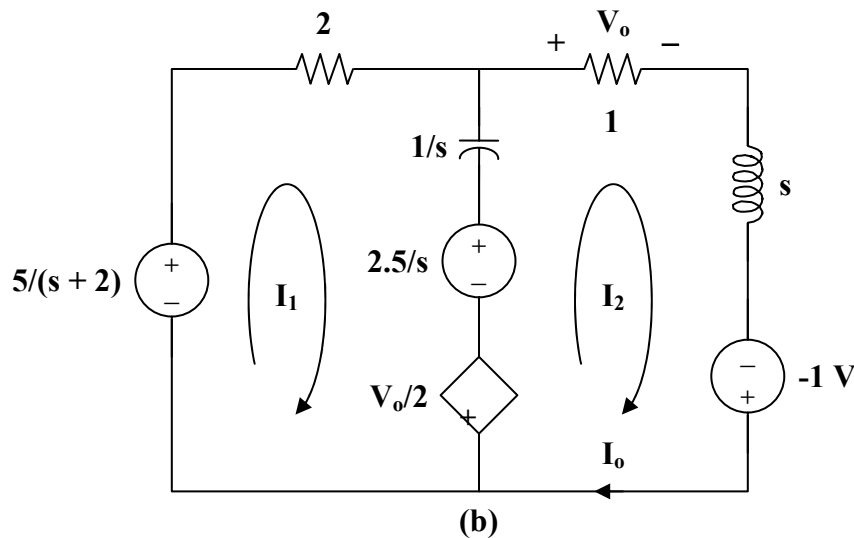


To dc, the capacitor acts like an open circuit and the inductor acts like a short circuit. Hence,

$$i_L(0) = i_o = \frac{-3}{3} = -1 \text{ A}, \quad v_o = -1 \text{ V}$$

$$v_c(0) = -(2)(-1) - \left(\frac{-1}{2}\right) = 2.5 \text{ V}$$

We now incorporate the initial conditions for  $t > 0$  as shown in Fig. (b).



For mesh 1,

$$\frac{-5}{s+2} + \left(2 + \frac{1}{s}\right)I_1 - \frac{1}{s}I_2 + \frac{2.5}{s} + \frac{V_o}{2} = 0$$

But,  $V_o = I_o = I_2$

$$\left(2 + \frac{1}{s}\right)I_1 + \left(\frac{1}{2} - \frac{1}{s}\right)I_2 = \frac{5}{s+2} - \frac{2.5}{s} \quad (1)$$

For mesh 2,

$$\begin{aligned} \left(1 + s + \frac{1}{s}\right)I_2 - \frac{1}{s}I_1 + 1 - \frac{V_o}{2} - \frac{2.5}{s} &= 0 \\ -\frac{1}{s}I_1 + \left(\frac{1}{2} + s + \frac{1}{s}\right)I_2 &= \frac{2.5}{s} - 1 \end{aligned} \quad (2)$$

Put (1) and (2) in matrix form.

$$\begin{bmatrix} 2 + \frac{1}{s} & \frac{1}{2} - \frac{1}{s} \\ -\frac{1}{s} & \frac{1}{2} + s + \frac{1}{s} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \frac{5}{s+2} - \frac{2.5}{s} \\ \frac{2.5}{s} - 1 \end{bmatrix}$$

$$\Delta = 2s + 2 + \frac{3}{s}, \quad \Delta_2 = -2 + \frac{4}{s} + \frac{5}{s(s+2)}$$

$$I_o = I_2 = \frac{\Delta_2}{\Delta} = \frac{-2s^2 + 13}{(s+2)(2s^2 + 2s + 3)} = \frac{A}{s+2} + \frac{Bs + C}{2s^2 + 2s + 3}$$

$$-2s^2 + 13 = A(2s^2 + 2s + 3) + B(s^2 + 2s) + C(s + 2)$$

Equating coefficients :

$$s^2: \quad -2 = 2A + B$$

$$s^1: \quad 0 = 2A + 2B + C$$

$$s^0: \quad 13 = 3A + 2C$$

Solving these equations leads to

$$A = 0.7143, \quad B = -3.429, \quad C = 5.429$$

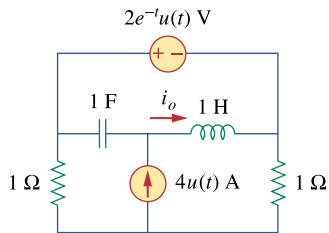
$$I_o = \frac{0.7143}{s+2} - \frac{3.429s - 5.429}{2s^2 + 2s + 3} = \frac{0.7143}{s+2} - \frac{1.7145s - 2.714}{s^2 + s + 1.5}$$

$$I_o = \frac{0.7143}{s+2} - \frac{1.7145(s+0.5)}{(s+0.5)^2 + 1.25} + \frac{(3.194)(\sqrt{1.25})}{(s+0.5)^2 + 1.25}$$

$$i_o(t) = \underline{\underline{\left[0.7143e^{-2t} - 1.7145e^{-0.5t} \cos(1.25t) + 3.194e^{-0.5t} \sin(1.25t)\right]u(t) \text{ A}}}$$

### Chapter 16, Problem 17.

Calculate  $i_o(t)$  for  $t > 0$  in the network of Fig. 16.51.

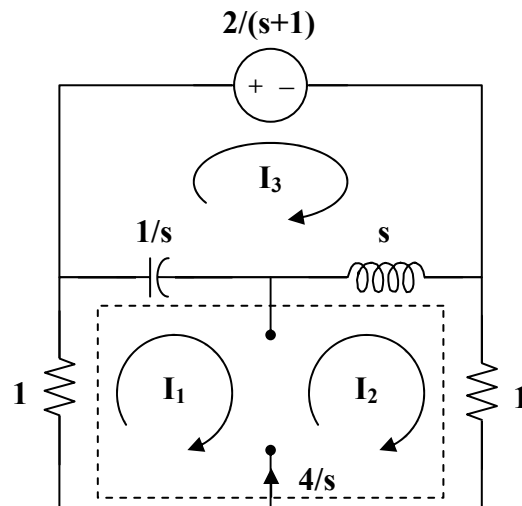


**Figure 16.51**

For Prob. 16.17.

### Chapter 16, Solution 17.

We apply mesh analysis to the s-domain form of the circuit as shown below.



For mesh 3,

$$\frac{2}{s+1} + \left(s + \frac{1}{s}\right)I_3 - \frac{1}{s}I_1 - sI_2 = 0 \quad (1)$$

For the supermesh,

$$\left(1 + \frac{1}{s}\right)I_1 + (1+s)I_2 - \left(\frac{1}{s} + s\right)I_3 = 0 \quad (2)$$

$$\text{Adding (1) and (2) we get, } I_1 + I_2 = -2/(s+1) \quad (3)$$

$$\text{But } -I_1 + I_2 = 4/s \quad (4)$$

$$\text{Adding (3) and (4) we get, } I_2 = (2/s) - 1/(s+1) \quad (5)$$

$$\text{Substituting (5) into (4) yields, } I_1 = -(2/s) - (1/(s+1)) \quad (6)$$

Substituting (5) and (6) into (1) we get,

$$\frac{2}{s^2} + \frac{1}{s(s+1)} - 2 + \frac{s}{s+1} + \left( \frac{s^2 + 1}{s} \right) I_3 = -\frac{2}{s+1}$$

$$I_3 = -\frac{2}{s} + \frac{1.5 - 0.5j}{s+j} + \frac{1.5 + 0.5j}{s-j}$$

Substituting (3) into (1) and (2) leads to

$$-\left(s + \frac{1}{s}\right) I_2 + \left(s + \frac{1}{s}\right) I_3 = \frac{2(-s^2 + 2s + 2)}{s^2(s+1)} \quad (4)$$

$$\left(2 + s + \frac{1}{s}\right) I_2 - \left(s + \frac{1}{s}\right) I_3 = -\frac{4(s+1)}{s^2} \quad (5)$$

We can now solve for  $I_o$ .

$$I_o = I_2 - I_3 = (4/s) - (1/(s+1)) + ((-1.5+0.5j)/(s+j)) + ((-1.5-0.5)/(s-j))$$

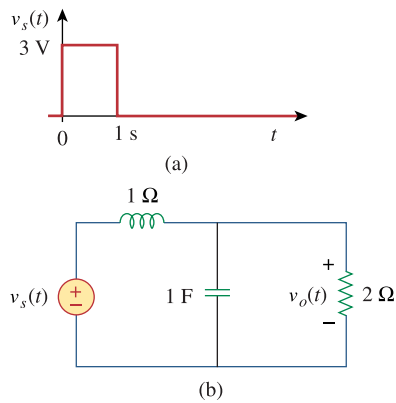
or

$$i_o(t) = \underline{4 - e^{-t} + 1.5811e^{-jt+161.57^\circ} + 1.5811e^{jt-161.57^\circ}} u(t) \text{ A}$$

This is a challenging problem. I did check it with using a Thevenin equivalent circuit and got the same exact answer.

### Chapter 16, Problem 18.

(a) Find the Laplace transform of the voltage shown in Fig. 16.52(a). (b) Using that value of  $v_s(t)$  in the circuit shown in Fig. 16.52(b), find the value of  $v_o(t)$ .

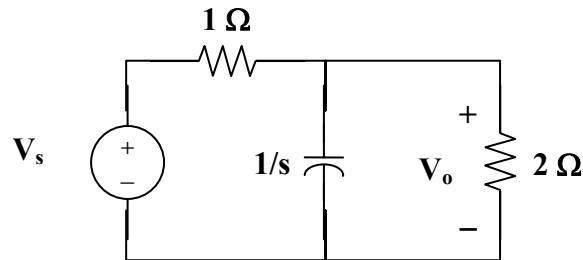


**Figure 16.52**

For Prob. 16.18.

### Chapter 16, Solution 18.

$$v_s(t) = 3u(t) - 3u(t-1) \quad \text{or} \quad V_s = \frac{3}{s} - \frac{e^{-s}}{s} = \frac{3}{s}(1 - e^{-s})$$



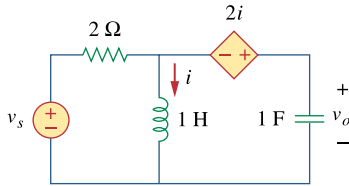
$$\frac{V_o - V_s}{1} + sV_o + \frac{V_o}{2} = 0 \rightarrow (s + 1.5)V_o = V_s$$

$$V_o = \frac{3}{s(s + 1.5)}(1 - e^{-s}) = \left( \frac{2}{s} - \frac{2}{s + 1.5} \right)(1 - e^{-s})$$

$$v_o(t) = \underline{[(2 - 2e^{-1.5t})u(t) - (2 - 2e^{-1.5(t-1)})u(t-1)]V}$$

### Chapter 16, Problem 19.

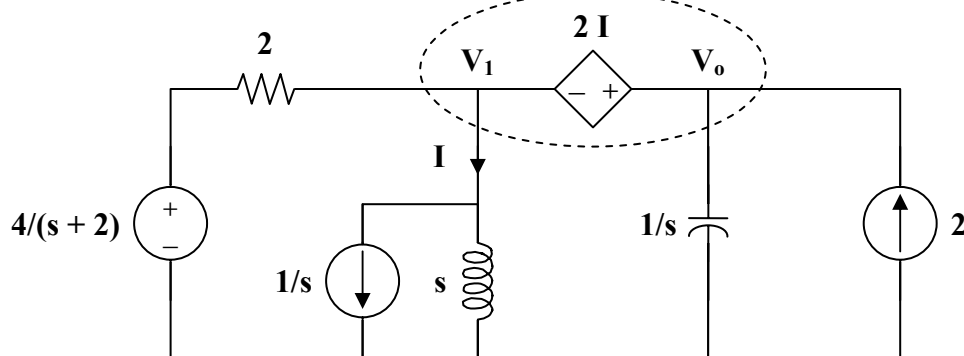
In the circuit of Fig. 16.53, let  $i(0) = 1$  A,  $v_o(0)$  and  $v_s = 4e^{-2t}u(t)$  V. Find  $v_o(t)$  for  $t > 0$ .



**Figure 16.53**  
For Prob. 16.19.

## Chapter 16, Solution 19.

We incorporate the initial conditions in the s-domain circuit as shown below.



At the supernode,

$$\begin{aligned} \frac{(4/(s+2)) - V_1}{2} + 2 &= \frac{V_1}{s} + \frac{1}{s} + sV_o \\ \frac{2}{s+2} + 2 &= \left(\frac{1}{2} + \frac{1}{s}\right)V_1 + \frac{1}{s} + sV_o \end{aligned} \quad (1)$$

But  $V_o = V_1 + 2I$  and  $I = \frac{V_1 + 1}{s}$

$$V_o = V_1 + \frac{2(V_1 + 1)}{s} \longrightarrow V_1 = \frac{V_o - 2/s}{(s+2)/s} = \frac{sV_o - 2}{s+2} \quad (2)$$

Substituting (2) into (1)

$$\begin{aligned} \frac{2}{s+2} + 2 - \frac{1}{s} &= \left(\frac{s+2}{2s}\right) \left[\left(\frac{s}{s+2}\right)V_o - \frac{2}{s+2}\right] + sV_o \\ \frac{2}{s+2} + 2 - \frac{1}{s} + \frac{1}{s} &= \left[\left(\frac{1}{2}\right) + s\right]V_o \\ \frac{2s+4+2}{(s+2)} &= \frac{2s+6}{s+2} = (s+1/2)V_o \end{aligned}$$

$$V_o = \frac{2s+6}{(s+2)(s+1/2)} = \frac{A}{s+1/2} + \frac{B}{s+2}$$

$$A = (-1+6)/(-0.5+2) = 3.333, \quad B = (-4+6)/(-2+1/2) = -1.3333$$

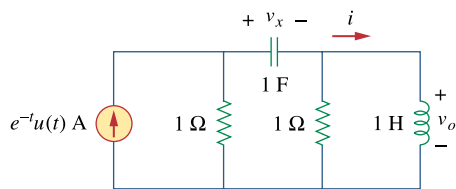
$$V_o = \frac{3.333}{s+1/2} - \frac{1.3333}{s+2}$$

Therefore,

$$v_o(t) = \underline{(3.333e^{-t/2} - 1.3333e^{-2t})u(t) \text{ V}}$$

### Chapter 16, Problem 20.

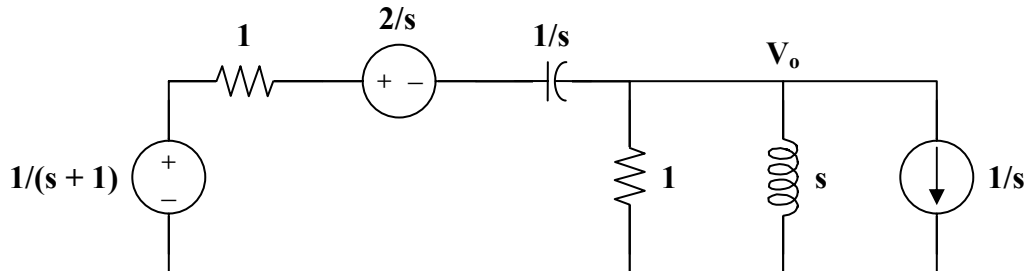
Find  $v_o(t)$  in the circuit of Fig. 16.54 if  $v_x(0) = 2 \text{ V}$  and  $i(0) = 1 \text{ A}$ .



**Figure 16.54**  
For Prob. 16.20.

## Chapter 16, Solution 20.

We incorporate the initial conditions and transform the current source to a voltage source as shown.



At the main non-reference node, KCL gives

$$\frac{1/(s+1) - 2/s - V_o}{1 + 1/s} = \frac{V_o}{1} + \frac{V_o}{s} + \frac{1}{s}$$

$$\frac{s}{s+1} - 2 - sV_o = (s+1)(1 + 1/s)V_o + \frac{s+1}{s}$$

$$\frac{s}{s+1} - \frac{s+1}{s} - 2 = (2s + 2 + 1/s)V_o$$

$$V_o = \frac{-2s^2 - 4s - 1}{(s+1)(2s^2 + 2s + 1)}$$

$$V_o = \frac{-s - 2s - 0.5}{(s+1)(s^2 + s + 0.5)} = \frac{A}{s+1} + \frac{Bs + C}{s^2 + s + 0.5}$$

$$A = (s+1)V_o \big|_{s=-1} = 1$$

$$-s^2 - 2s - 0.5 = A(s^2 + s + 0.5) + B(s^2 + s) + C(s+1)$$

Equating coefficients :

$$s^2: \quad -1 = A + B \quad \longrightarrow \quad B = -2$$

$$s^1: \quad -2 = A + B + C \quad \longrightarrow \quad C = -1$$

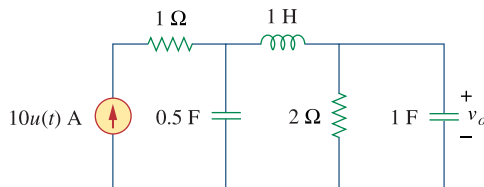
$$s^0: \quad -0.5 = 0.5A + C = 0.5 - 1 = -0.5$$

$$V_o = \frac{1}{s+1} - \frac{2s+1}{s^2 + s + 0.5} = \frac{1}{s+1} - \frac{2(s+0.5)}{(s+0.5)^2 + (0.5)^2}$$

$$v_o(t) = \underline{\underline{[e^{-t} - 2e^{-t/2} \cos(t/2)]u(t) \text{ V}}}$$

### Chapter 16, Problem 21.

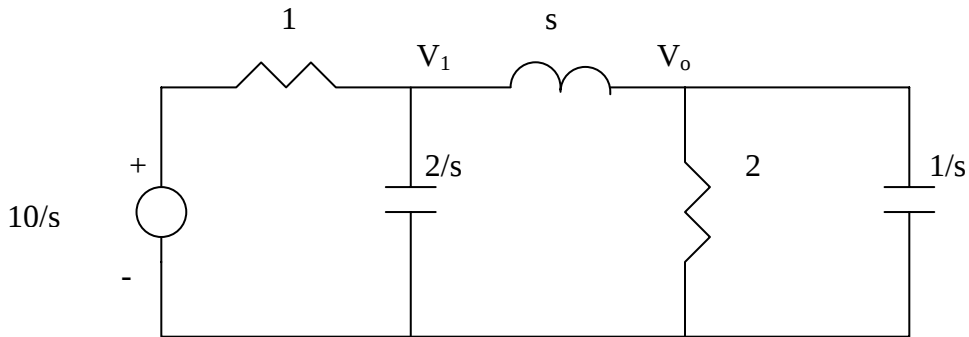
Find the voltage  $v_o(t)$  in the circuit of Fig. 16.55 by means of the Laplace transform.



**Figure 16.55**  
For Prob. 16.21.

## Chapter 16, Solution 21.

The s-domain version of the circuit is shown below.



At node 1,

$$\frac{\frac{10}{s} - V_1}{1} = \frac{V_1 - V_o}{s} + \frac{s}{2} V_o \quad \longrightarrow \quad 10 = (s+1)V_1 + \left(\frac{s^2}{2} - 1\right)V_o \quad (1)$$

At node 2,

$$\frac{V_1 - V_o}{s} = \frac{V_o}{2} + sV_o \quad \longrightarrow \quad V_1 = V_o \left(\frac{s}{2} + s^2 + 1\right) \quad (2)$$

Substituting (2) into (1) gives

$$10 = (s+1)(s^2 + s/2 + 1)V_o + \left(\frac{s^2}{2} - 1\right)V_o = s(s^2 + 2s + 1.5)V_o$$

$$V_o = \frac{10}{s(s^2 + 2s + 1.5)} = \frac{A}{s} + \frac{Bs + C}{s^2 + 2s + 1.5}$$

$$10 = A(s^2 + 2s + 1.5) + Bs^2 + Cs$$

$$s^2 : \quad 0 = A + B$$

$$s : \quad 0 = 2A + C$$

$$\text{constant :} \quad 10 = 1.5A \quad \longrightarrow \quad A = 20/3, \quad B = -20/3, \quad C = -40/3$$

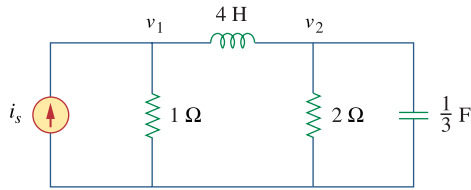
$$V_o = \frac{20}{3} \left[ \frac{1}{s} - \frac{s+2}{s^2 + 2s + 1.5} \right] = \frac{20}{3} \left[ \frac{1}{s} - \frac{s+1}{(s+1)^2 + 0.7071^2} - 1.414 \frac{0.7071}{(s+1)^2 + 0.7071^2} \right]$$

Taking the inverse Laplace transform finally yields

$$v_o(t) = \frac{20}{3} \left[ 1 - e^{-t} \cos 0.7071t - 1.414 e^{-t} \sin 0.7071t \right] u(t) \text{ V}$$

### Chapter 16, Problem 22.

Find the node voltages  $v_1$  and  $v_2$  in the circuit of Fig. 16.56 using the Laplace transform technique. Assume that  $i_s = 12e^{-t}u(t)$  A and that all initial conditions are zero.

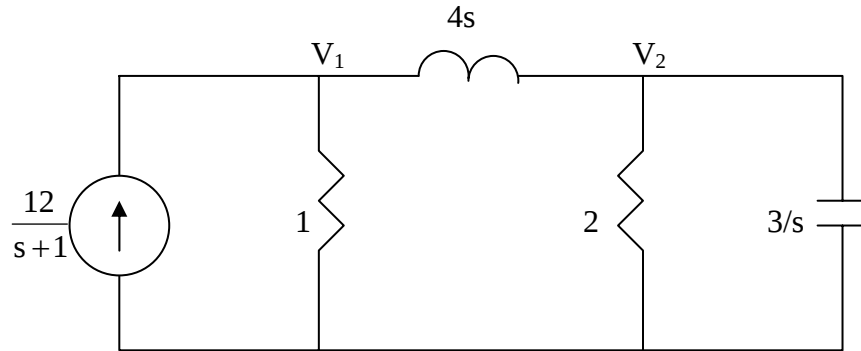


**Figure 16.56**

For Prob. 16.22.

### Chapter 16, Solution 22.

The s-domain version of the circuit is shown below.



At node 1,

$$\frac{12}{s+1} = \frac{V_1}{1} + \frac{V_1 - V_2}{4s} \longrightarrow \frac{12}{s+1} = V_1 \left( 1 + \frac{1}{4s} \right) - \frac{V_2}{4s} \quad (1)$$

At node 2,

$$\frac{V_1 - V_2}{4s} = \frac{V_2}{2} + \frac{s}{3} V_2 \longrightarrow V_1 = V_2 \left( \frac{4}{3}s^2 + 2s + 1 \right) \quad (2)$$

Substituting (2) into (1),

$$\frac{12}{s+1} = V_2 \left[ \left( \frac{4}{3}s^2 + 2s + 1 \right) \left( 1 + \frac{1}{4s} \right) - \frac{1}{4s} \right] = \left( \frac{4}{3}s^2 + \frac{7}{3}s + \frac{3}{2} \right) V_2$$

$$V_2 = \frac{9}{(s+1)(s^2 + \frac{7}{4}s + \frac{9}{8})} = \frac{A}{(s+1)} + \frac{Bs + C}{(s^2 + \frac{7}{4}s + \frac{9}{8})}$$

$$9 = A(s^2 + \frac{7}{4}s + \frac{9}{8}) + B(s^2 + s) + C(s+1)$$

Equating coefficients:

$$s^2 : \quad 0 = A + B$$

$$s : \quad 0 = \frac{7}{4}A + B + C = \frac{3}{4}A + C \quad \longrightarrow \quad C = -\frac{3}{4}A$$

$$\text{constant :} \quad 9 = \frac{9}{8}A + C = \frac{3}{8}A \quad \longrightarrow \quad A = 24, B = -24, C = -18$$

$$V_2 = \frac{24}{(s+1)} - \frac{24s+18}{(s^2 + \frac{7}{4}s + \frac{9}{8})} = \frac{24}{(s+1)} - \frac{24(s + 7/8)}{(s + \frac{7}{8})^2 + \frac{23}{64}} + \frac{3}{(s + \frac{7}{8})^2 + \frac{23}{64}}$$

Taking the inverse of this produces:

$$\underline{v_2(t) = [24e^{-t} - 24e^{-0.875t} \cos(0.5995t) + 5.004e^{-0.875t} \sin(0.5995t)]u(t)}$$

Similarly,

$$V_1 = \frac{9\left(\frac{4}{3}s^2 + 2s + 1\right)}{(s+1)(s^2 + \frac{7}{4}s + \frac{9}{8})} = \frac{D}{(s+1)} + \frac{Es + F}{(s^2 + \frac{7}{4}s + \frac{9}{8})}$$

$$9\left(\frac{4}{3}s^2 + 2s + 1\right) = D(s^2 + \frac{7}{4}s + \frac{9}{8}) + E(s^2 + s) + F(s+1)$$

Equating coefficients:

$$s^2 : \quad 12 = D + E$$

$$s : \quad 18 = \frac{7}{4}D + E + F \text{ or } 6 = \frac{3}{4}D + F \quad \longrightarrow \quad F = 6 - \frac{3}{4}D$$

$$\text{constant :} \quad 9 = \frac{9}{8}D + F \text{ or } 3 = \frac{3}{8}D \quad \longrightarrow \quad D = 8, E = 4, F = 0$$

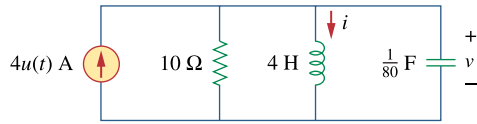
$$V_1 = \frac{8}{(s+1)} + \frac{4s}{(s^2 + \frac{7}{4}s + \frac{9}{8})} = \frac{8}{(s+1)} + \frac{4(s + 7/8)}{(s + \frac{7}{8})^2 + \frac{23}{64}} - \frac{7/2}{(s + \frac{7}{8})^2 + \frac{23}{64}}$$

Thus,

$$\underline{v_1(t) = [8e^{-t} + 4e^{-0.875t} \cos(0.5995t) - 5.838e^{-0.875t} \sin(0.5995t)]u(t)}$$

### Chapter 16, Problem 23.

Consider the parallel  $RLC$  circuit of Fig. 16.57. Find  $v(t)$  and  $i(t)$  given that  $v(0) = 5$  and  $i(0) = -2$  A.

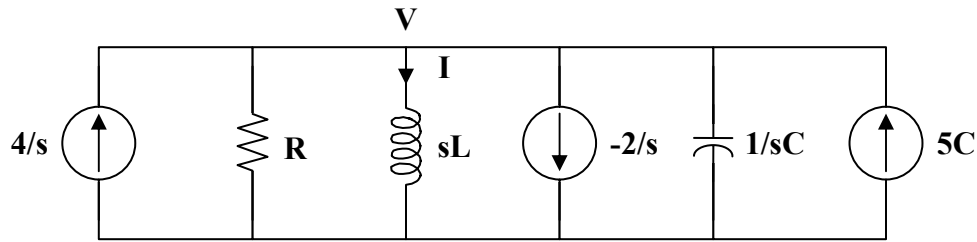


**Figure 16.57**

For Prob. 16.23.

### Chapter 16, Solution 23.

The s-domain form of the circuit with the initial conditions is shown below.



At the non-reference node,

$$\begin{aligned}\frac{4}{s} + \frac{2}{s} + 5C &= \frac{V}{R} + \frac{V}{sL} + sCV \\ \frac{6 + 5sC}{s} &= \frac{CV}{s} \left( s^2 + \frac{s}{RC} + \frac{1}{LC} \right) \\ V &= \frac{5s + 6/C}{s^2 + s/RC + 1/LC}\end{aligned}$$

But  $\frac{1}{RC} = \frac{1}{10/80} = 8$ ,  $\frac{1}{LC} = \frac{1}{4/80} = 20$

$$V = \frac{5s + 480}{s^2 + 8s + 20} = \frac{5(s + 4)}{(s + 4)^2 + 2^2} + \frac{(230)(2)}{(s + 4)^2 + 2^2}$$

$$\underline{v(t) = (5e^{-4t} \cos(2t) + 230e^{-4t} \sin(2t))u(t) \text{ V}}$$

$$I = \frac{V}{sL} = \frac{5s + 480}{4s(s^2 + 8s + 20)}$$

$$I = \frac{1.25s + 120}{s(s^2 + 8s + 20)} = \frac{A}{s} + \frac{Bs + C}{s^2 + 8s + 20}$$

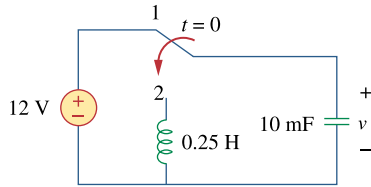
$$A = 6, \quad B = -6, \quad C = -46.75$$

$$I = \frac{6}{s} - \frac{6s + 46.75}{s^2 + 8s + 20} = \frac{6}{s} - \frac{6(s + 4)}{(s + 4)^2 + 2^2} - \frac{(11.375)(2)}{(s + 4)^2 + 2^2}$$

$$\underline{i(t) = (6 - 6e^{-4t} \cos(2t) - 11.375e^{-4t} \sin(2t))u(t), \quad t > 0}$$

### Chapter 16, Problem 24.

The switch in Fig. 16.58 moves from position 1 to position 2 at  $t = 0$ . Find  $v(t)$ , for all  $t > 0$ .

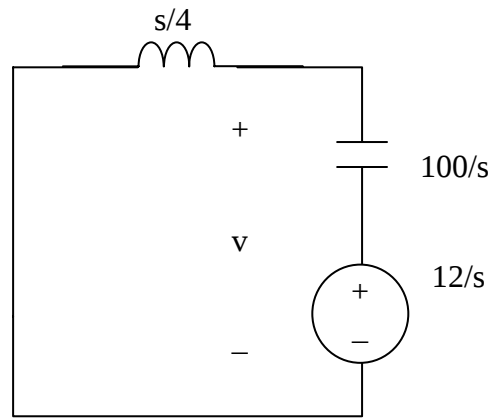


**Figure 16.58**

For Prob. 16.24.

### Chapter 16, Solution 24.

When the switch is position 1,  $v(0)=12$ , and  $i_L(0) = 0$ . When the switch is in position 2, we have the circuit as shown below.



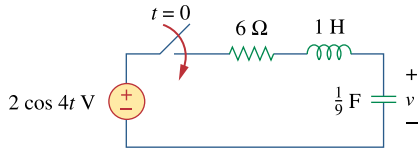
$$10\text{mF} = 0.01\text{F} \quad \longrightarrow \quad \frac{1}{sC} = \frac{100}{s}$$

$$I = \frac{12/s}{s/4 + 100/s} = \frac{48}{s^2 + 400}, \quad V = sLI = \frac{s}{4} I = \frac{12s}{s^2 + 400}$$

$$\underline{v(t) = 12 \cos 20t, \quad t > 0}$$

### Chapter 16, Problem 25.

For the *RLC* circuit shown in Fig. 16.59, find the complete response if  $v(0) = 2$  V when the switch is closed.

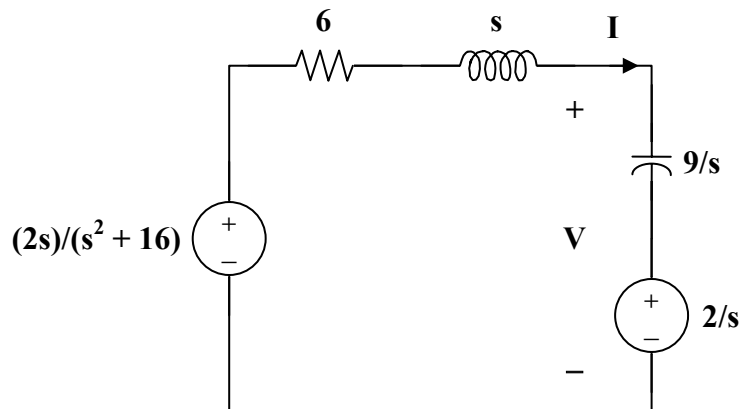


**Figure 16.59**

For Prob. 16.25.

### Chapter 16, Solution 25.

For  $t > 0$ , the circuit in the  $s$ -domain is shown below.



Applying KVL,

$$\frac{-2s}{s^2 + 16} + \left(6 + s + \frac{9}{s}\right)I + \frac{2}{s} = 0$$

$$I = \frac{-32}{(s^2 + 6s + 9)(s^2 + 16)}$$

$$V = \frac{9}{s}I + \frac{2}{s} = \frac{2}{s} + \frac{-288}{s(s+3)^2(s^2 + 16)}$$

$$= \frac{2}{s} + \frac{A}{s} + \frac{B}{s+3} + \frac{C}{(s+3)^2} + \frac{Ds + E}{s^2 + 16}$$

$$\begin{aligned} -288 &= A(s^4 + 6s^3 + 25s^2 + 96s + 144) + B(s^4 + 3s^3 + 16s^2 + 48s) \\ &\quad + C(s^3 + 16s) + D(s^4 + 6s^3 + 9s^2) + E(s^3 + 6s^2 + 9s) \end{aligned}$$

Equating coefficients :

$$s^0: \quad -288 = 144A \quad (1)$$

$$s^1: \quad 0 = 96A + 48B + 16C + 9E \quad (2)$$

$$s^2: \quad 0 = 25A + 16B + 9D + 6E \quad (3)$$

$$s^3: \quad 0 = 6A + 3B + C + 6D + E \quad (4)$$

$$s^4: \quad 0 = A + B + D \quad (5)$$

Solving equations (1), (2), (3), (4) and (5) gives

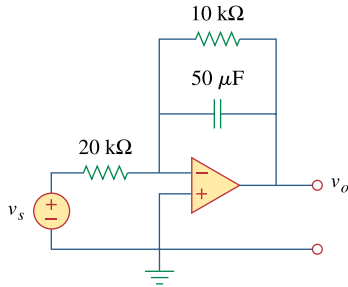
$$A = -2, \quad B = 2.202, \quad C = 3.84, \quad D = -0.202, \quad E = 2.766$$

$$V(s) = \frac{2.202}{s+3} + \frac{3.84}{(s+3)^2} - \frac{0.202s}{s^2 + 16} + \frac{(0.6915)(4)}{s^2 + 16}$$

$$v(t) = \underline{\underline{\{2.202e^{-3t} + 3.84te^{-3t} - 0.202\cos(4t) + 0.6915\sin(4t)\}u(t) \text{ V}}}$$

### Chapter 16, Problem 26.

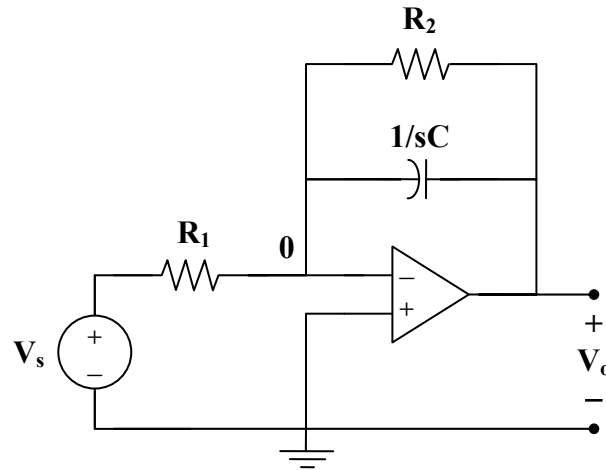
For the op amp circuit in Fig. 16.60, find  $v_o(t)$  for  $t > 0$ . Take  $v_s = 3e^{-5t}u(t)$  V.



**Figure 16.60**  
For Prob. 16.26.

### Chapter 16, Solution 26.

Consider the op-amp circuit below.



At node 0,

$$\frac{V_s - 0}{R_1} = \frac{0 - V_o}{R_2} + (0 - V_o)sC$$

$$V_s = R_1 \left( \frac{1}{R_2} + sC \right) (-V_o)$$

$$\frac{V_o}{V_s} = \frac{-1}{sR_1C + R_1/R_2}$$

But  $\frac{R_1}{R_2} = \frac{20}{10} = 2$ ,  $R_1C = (20 \times 10^3)(50 \times 10^{-6}) = 1$

So,  $\frac{V_o}{V_s} = \frac{-1}{s+2}$

$$V_s = 3e^{-5t} \longrightarrow V_s = 3/(s+5)$$

$$V_o = \frac{-3}{(s+2)(s+5)}$$

$$-V_o = \frac{3}{(s+2)(s+5)} = \frac{A}{s+2} + \frac{B}{s+5}$$

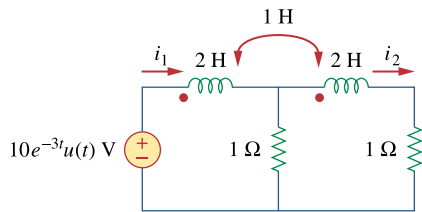
$$A = 1, \quad B = -1$$

$$V_o = \frac{1}{s+5} - \frac{1}{s+2}$$

$$v_o(t) = \underline{(e^{-5t} - e^{-2t})u(t)}$$

**Chapter 16, Problem 27.**

Find  $I_1(s)$  and  $I_2(s)$  in the circuit of Fig. 16.61.

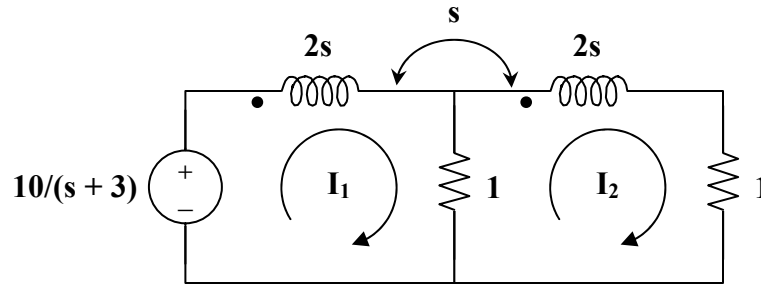


**Figure 16.61**

For Prob. 16.27.

## Chapter 16, Solution 27.

Consider the following circuit.



For mesh 1,

$$\frac{10}{s+3} = (1+2s)I_1 - I_2 - sI_2$$

$$\frac{10}{s+3} = (1+2s)I_1 - (1+s)I_2 \quad (1)$$

For mesh 2,

$$0 = (2+2s)I_2 - I_1 - sI_1$$

$$0 = -(1+s)I_1 + 2(s+1)I_2 \quad (2)$$

(1) and (2) in matrix form,

$$\begin{bmatrix} 10/(s+3) \\ 0 \end{bmatrix} = \begin{bmatrix} 2s+1 & -(s+1) \\ -(s+1) & 2(s+1) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\Delta = 3s^2 + 4s + 1$$

$$\Delta_1 = \frac{20(s+1)}{s+3}$$

$$\Delta_2 = \frac{10(s+1)}{s+3}$$

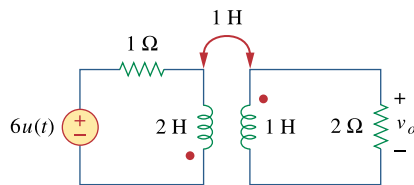
Thus

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{20(s+1)}{(s+3)(3s^2 + 4s + 1)}$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{10(s+1)}{(s+3)(3s^2 + 4s + 1)} = \frac{I_1}{2}$$

### Chapter 16, Problem 28.

For the circuit in Fig. 16.62, find  $v_o(t)$  for  $t > 0$ .

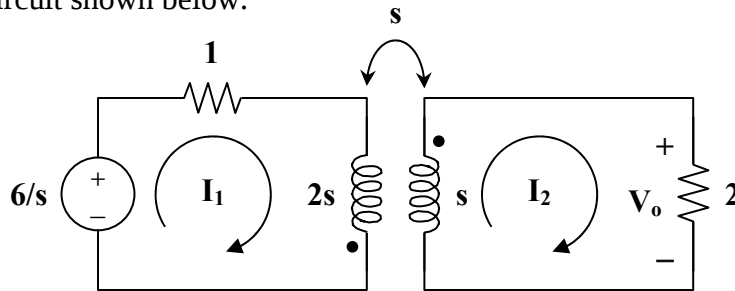


**Figure 16.62**

For Prob. 16.28.

## Chapter 16, Solution 28.

Consider the circuit shown below.



For mesh 1,

$$\frac{6}{s} = (1 + 2s)I_1 + sI_2 \quad (1)$$

For mesh 2,

$$0 = sI_1 + (2 + s)I_2$$

$$I_1 = -\left(1 + \frac{2}{s}\right)I_2 \quad (2)$$

Substituting (2) into (1) gives

$$\frac{6}{s} = -(1 + 2s)\left(1 + \frac{2}{s}\right)I_2 + sI_2 = \frac{-(s^2 + 5s + 2)}{s}I_2$$

or 
$$I_2 = \frac{-6}{s^2 + 5s + 2}$$

$$V_o = 2I_2 = \frac{-12}{s^2 + 5s + 2} = \frac{-12}{(s + 0.438)(s + 4.561)}$$

Since the roots of  $s^2 + 5s + 2 = 0$  are -0.438 and -4.561,

$$V_o = \frac{A}{s + 0.438} + \frac{B}{s + 4.561}$$

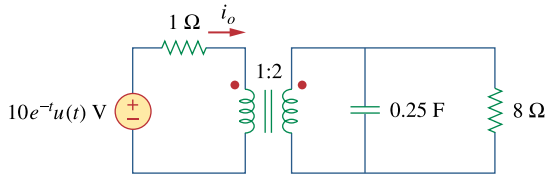
$$A = \frac{-12}{4.123} = -2.91, \quad B = \frac{-12}{-4.123} = 2.91$$

$$V_o(s) = \frac{-2.91}{s + 0.438} + \frac{2.91}{s + 4.561}$$

$$v_o(t) = \underline{2.91[e^{-4.561t} - e^{0.438t}]}u(t) \text{ V}$$

### Chapter 16, Problem 29.

For the ideal transformer circuit in Fig. 16.63, determine  $i_o(t)$ .

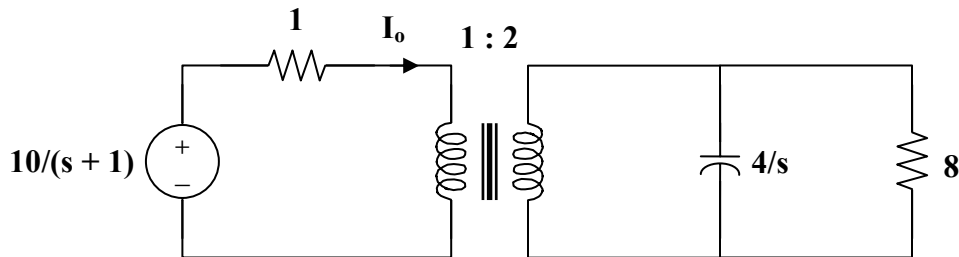


**Figure 16.63**

For Prob. 16.29.

### Chapter 16, Solution 29.

Consider the following circuit.



$$\text{Let } Z_L = 8 \parallel \frac{4}{s} = \frac{(8)(4/s)}{8 + 4/s} = \frac{8}{2s+1}$$

When this is reflected to the primary side,

$$Z_{in} = 1 + \frac{Z_L}{n^2}, \quad n = 2$$

$$Z_{in} = 1 + \frac{2}{2s+1} = \frac{2s+3}{2s+1}$$

$$I_o = \frac{10}{s+1} \cdot \frac{1}{Z_{in}} = \frac{10}{s+1} \cdot \frac{2s+1}{2s+3}$$

$$I_o = \frac{10s+5}{(s+1)(s+1.5)} = \frac{A}{s+1} + \frac{B}{s+1.5}$$

$$A = -10, \quad B = 20$$

$$I_o(s) = \frac{-10}{s+1} + \frac{20}{s+1.5}$$

$$i_o(t) = \underline{10[2e^{-1.5t} - e^{-t}]u(t) \text{ A}}$$

### Chapter 16, Problem 30.

The transfer function of a system is

$$H(s) = \frac{s^2}{3s+1}$$

Find the output when the system has an input of  $4e^{-t/3}u(t)$ .

### Chapter 16, Solution 30.

$$Y(s) = H(s)X(s), \quad X(s) = \frac{4}{s+1/3} = \frac{12}{3s+1}$$

$$Y(s) = \frac{12s^2}{(3s+1)^2} = \frac{4}{3} - \frac{8s+4/3}{(3s+1)^2}$$

$$Y(s) = \frac{4}{3} - \frac{8}{9} \cdot \frac{s}{(s+1/3)^2} - \frac{4}{27} \cdot \frac{1}{(s+1/3)^2}$$

$$\text{Let } G(s) = \frac{-8}{9} \cdot \frac{s}{(s+1/3)^2}$$

Using the time differentiation property,

$$g(t) = \frac{-8}{9} \cdot \frac{d}{dt}(te^{-t/3}) = \frac{-8}{9} \left( \frac{-1}{3}te^{-t/3} + e^{-t/3} \right)$$

$$g(t) = \frac{8}{27}te^{-t/3} - \frac{8}{9}e^{-t/3}$$

Hence,

$$y(t) = \frac{4}{3}u(t) + \frac{8}{27}te^{-t/3} - \frac{8}{9}e^{-t/3} - \frac{4}{27}te^{-t/3}$$

$$y(t) = \underline{\underline{\frac{4}{3}u(t) - \frac{8}{9}e^{-t/3} + \frac{4}{27}te^{-t/3}}}$$

### Chapter 16, Problem 31.

When the input to a system is a unit step function, the response is  $10 \cos 2tu(t)$ . Obtain the transfer function of the system.

### Chapter 16, Solution 31.

$$x(t) = u(t) \longrightarrow X(s) = \frac{1}{s}$$

$$y(t) = 10 \cos(2t) \longrightarrow Y(s) = \frac{10s}{s^2 + 4}$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{10s^2}{\underline{s^2 + 4}}$$

### Chapter 16, Problem 32.

A circuit is known to have its transfer function as

$$H(s) = \frac{s+3}{s^2+4s+5}$$

Find its output when:

- (a) the input is a unit step function
- (b) the input is  $6te^{-2t}u(t)$ .

### Chapter 16, Solution 32.

$$(a) \quad Y(s) = H(s)X(s)$$

$$\begin{aligned} &= \frac{s+3}{s^2+4s+5} \cdot \frac{1}{s} \\ &= \frac{s+3}{s(s^2+4s+5)} = \frac{A}{s} + \frac{Bs+C}{s^2+4s+5} \end{aligned}$$

$$s+3 = A(s^2+4s+5) + Bs^2 + Cs$$

Equating coefficients :

$$s^0: \quad 3 = 5A \quad \longrightarrow \quad A = 3/5$$

$$s^1: \quad 1 = 4A + C \quad \longrightarrow \quad C = 1 - 4A = -7/5$$

$$s^2: \quad 0 = A + B \quad \longrightarrow \quad B = -A = -3/5$$

$$Y(s) = \frac{3/5}{s} - \frac{1}{5} \cdot \frac{3s+7}{s^2+4s+5}$$

$$Y(s) = \frac{0.6}{s} - \frac{1}{5} \cdot \frac{3(s+2)+1}{(s+2)^2+1}$$

$$y(t) = \underline{\underline{[0.6 - 0.6e^{-2t} \cos(t) - 0.2e^{-2t} \sin(t)]u(t)}}$$

$$(b) \quad x(t) = 6t e^{-2t} \longrightarrow X(s) = \frac{6}{(s+2)^2}$$

$$Y(s) = H(s)X(s) = \frac{s+3}{s^2+4s+5} \cdot \frac{6}{(s+2)^2}$$

$$Y(s) = \frac{6(s+3)}{(s+2)^2(s^2+4s+5)} = \frac{A}{s+2} + \frac{B}{(s+2)^2} + \frac{Cs+D}{s^2+4s+5}$$

Equating coefficients :

$$s^3: \quad 0 = A + C \longrightarrow C = -A \quad (1)$$

$$s^2: \quad 0 = 6A + B + 4C + D = 2A + B + D \quad (2)$$

$$s^1: \quad 6 = 13A + 4B + 4C + 4D = 9A + 4B + 4D \quad (3)$$

$$s^0: \quad 18 = 10A + 5B + 4D = 2A + B \quad (4)$$

Solving (1), (2), (3), and (4) gives

$$A = 6, \quad B = 6, \quad C = -6, \quad D = -18$$

$$Y(s) = \frac{6}{s+2} + \frac{6}{(s+2)^2} - \frac{6s+18}{(s+2)^2+1}$$

$$Y(s) = \frac{6}{s+2} + \frac{6}{(s+2)^2} - \frac{6(s+2)}{(s+2)^2+1} - \frac{6}{(s+2)^2+1}$$

$$y(t) = \underline{\underline{[6e^{-2t} + 6te^{-2t} - 6e^{-2t} \cos(t) - 6e^{-2t} \sin(t)]u(t)}}$$

### Chapter 16, Problem 33.

When a unit step is applied to a system at  $t = 0$  its response is

$$y(t) = \left[ 4 + \frac{1}{2}e^{-3t} - e^{-2t} (2 \cos 4t + 3 \sin 4t) \right] u(t)$$

What is the transfer function of the system?

### Chapter 16, Solution 33.

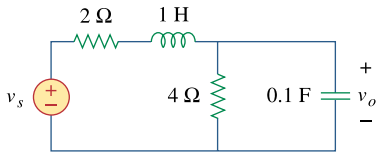
$$H(s) = \frac{Y(s)}{X(s)}, \quad X(s) = \frac{1}{s}$$

$$Y(s) = \frac{4}{s} + \frac{1}{2(s+3)} - \frac{2s}{(s+2)^2 + 16} - \frac{(3)(4)}{(s+2)^2 + 16}$$

$$H(s) = s Y(s) = 4 + \frac{s}{2(s+3)} - \frac{2s(s+2)}{s^2 + 4s + 20} - \frac{12s}{s^2 + 4s + 20}$$

### Chapter 16, Problem 34.

For the circuit in Fig. 16.64, find  $H(s) = V_o(s)/V_s(s)$ . Assume zero initial conditions.

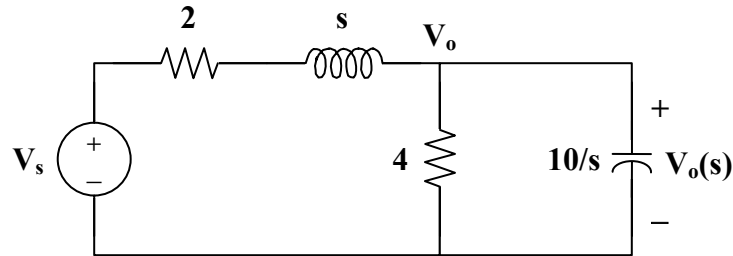


**Figure 16.64**

For Prob. 16.34.

### Chapter 16, Solution 34.

Consider the following circuit.



Using nodal analysis,

$$\frac{V_s - V_o}{s + 2} = \frac{V_o}{4} + \frac{V_o}{10/s}$$

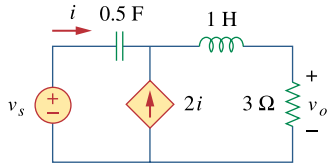
$$V_s = (s + 2) \left( \frac{1}{s + 2} + \frac{1}{4} + \frac{s}{10} \right) V_o = \left( 1 + \frac{1}{4}(s + 2) + \frac{1}{10}(s^2 + 2s) \right) V_o$$

$$V_s = \frac{1}{20} (2s^2 + 9s + 30) V_o$$

$$\frac{V_o}{V_s} = \frac{20}{2s^2 + 9s + 30}$$

### Chapter 16, Problem 35.

Obtain the transfer function  $H(s) = V_o/V_s$  for the circuit of Fig. 16.65.

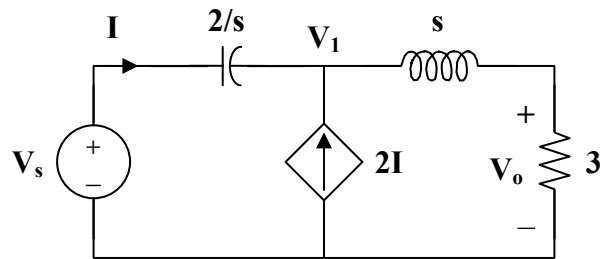


**Figure 16.65**

For Prob. 16.35.

### Chapter 16, Solution 35.

Consider the following circuit.



At node 1,

$$2I + I = \frac{V_1}{s+3}, \quad \text{where } I = \frac{V_s - V_1}{2/s}$$

$$3 \cdot \frac{V_s - V_1}{2/s} = \frac{V_1}{s+3}$$

$$\frac{V_1}{s+3} = \frac{3s}{2} V_s - \frac{3s}{2} V_1$$

$$\left( \frac{1}{s+3} + \frac{3s}{2} \right) V_1 = \frac{3s}{2} V_s$$

$$V_1 = \frac{3s(s+3)}{3s^2 + 9s + 2} V_s$$

$$V_o = \frac{3}{s+3} V_1 = \frac{9s}{3s^2 + 9s + 2} V_s$$

$$H(s) = \frac{V_o}{V_s} = \frac{9s}{\underline{3s^2 + 9s + 2}}$$

### Chapter 16, Problem 36.

The transfer function of a certain circuit is

$$H(s) = \frac{5}{s+1} - \frac{3}{s+2} + \frac{6}{s+4}$$

Find the impulse response of the circuit.

### Chapter 16, Solution 36.

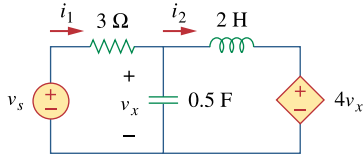
Taking the inverse Laplace transform of each term gives

$$h(t) = \underline{(5e^{-t} - 3e^{-2t} + 6e^{-4t})u(t)}$$

### Chapter 16, Problem 37.

For the circuit in Fig. 16.66, find:

- (a)  $I_1/V_s$       (b)  $I_2/V_x$

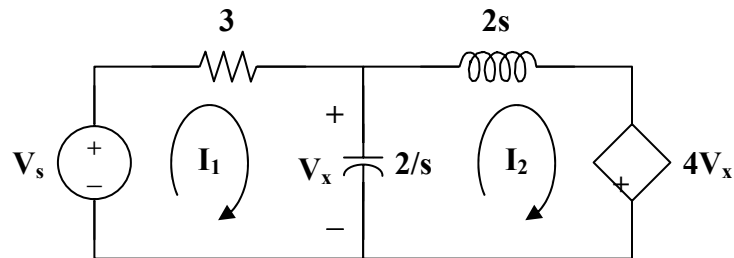


**Figure 16.66**

For Prob. 16.37.

### Chapter 16, Solution 37.

- (a) Consider the circuit shown below.



For loop 1,

$$V_s = \left(3 + \frac{2}{s}\right)I_1 - \frac{2}{s}I_2 \quad (1)$$

For loop 2,

$$4V_x + \left(2s + \frac{2}{s}\right)I_2 - \frac{2}{s}I_1 = 0$$

$$\text{But, } V_x = (I_1 - I_2)\left(\frac{2}{s}\right)$$

$$\text{So, } \frac{8}{s}(I_1 - I_2) + \left(2s + \frac{2}{s}\right)I_2 - \frac{2}{s}I_1 = 0$$

$$0 = \frac{-6}{s}I_1 + \left(\frac{6}{s} - 2s\right)I_2 \quad (2)$$

In matrix form, (1) and (2) become

$$\begin{bmatrix} V_s \\ 0 \end{bmatrix} = \begin{bmatrix} 3 + 2/s & -2/s \\ -6/s & 6/s - 2s \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\Delta = \left(3 + \frac{2}{s}\right)\left(\frac{6}{s} - 2s\right) - \left(\frac{6}{s}\right)\left(\frac{2}{s}\right)$$

$$\Delta = \frac{18}{s} - 6s - 4$$

$$\Delta_1 = \left(\frac{6}{s} - 2s\right)V_s, \quad \Delta_2 = \frac{6}{s}V_s$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{(6/s - 2s)}{18/s - 4 - 6s} V_s$$

$$\frac{I_1}{V_s} = \frac{3/s - s}{9/s - 2 - 3} = \frac{s^2 - 3}{\underline{\underline{3s^2 + 2s - 9}}}$$

$$(b) \quad I_2 = \frac{\Delta_2}{\Delta}$$

$$V_x = \frac{2}{s}(I_1 - I_2) = \frac{2}{s}\left(\frac{\Delta_1 - \Delta_2}{\Delta}\right)$$

$$V_x = \frac{2/s V_s (6/s - 2s - 6/s)}{\Delta} = \frac{-4V_s}{\Delta}$$

$$\frac{I_2}{V_x} = \frac{6/s V_s}{-4V_s} = \frac{-3}{\underline{\underline{2s}}}$$

### Chapter 16, Problem 38.

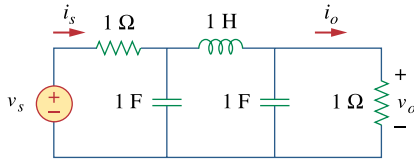
Refer to the network in Fig. 16.67. Find the following transfer functions:

(a)  $H_1(s) = V_o(s)/V_s(s)$

(b)  $H_2(s) = V_o(s)/I_s(s)$

(c)  $H_3(s) = I_o(s)/I_s(s)$

(d)  $H_4(s) = I_o(s)/V_s(s)$

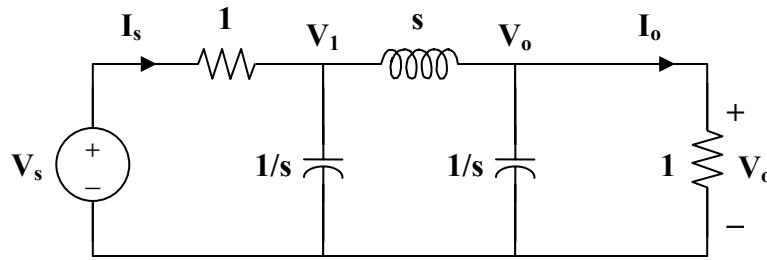


**Figure 16.67**

For Prob. 16.38.

### Chapter 16, Solution 38.

(a) Consider the following circuit.



At node 1,

$$\frac{V_s - V_1}{1} = s V_1 + \frac{V_1 - V_o}{s}$$

$$V_s = \left(1 + s + \frac{1}{s}\right) V_1 - \frac{1}{s} V_o \quad (1)$$

At node o,

$$\frac{V_1 - V_o}{s} = s V_o + V_o = (s + 1) V_o$$

$$V_1 = (s^2 + s + 1) V_o \quad (2)$$

Substituting (2) into (1)

$$V_s = (s + 1 + 1/s)(s^2 + s + 1) V_o - 1/s V_o$$

$$V_s = (s^3 + 2s^2 + 3s + 2) V_o$$

$$H_1(s) = \frac{V_o}{V_s} = \frac{1}{\underline{s^3 + 2s^2 + 3s + 2}}$$

(b)  $I_s = V_s - V_1 = (s^3 + 2s^2 + 3s + 2) V_o - (s^2 + s + 1) V_o$

$$I_s = (s^3 + s^2 + 2s + 1) V_o$$

$$H_2(s) = \frac{V_o}{I_s} = \frac{1}{\underline{s^3 + s^2 + 2s + 1}}$$

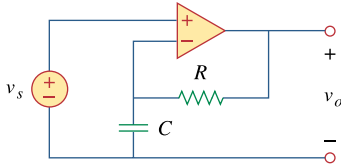
(c)  $I_o = \frac{V_o}{1}$

$$H_3(s) = \frac{I_o}{I_s} = \frac{V_o}{I_s} = H_2(s) = \frac{1}{\underline{s^3 + s^2 + 2s + 1}}$$

(d)  $H_4(s) = \frac{I_o}{V_s} = \frac{V_o}{V_s} = H_1(s) = \frac{1}{\underline{s^3 + 2s^2 + 3s + 2}}$

### Chapter 16, Problem 39.

Calculate the gain  $H(s) = V_o/V_s$  in the op amp circuit of Fig. 16.68.

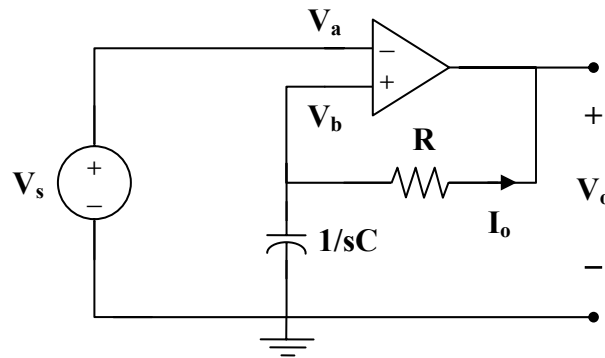


**Figure 16.68**

For Prob. 16.39.

### Chapter 16, Solution 39.

Consider the circuit below.



Since no current enters the op amp,  $I_o$  flows through both  $R$  and  $C$ .

$$V_o = -I_o \left( R + \frac{1}{sC} \right)$$

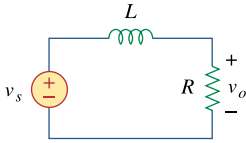
$$V_a = V_b = V_s = \frac{-I_o}{sC}$$

$$H(s) = \frac{V_o}{V_s} = \frac{R + 1/sC}{1/sC} = \underline{\underline{sRC + 1}}$$

### Chapter 16, Problem 40.

Refer to the  $RL$  circuit in Fig. 16.69. Find:

- (a) the impulse response  $h(t)$  of the circuit.
- (b) the unit step response of the circuit.



**Figure 16.69**

For Prob. 16.40.

### Chapter 16, Solution 40.

$$(a) \quad H(s) = \frac{V_o}{V_s} = \frac{R}{R + sL} = \frac{R/L}{s + R/L}$$

$$h(t) = \underline{\underline{\frac{R}{L} e^{-Rt/L} u(t)}}$$

$$(b) \quad v_s(t) = u(t) \longrightarrow V_s(s) = 1/s$$

$$V_o = \frac{R/L}{s + R/L} V_s = \frac{R/L}{s(s + R/L)} = \frac{A}{s} + \frac{B}{s + R/L}$$

$$A = 1, \quad B = -1$$

$$V_o = \frac{1}{s} - \frac{1}{s + R/L}$$

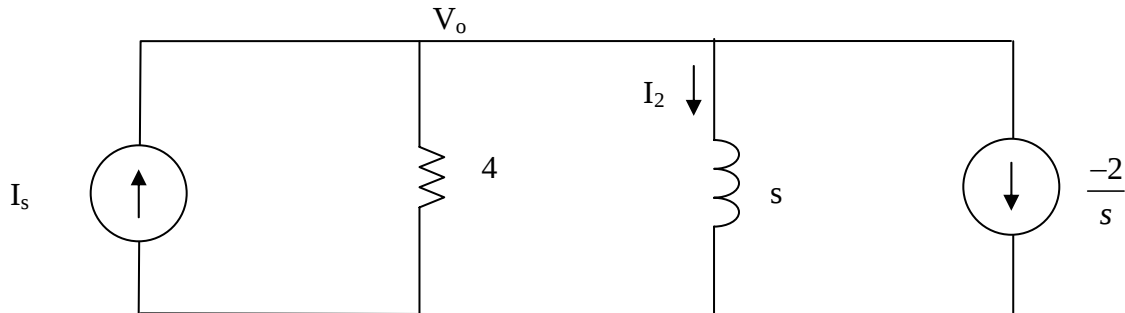
$$v_o(t) = u(t) - e^{-Rt/L} u(t) = \underline{\underline{(1 - e^{-Rt/L}) u(t)}}$$

### Chapter 16, Problem 41.

A parallel  $RL$  circuit has  $R = 4\Omega$  and  $L = 1$  H. The input to the circuit is  $i_s(t) = 2e^{-t}u(t)$  A. Find the inductor current  $i_L(t)$  for all  $t > 0$  and assume that  $i_L(0) = -2$  A.

### Chapter 16, Solution 41.

Consider the circuit as shown below.



$$I_s = \frac{V_o}{4} + \frac{V_o}{s} - \frac{2}{s}$$

But  $I_s = \frac{2}{s+1}$

$$\frac{2}{s+1} = V_o \left( \frac{1}{4} + \frac{1}{s} \right) - \frac{2}{s} \longrightarrow V_o \left( \frac{s+4}{4s} \right) = \frac{2}{s+1} + \frac{2}{s} = \frac{4s+2}{s(s+1)}$$

$$V_o = \frac{8(2s+1)}{(s+1)(s+4)}$$

$$I_L = \frac{V_o}{s} = \frac{8(2s+1)}{s(s+1)(s+4)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+4}$$

$$A = \frac{8(1)}{(1)(4)} = 2, \quad B = \frac{8(-2+1)}{(-1)(2)} = 8/3, \quad C = \frac{8(-8+1)}{(-4)(-3)} = -14/3$$

$$I_L = \frac{V_o}{s} = \frac{2}{s} + \frac{8/3}{s+1} + \frac{-14/3}{s+4}$$

$$i_L(t) = \left( 2 + \frac{8}{3}e^{-t} - \frac{14}{3}e^{-4t} \right) u(t) = \left( 2 + \frac{8}{3}e^{-t} - \frac{14}{3}e^{-4t} \right) u(t) \text{ A}$$

### Chapter 16, Problem 42.

A circuit has a transfer function

$$H(s) = \frac{s+4}{(s+1)(s+2)^2}$$

Find the impulse response.

### Chapter 16, Solution 42.

$$H(s) = \frac{s+4}{(s+1)(s+2)^2} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{(s+2)^2}$$

$$s+4 = A(s+2)^2 + B(s+1)(s+2) + C(s+1) = A(s^2+2s+4) + B(s^2+3s+2) + C(s+1)$$

We equate coefficients.

$$s^2: \quad 0 = A+B \text{ or } B = -A$$

$$s: \quad 1 = 4A + 3B + C = B + C$$

$$\text{constant: } 4 = 4A + 2B + C = 2A + C$$

Solving these gives  $A=3$ ,  $B=-3$ ,  $C=-2$

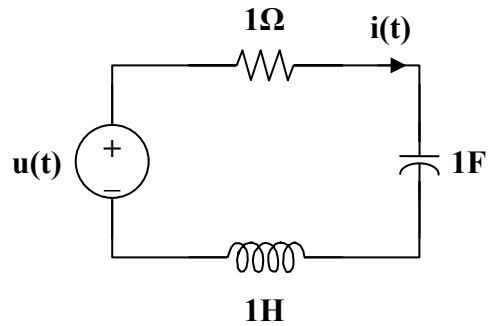
$$H(s) = \frac{3}{s+1} - \frac{3}{s+2} - \frac{2}{(s+2)^2}$$

$$h(t) = \underline{(3e^{-t} - 3e^{-2t} - 2te^{-2t})u(t)}$$

### Chapter 16, Problem 43.

Develop the state equations for Prob. 16.1.

### Chapter 16, Solution 43.



First select the inductor current  $i_L$  and the capacitor voltage  $v_C$  to be the state variables.

Applying KVL we get:

$$-u(t) + i + v_C + i' = 0; \quad i = v_C'$$

Thus,

$$v_C' = i$$

$$i' = -v_C - i + u(t)$$

Finally we get,

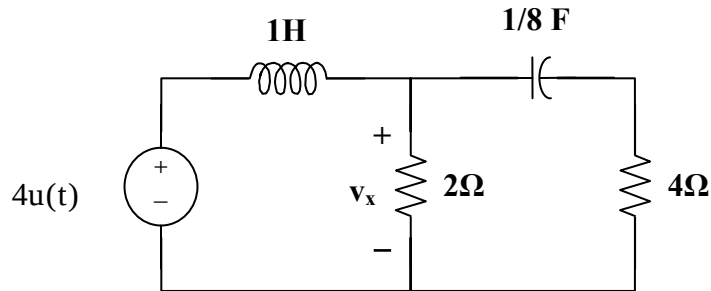
$$\begin{bmatrix} v_C' \\ i' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} v_C \\ i \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t); \quad i(t) = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} v_C \\ i \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} u(t)$$

---

### Chapter 16, Problem 44.

Develop the state equations for Prob. 16.2.

### Chapter 16, Solution 44.



First select the inductor current  $i_L$  and the capacitor voltage  $v_C$  to be the state variables.

Applying KCL we get:

$$-i_L + \frac{v_x}{2} + \frac{v_C'}{8} = 0; \text{ or } v_C' = 8i_L - 4v_x$$

$$i_L' = 4u(t) - v_x$$

$$v_x = v_C + 4 \frac{v_C'}{8} = v_C + \frac{v_C'}{2} = v_C + 4i_L - 2v_x; \text{ or } v_x = 0.3333v_C + 1.3333i_L$$

$$v_C' = 8i_L - 1.3333v_C - 5.3333i_L = -1.3333v_C + 2.666i_L$$

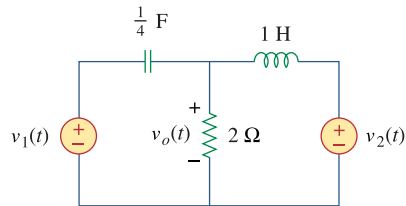
$$i_L' = 4u(t) - 0.3333v_C - 1.3333i_L$$

Now we can write the state equations.

$$\begin{bmatrix} v_C' \\ i_L' \end{bmatrix} = \begin{bmatrix} -1.3333 & 2.666 \\ -0.3333 & -1.3333 \end{bmatrix} \begin{bmatrix} v_C \\ i_L \end{bmatrix} + \begin{bmatrix} 0 \\ 4 \end{bmatrix} u(t); \quad v_x = \begin{bmatrix} 0.3333 \\ 1.3333 \end{bmatrix} \begin{bmatrix} v_C \\ i_L \end{bmatrix}$$

### Chapter 16, Problem 45.

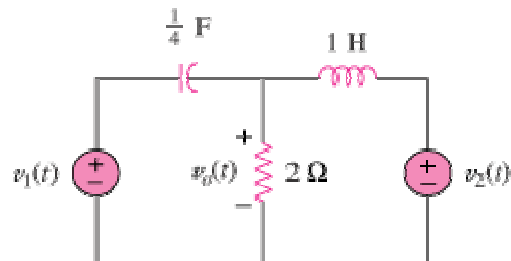
Develop the state equations for the circuit shown in Fig. 16.70.



**Figure 16.70**

For Prob. 16.45.

### Chapter 16, Solution 45.



First select the inductor current  $i_L$  (current flowing left to right) and the capacitor voltage  $v_C$  (voltage positive on the left and negative on the right) to be the state variables.

Applying KCL we get:

$$-\frac{v_C'}{4} + \frac{v_o}{2} + i_L = 0 \text{ or } v_C' = 4i_L + 2v_o$$

$$i_L' = v_o - v_2$$

$$v_o = -v_C + v_1$$

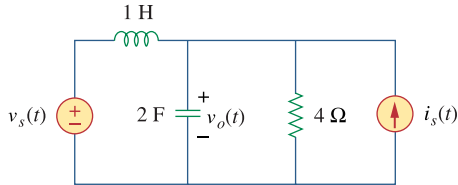
$$v_C' = 4i_L - 2v_C + 2v_1$$

$$i_L' = -v_C + v_1 - v_2$$

$$\begin{bmatrix} i_L' \\ v_C' \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} i_L \\ v_C \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix}; \quad \mathbf{v_o}(t) = \begin{bmatrix} 0 & -1 \end{bmatrix} \begin{bmatrix} i_L \\ v_C \end{bmatrix} + \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix}$$

## Chapter 16, Problem 46.

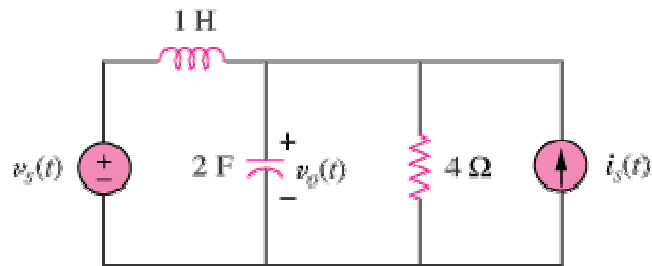
Develop the state equations for the circuit shown in Fig. 16.71.



**Figure 16.71**

For Prob. 16.46.

## Chapter 16, Solution 46.



First select the inductor current  $i_L$  (left to right) and the capacitor voltage  $v_C$  to be the state variables.

Letting  $v_o = v_C$  and applying KCL we get:

$$-i_L + \dot{v}_C + \frac{v_C}{4} - i_s = 0 \text{ or } \dot{v}_C = -0.25v_C + i_L + i_s$$

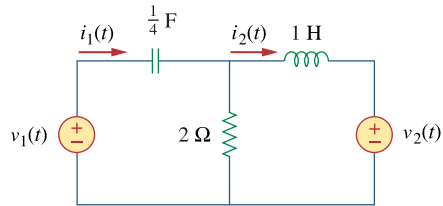
$$\dot{i}_L = -v_C + v_s$$

Thus,

$$\begin{bmatrix} \dot{v}_C \\ \dot{i}_L \end{bmatrix} = \begin{bmatrix} -0.25 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} v_C \\ i_L \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} v_s \\ i_s \end{bmatrix}; \quad v_o(t) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} v_C \\ i_L \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} v_s \\ i_s \end{bmatrix}$$

### Chapter 16, Problem 47.

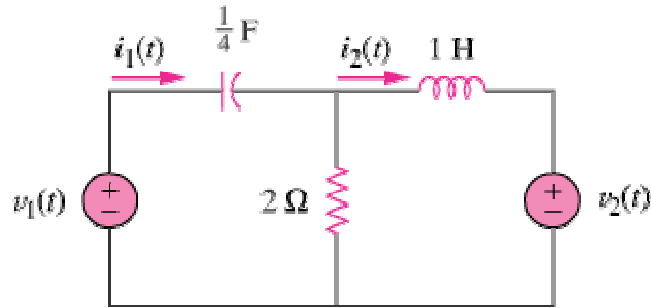
Develop the state equations for the circuit shown in Fig. 16.72.



**Figure 16.72**

For Prob. 16.47.

Chapter 16, Solution 47.



First select the inductor current  $i_L$  (left to right) and the capacitor voltage  $v_C$  (+ on the left) to be the state variables.

Letting  $i_1 = \frac{v_C'}{4}$  and  $i_2 = i_L$  and applying KVL we get:

Loop 1:

$$-v_1 + v_C + 2\left(\frac{v_C'}{4} - i_L\right) = 0 \text{ or } v_C' = 4i_L - 2v_C + 2v_1$$

Loop 2:

$$2\left(i_L - \frac{v_C'}{4}\right) + i_L' + v_2 = 0 \text{ or}$$

$$i_L' = -2i_L + \frac{4i_L - 2v_C + 2v_1}{2} - v_2 = -v_C + v_1 - v_2$$

$$i_1 = \frac{4i_L - 2v_C + 2v_1}{4} = i_L - 0.5v_C + 0.5v_1$$

$$\begin{bmatrix} i_L' \\ v_C' \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} i_L \\ v_C \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix}; \begin{bmatrix} i_1(t) \\ i_2(t) \end{bmatrix} = \begin{bmatrix} 1 & -0.5 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} i_L \\ v_C \end{bmatrix} + \begin{bmatrix} 0.5 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} v_1(t) \\ v_2(t) \end{bmatrix}$$

**Chapter 16, Problem 48.**

Develop the state equations for the following differential equation.

$$\frac{d^2 y(t)}{dt^2} + \frac{4dy(t)}{dt} + 3y(t) = z(t)$$

**Chapter 16, Solution 48.**

Let  $x_1 = y(t)$ . Thus,  $x_1' = y' = x_2$  and  $x_2' = y'' = -3x_1 - 4x_2 + z(t)$

This gives our state equations.

$$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -3 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} z(t); \quad y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} z(t)$$

**Chapter 16, Problem 49.**

\* Develop the state equations for the following differential equation.

$$\frac{d^2 y(t)}{dt^2} + \frac{5dy(t)}{dt} + 6y(t) = \frac{dz(t)}{dt} z(t)$$

\* An asterisk indicates a challenging problem.

**Chapter 16, Solution 49.**

Let  $x_1 = y(t)$  and  $x_2 = x_1' - z = y' - z$  or  $y' = x_2 + z$

Thus,

$$x_2' = y'' - z' = -6x_1 - 5(x_2 + z) + z' + 2z - z' = -6x_1 - 5x_2 - 3z$$

This now leads to our state equations,

$$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ -3 \end{bmatrix} z(t); \quad y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} z(t)$$

### Chapter 16, Problem 50.

\* Develop the state equations for the following differential equation.

$$\frac{d^3 y(t)}{dt^3} + \frac{6d^2 y(t)}{dt^2} + \frac{11dy(t)}{dt} + 6y(t) = z(t)$$

\* An asterisk indicates a challenging problem.

### Chapter 16, Solution 50.

Let  $x_1 = y(t)$ ,  $x_2 = \dot{x}_1$ , and  $x_3 = \dot{x}_2$ .

Thus,

$$\ddot{x}_3 = -6x_1 - 11x_2 - 6x_3 + z(t)$$

We can now write our state equations.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} z(t); \quad y(t) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} z(t)$$

### Chapter 16, Problem 51.

\* Given the following state equation, solve for  $y(t)$ :

$$\dot{\mathbf{x}} = \begin{bmatrix} -4 & 4 \\ -2 & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \mathbf{x}$$

\* An asterisk indicates a challenging problem.

### Chapter 16, Solution 51.

We transform the state equations into the s-domain and solve using Laplace transforms.

$$s\mathbf{X}(s) - \mathbf{x}(0) = \mathbf{A}\mathbf{X}(s) + \mathbf{B}\left(\frac{1}{s}\right)$$

Assume the initial conditions are zero.

$$(s\mathbf{I} - \mathbf{A})\mathbf{X}(s) = \mathbf{B}\left(\frac{1}{s}\right)$$

$$\mathbf{X}(s) = \begin{bmatrix} s+4 & -4 \\ 2 & s \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 2 \end{bmatrix} \left(\frac{1}{s}\right) = \frac{1}{s^2 + 4s + 8} \begin{bmatrix} s & 4 \\ 2 & s+4 \end{bmatrix} \begin{bmatrix} 0 \\ 2/s \end{bmatrix}$$

$$\begin{aligned} Y(s) = X_1(s) &= \frac{8}{s(s^2 + 4s + 8)} = \frac{1}{s} + \frac{-s-4}{s^2 + 4s + 8} \\ &= \frac{1}{s} + \frac{-s-4}{(s+2)^2 + 2^2} = \frac{1}{s} + \frac{-(s+2)}{(s+2)^2 + 2^2} + \frac{-2}{(s+2)^2 + 2^2} \end{aligned}$$

$$\underline{y(t) = \left(1 - e^{-2t}(\cos 2t + \sin 2t)\right)u(t)}$$

**Chapter 16, Problem 52.**

\* Given the following state equation, solve for  $y_1(t)$ .

$$\dot{\mathbf{x}} = \begin{bmatrix} -2 & -1 \\ 2 & -4 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 1 & 1 \\ 4 & 0 \end{bmatrix} \begin{bmatrix} u(t) \\ 2u(t) \end{bmatrix}$$

$$\mathbf{y} = \begin{bmatrix} -2 & -2 \\ 1 & -0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} u(t) \\ 2u(t) \end{bmatrix}$$

\* An asterisk indicates a challenging problem.

**Chapter 16, Solution 52.**

Assume that the initial conditions are zero. Using Laplace transforms we get,

$$\mathbf{X}(s) = \begin{bmatrix} s+2 & 1 \\ -2 & s+4 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 1 \\ 4 & 0 \end{bmatrix} \begin{bmatrix} 1/s \\ 2/s \end{bmatrix} = \frac{1}{s^2 + 6s + 10} \begin{bmatrix} s+4 & -1 \\ 2 & s+2 \end{bmatrix} \begin{bmatrix} 3/s \\ 4/s \end{bmatrix}$$

$$\begin{aligned} X_1 &= \frac{3s+8}{s((s+3)^2 + 1^2)} = \frac{0.8}{s} + \frac{-0.8s-1.8}{(s+3)^2 + 1^2} \\ &= \frac{0.8}{s} - 0.8 \frac{s+3}{(s+3)^2 + 1^2} + .6 \frac{1}{(s+3)^2 + 1^2} \end{aligned}$$

$$x_1(t) = (0.8 - 0.8e^{-3t} \cos t + 0.6e^{-3t} \sin t)u(t)$$

$$\begin{aligned} X_2 &= \frac{4s+14}{s((s+3)^2 + 1^2)} = \frac{1.4}{s} + \frac{-1.4s-4.4}{(s+3)^2 + 1^2} \\ &= \frac{1.4}{s} - 1.4 \frac{s+3}{(s+3)^2 + 1^2} - 0.2 \frac{1}{(s+3)^2 + 1^2} \end{aligned}$$

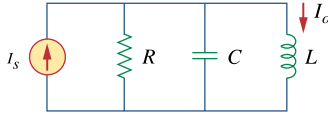
$$x_2(t) = (1.4 - 1.4e^{-3t} \cos t - 0.2e^{-3t} \sin t)u(t)$$

$$\begin{aligned} y_1(t) &= -2x_1(t) - 2x_2(t) + 2u(t) \\ &= \underline{(-2.4 + 4.4e^{-3t} \cos t - 0.8e^{-3t} \sin t)u(t)} \end{aligned}$$

$$y_2(t) = x_1(t) - 2u(t) = \underline{(-1.2 - 0.8e^{-3t} \cos t + 0.6e^{-3t} \sin t)u(t)}$$

### Chapter 16, Problem 53.

Show that the parallel  $RLC$  circuit shown in Fig. 16.73 is stable.



**Figure 16.73**

For Prob. 16.53.

### Chapter 16, Solution 53.

If  $V_o$  is the voltage across  $R$ , applying KCL at the non-reference node gives

$$I_s = \frac{V_o}{R} + sC V_o + \frac{V_o}{sL} = \left( \frac{1}{R} + sC + \frac{1}{sL} \right) V_o$$

$$V_o = \frac{I_s}{\frac{1}{R} + sC + \frac{1}{sL}} = \frac{sRL I_s}{sL + R + s^2RLC}$$

$$I_o = \frac{V_o}{R} = \frac{sL I_s}{s^2RLC + sL + R}$$

$$H(s) = \frac{I_o}{I_s} = \frac{sL}{s^2RLC + sL + R} = \frac{s/RC}{s^2 + s/RC + 1/LC}$$

The roots

$$s_{1,2} = \frac{-1}{2RC} \pm \sqrt{\frac{1}{(2RC)^2} - \frac{1}{LC}}$$

both lie in the left half plane since  $R$ ,  $L$ , and  $C$  are positive quantities.

Thus, **the circuit is stable.**

### Chapter 16, Problem 54.

A system is formed by cascading two systems as shown in Fig. 16.74. Given that the impulse response of the systems are

$$h_1(t) = 3e^{-t} u(t), \quad h_2(t) = e^{-4t} u(t)$$

- (a) Obtain the impulse response of the overall system.
- (b) Check if the overall system is stable.



**Figure 16.74**

For Prob. 16.54.

### Chapter 16, Solution 54.

$$(a) \quad H_1(s) = \frac{3}{s+1}, \quad H_2(s) = \frac{1}{s+4}$$

$$H(s) = H_1(s)H_2(s) = \frac{3}{(s+1)(s+4)}$$

$$h(t) = \mathcal{L}^{-1}[H(s)] = \mathcal{L}^{-1}\left[\frac{A}{s+1} + \frac{B}{s+4}\right]$$

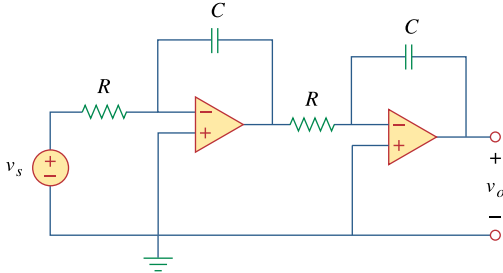
$$A = 1, \quad B = -1$$

$$h(t) = \underline{(e^{-t} - e^{-4t})u(t)}$$

- (b) Since the poles of  $H(s)$  all lie in the left half  $s$ -plane, **the system is stable.**

### Chapter 16, Problem 55.

Determine whether the op amp circuit in Fig. 16.75 is stable.



**Figure 16.75**  
For Prob. 16.55.

### Chapter 16, Solution 55.

Let  $V_{o1}$  be the voltage at the output of the first op amp.

$$\frac{V_{o1}}{V_s} = \frac{-1/sC}{R} = \frac{-1}{sRC}, \quad \frac{V_o}{V_{o1}} = \frac{-1}{sRC}$$

$$H(s) = \frac{V_o}{V_s} = \frac{1}{s^2 R^2 C^2}$$

$$h(t) = \frac{t}{R^2 C^2}$$

$$\lim_{t \rightarrow \infty} h(t) = \infty, \text{ i.e. the output is unbounded.}$$

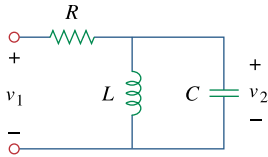
Hence, **the circuit is unstable.**

### Chapter 16, Problem 56.

It is desired to realize the transfer function

$$\frac{V_2(s)}{V_1(s)} = \frac{2s}{s^2 + 2s + 6}$$

using the circuit in Fig. 16.76. Choose  $R = 1 \text{ k}\Omega$  and find  $L$  and  $C$ .



**Figure 16.76**

For Prob. 16.56.

### Chapter 16, Solution 56.

$$sL \parallel \frac{1}{sC} = \frac{sL \cdot \frac{1}{sC}}{sL + \frac{1}{sC}} = \frac{sL}{1 + s^2LC}$$

$$\frac{V_2}{V_1} = \frac{\frac{sL}{1 + s^2LC}}{R + \frac{sL}{1 + s^2LC}} = \frac{sL}{s^2RLC + sL + R}$$

$$\frac{V_2}{V_1} = \frac{s \cdot \frac{1}{RC}}{s^2 + s \cdot \frac{1}{RC} + \frac{1}{LC}}$$

Comparing this with the given transfer function,

$$2 = \frac{1}{RC}, \quad 6 = \frac{1}{LC}$$

$$\text{If } R = 1 \text{ k}\Omega, \quad C = \frac{1}{2R} = \underline{\underline{500 \mu\text{F}}}$$

$$L = \frac{1}{6C} = \underline{\underline{333.3 \text{ H}}}$$

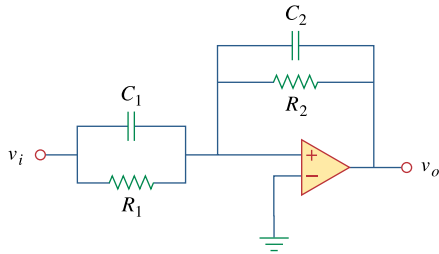
**Chapter 16, Problem 57.**

**ed**

Design an op amp circuit, using Fig. 16.77, that will realize the following transfer function:

$$\frac{V_o(s)}{V_i(s)} = -\frac{s + 1,000}{2(s + 4,000)}$$

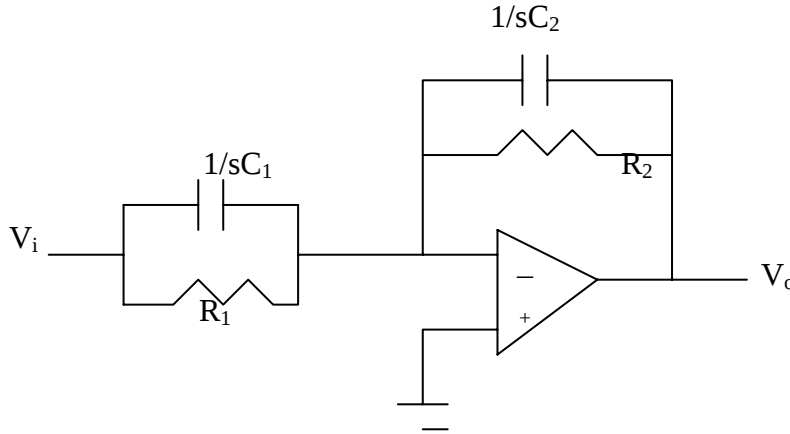
Choose  $C_1 = 10 \mu\text{F}$ ; determine  $R_1$ ,  $R_2$ , and  $C_2$



**Figure 16.77**  
For Prob. 16.57.

### Chapter 16, Solution 57.

The circuit is transformed in the s-domain as shown below.



$$\text{Let } Z_1 = R_1 // \frac{1}{sC_1} = \frac{R_1 \frac{1}{sC_1}}{R_1 + \frac{1}{sC_1}} = \frac{R_1}{1 + sR_1C_1}$$

$$Z_2 = R_2 // \frac{1}{sC_2} = \frac{R_2 \frac{1}{sC_2}}{R_2 + \frac{1}{sC_2}} = \frac{R_2}{1 + sR_2C_2}$$

This is an inverting amplifier.

$$H(s) = \frac{V_o}{V_i} = -\frac{Z_2}{Z_1} = -\frac{\frac{R_2}{1 + sR_2C_2}}{\frac{R_1}{1 + sR_1C_1}} = -\frac{R_2}{R_1} \frac{R_1C_1}{R_2C_2} \left[ \frac{s + \frac{1}{R_1C_1}}{s + \frac{1}{R_2C_2}} \right] = \frac{-C_1}{C_2} \left[ \frac{s + \frac{1}{R_1C_1}}{s + \frac{1}{R_2C_2}} \right]$$

Comparing this with

$$H(s) = -\frac{(s+1000)}{2(s+4000)}$$

we obtain:

$$\frac{C_1}{C_2} = 1/2 \quad \longrightarrow \quad C_2 = 2C_1 = \underline{20\mu F}$$

$$\frac{1}{R_1C_1} = 1000 \quad \longrightarrow \quad R_1 = \frac{1}{1000C_1} = \frac{1}{10^3 \times 10 \times 10^{-6}} = \underline{100\Omega}$$

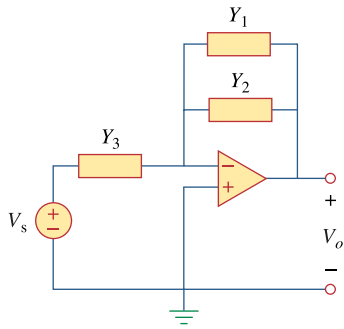
$$\frac{1}{R_2C_2} = 4000 \quad \longrightarrow \quad R_2 = \frac{1}{4000C_2} = \frac{1}{4 \times 10^3 \times 20 \times 10^{-6}} = \underline{12.5\Omega}$$

### Chapter 16, Problem 58.

Realize the transfer function

$$\frac{V_o(s)}{V_s(s)} = \frac{s}{s+10}$$

using the circuit in Fig. 16.78. Let  $Y_1 = sC_1$ ,  $Y_2 = 1/R_1$ ,  $Y_3 = sC_2$ . Choose  $R_1 = 1\text{ k}\Omega$  and determine  $C_1$  and  $C_2$ .



**Figure 16.78**

For Prob. 16.58.

### Chapter 16, Solution 58.

We apply KCL at the noninverting terminal at the op amp.

$$(V_s - 0)Y_3 = (0 - V_o)(Y_1 - Y_2)$$

$$Y_3 V_s = -(Y_1 + Y_2)V_o$$

$$\frac{V_o}{V_s} = \frac{-Y_3}{Y_1 + Y_2}$$

$$\text{Let } Y_1 = sC_1, \quad Y_2 = 1/R_1, \quad Y_3 = sC_2$$

$$\frac{V_o}{V_s} = \frac{-sC_2}{sC_1 + 1/R_1} = \frac{-sC_2/C_1}{s + 1/R_1C_1}$$

Comparing this with the given transfer function,

$$\frac{C_2}{C_1} = 1, \quad \frac{1}{R_1C_1} = 10$$

If  $R_1 = 1\text{ k}\Omega$ ,

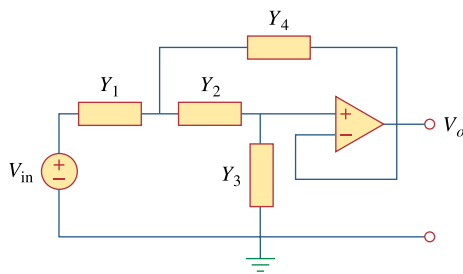
$$C_1 = C_2 = \frac{1}{10^4} = \underline{\underline{100\text{ }\mu\text{F}}}$$

## Chapter 16, Problem 59.

Synthesize the transfer function

$$\frac{V_o(s)}{V_{in}(s)} = \frac{10^6}{s^2 + 100s + 10^6}$$

using the topology of Fig. 16.79. Let  $Y_1 = 1/R_1$ ,  $Y_2 = 1/R_2$ ,  $Y_3 = sC_1$ ,  $Y_4 = sC_2$ . Choose  $R_1 = 1\text{k}\Omega$  and determine  $C_1$ ,  $C_2$ , and  $R_2$ .

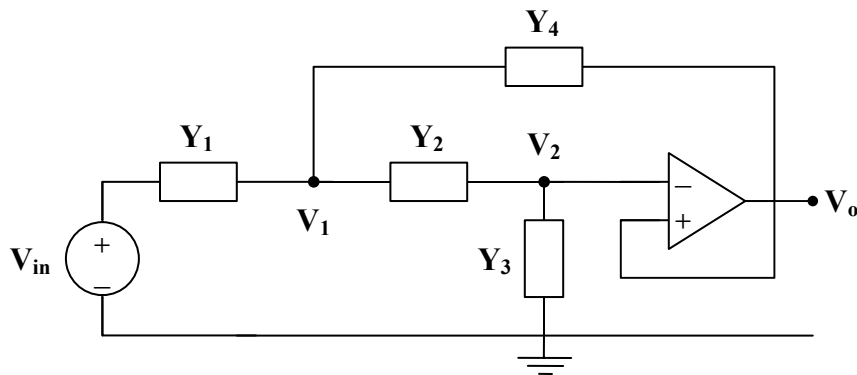


**Figure 16.79**

For Prob. 16.59.

## Chapter 16, Solution 59.

Consider the circuit shown below. We notice that  $V_3 = V_o$  and  $V_2 = V_3 = V_o$ .



At node 1,

$$\begin{aligned}(V_{in} - V_1) Y_1 &= (V_1 - V_o) Y_2 + (V_1 - V_o) Y_4 \\ V_{in} Y_1 &= V_1(Y_1 + Y_2 + Y_4) - V_o(Y_2 + Y_4)\end{aligned}\quad (1)$$

At node 2,

$$\begin{aligned}(V_1 - V_o) Y_2 &= (V_o - 0) Y_3 \\ V_1 Y_2 &= (Y_2 + Y_3) V_o \\ V_1 &= \frac{Y_2 + Y_3}{Y_2} V_o\end{aligned}\quad (2)$$

Substituting (2) into (1),

$$\begin{aligned}V_{in} Y_1 &= \frac{Y_2 + Y_3}{Y_2} \cdot (Y_1 + Y_2 + Y_4) V_o - V_o(Y_2 + Y_4) \\ V_{in} Y_1 Y_2 &= V_o(Y_1 Y_2 + Y_2^2 + Y_2 Y_4 + Y_1 Y_3 + Y_2 Y_3 + Y_3 Y_4 - Y_2^2 - Y_2 Y_4) \\ \frac{V_o}{V_{in}} &= \frac{Y_1 Y_2}{Y_1 Y_2 + Y_1 Y_3 + Y_2 Y_3 + Y_3 Y_4}\end{aligned}$$

$Y_1$  and  $Y_2$  must be resistive, while  $Y_3$  and  $Y_4$  must be capacitive.

$$\text{Let } Y_1 = \frac{1}{R_1}, \quad Y_2 = \frac{1}{R_2}, \quad Y_3 = sC_1, \quad Y_4 = sC_2$$

$$\begin{aligned}\frac{V_o}{V_{in}} &= \frac{\frac{1}{R_1 R_2}}{\frac{1}{R_1 R_2} + \frac{sC_1}{R_1} + \frac{sC_1}{R_2} + s^2 C_1 C_2} \\ \frac{V_o}{V_{in}} &= \frac{\frac{1}{R_1 R_2 C_1 C_2}}{s^2 + s \cdot \left( \frac{R_1 + R_2}{R_1 R_2 C_2} \right) + \frac{1}{R_1 R_2 C_1 C_2}}\end{aligned}$$

Choose  $R_1 = 1 \text{ k}\Omega$ , then

$$\frac{1}{R_1 R_2 C_1 C_2} = 10^6 \quad \text{and} \quad \frac{R_1 + R_2}{R_1 R_2 C_2} = 100$$

We have three equations and four unknowns. Thus, there is a family of solutions. One such solution is

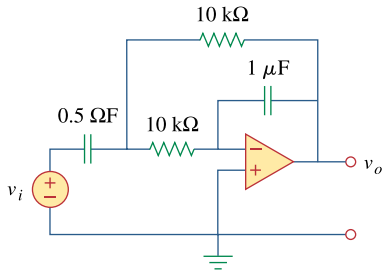
$$R_2 = \underline{\underline{1 \text{ k}\Omega}}, \quad C_1 = \underline{\underline{50 \text{ nF}}}, \quad C_2 = \underline{\underline{20 \text{ }\mu\text{F}}}$$

### Chapter 16, Problem 60.

Obtain the transfer function of the op amp circuit in Fig. 16.80 in the form of

$$\frac{V_o(s)}{V_i(s)} = \frac{as}{s^2 + bs + c}$$

where  $a$ ,  $b$ , and  $c$  are constants. Determine the constants.

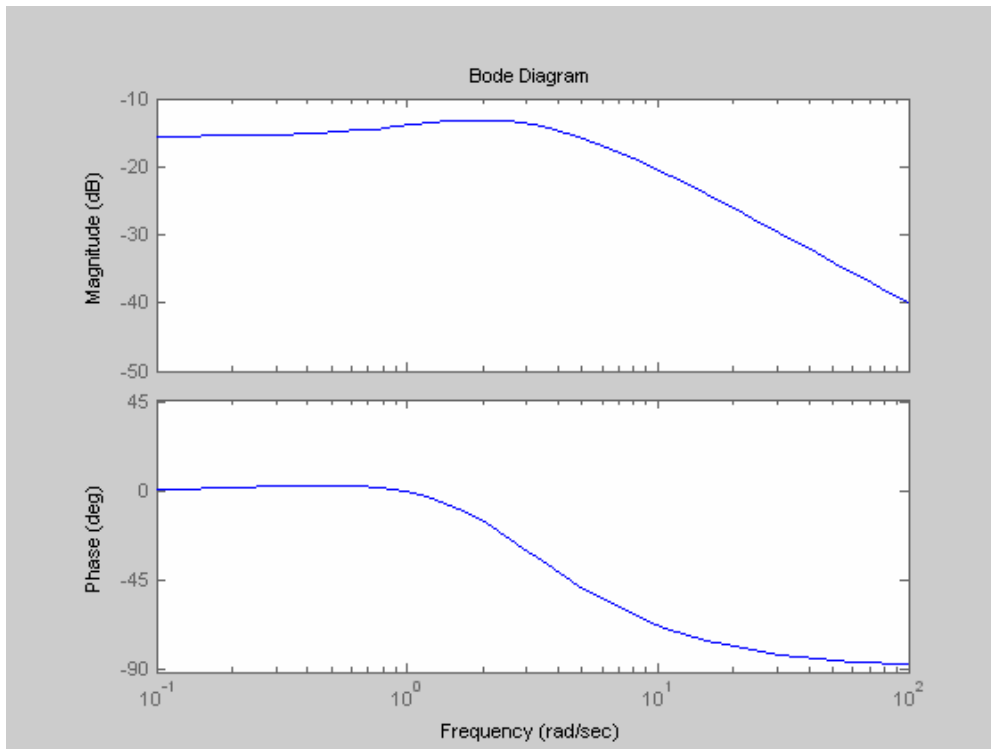


**Figure 16.80**  
For Prob. 16.67.

## Chapter 16, Solution 60.

With the following MATLAB codes, the Bode plots are generated as shown below.

```
num=[1 1];  
den=[1 5 6];  
bode(num,den);
```



### Chapter 16, Problem 61.

A certain network has an input admittance  $Y(s)$ . The admittance has a pole at  $s = -3$ , a zero at  $s = -1$ , and  $Y(\infty) = 0.25 \text{ S}$ .

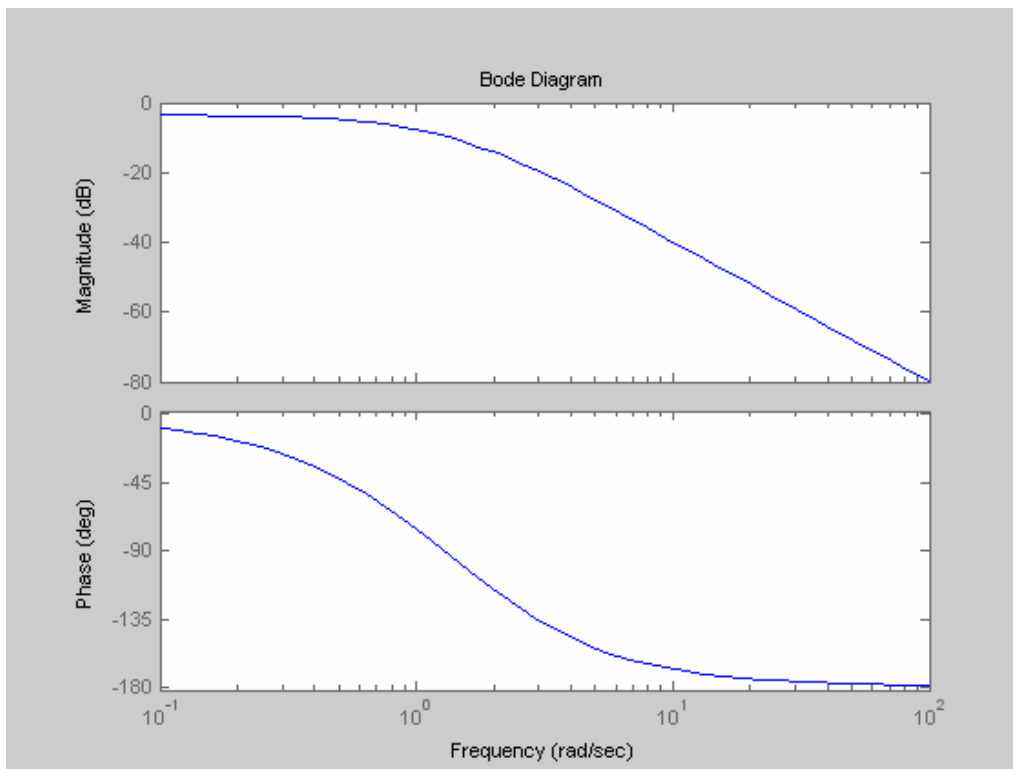
(a) Find  $Y(s)$ .

(b) An 8-V battery is connected to the network via a switch. If the switch is closed at  $t = 0$ , find the current  $i(t)$  through  $Y(s)$  using the Laplace transform.

### Chapter 16, Solution 61.

We use the following codes to obtain the Bode plots below.

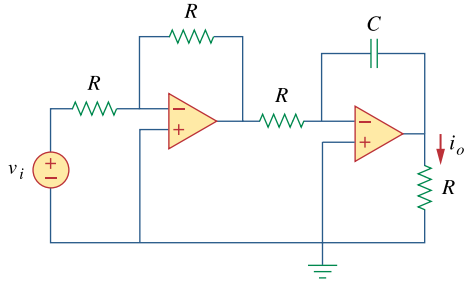
```
num=[1 4];  
den=[1 6 11 6];  
bode(num,den);
```



## Chapter 16, Problem 62.



A gyrator is a device for simulating an inductor in a network. A basic gyrator circuit is shown in Fig. 16.81. By finding  $V_i(s)/I_o(s)$ , show that the inductance produced by the gyrator is  $L = CR^2$ .



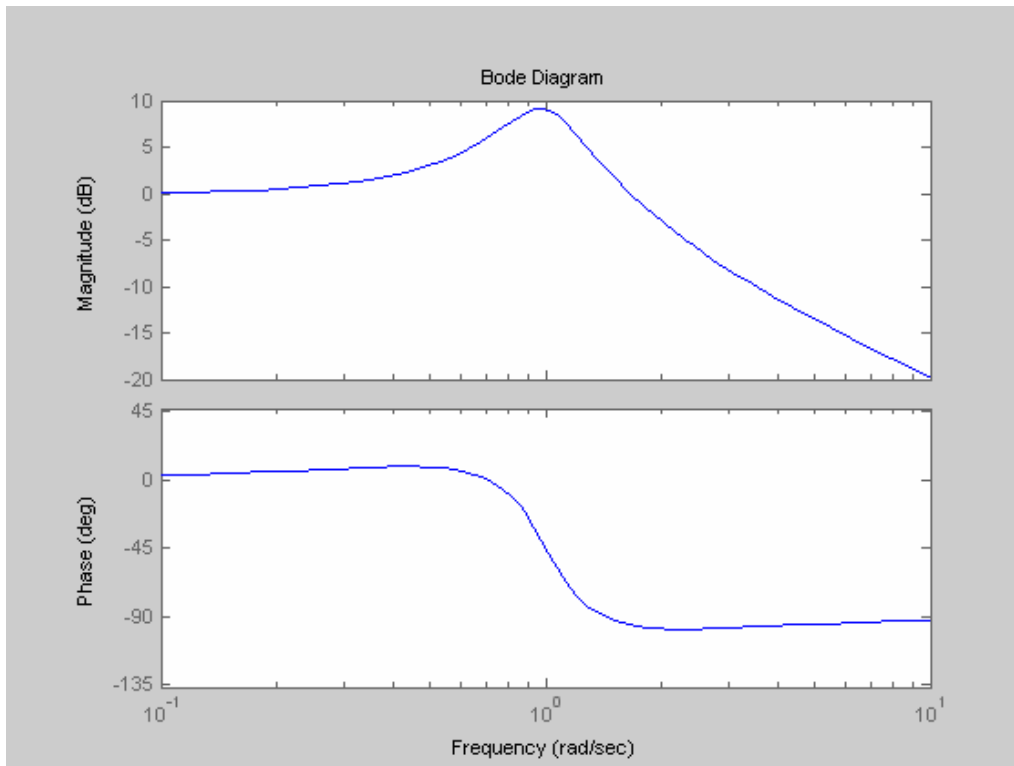
**Figure 16.81**

For Prob. 16.69.

## Chapter 16, Solution 62.

The following codes are used to obtain the Bode plots below.

```
num=[1 1];  
den=[1 0.5 1];  
bode(num,den);
```



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