CHAPTER 2

Circuit Elements

IN THIS CHAPTER:

2.1 Introduction
2.2 Engineering and Linear Models
2.3 Active and Passive Circuit Elements
2.4 Resistors
2.5 Independent Sources
2.6 Voltmeters and Ammeters
2.7 Dependent Sources
2.8 Transducers
2.9 Switches
2.10 How Can We Check . . . ?
2.11 DESIGN EXAMPLE—Temperature Sensor
2.12 Summary
Problems
Design Problems

2.1 Introduction

Not surprisingly, the behavior of an electric circuit depends on the behaviors of the individual circuit elements that comprise the circuit. Of course, different types of circuit elements behave differently. The equations that describe the behaviors of the various types of circuit elements are called the constitutive equations. Frequently, the constitutive equations describe a relationship between the current and voltage of the element. Ohm's law is a well-known example of a constitutive equation.

In this chapter we will investigate the behavior of several common types of circuit element:

- Resistors
- Independent voltage and current sources
- Open circuits and short circuits
- Voltmeters and ammeters
- Dependent sources
- Transducers
- Switches

2.2 Engineering and Linear Models

The art of engineering is to take a bright idea and, using money, materials, knowledgeable people, and a regard for the environment, produce something the buyer wants at an affordable price.

Engineers use *models* to represent the elements of an electric circuit. A model is a description of those properties of a device that we think are important. Frequently the model will consist of an equation relating the element voltage and current. Though the model is different from the electric device, the model can be used in pencil-and-paper calculations that will predict how a circuit comprised of actual devices will operate. Engineers frequently face a trade-off when selecting a model for a device. Simple models are easy to work with but may not be accurate. Accurate models are usually more complicated and harder to use. The conventional wisdom suggests that simple models be used first. The results obtained using the models must be checked to verify that use of these simple models is appropriate. More accurate models are used when necessary.

The idealized models of electric devices are precisely defined. It is important to distinguish between actual devices and their idealized models, which we call circuit elements. The goal of circuit analysis is to predict the quantitative electrical behavior of physical circuits. Its aim is to predict and to explain the terminal voltages and terminal currents of the circuit elements and thus the overall operation of the circuit.

Models of circuit elements can be categorized in a variety of ways. For example, it is important to distinguish linear models from nonlinear models because circuits that consist entirely of linear circuit elements are easier to analyze than circuits that contain some nonlinear elements.

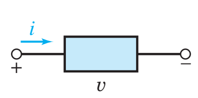


FIGURE 2.2-1 An element with an excitation current i and a response v.

An element or circuit is *linear* if the element's excitation and response satisfy certain properties. Consider the element shown in Figure 2.2-1. Suppose that the excitation is the current i and the response is the voltage v. When the element is subjected to a current i_1, it provides a response v_1. Furthermore, when the element is subjected to a current i_2, it provides a response v_2. For a linear element, it is necessary that the excitation $i_1 + i_2$ result in a response $v_1 + v_2$. This is usually called the *principle of superposition.*

Furthermore, it is necessary that the magnitude scale factor be preserved for a linear element. If the element is subjected to an excitation of ki, where k is a constant multiplier, then it is necessary that the response of a linear device be equal to kv. This is called the *property of homogeneity.* An element is linear if, and only if, the properties of superposition and homogeneity are satisfied for all excitations and responses.

A **linear element** satisfies both the properties of superposition and homogeneity.

Let us restate mathematically the two required properties of a linear circuit, using the arrow notation to imply the transition from excitation to response:

$$i \longrightarrow v$$

Then we may state the two properties required as follows.

Superposition:

$$i_1 \longrightarrow v_1$$
$$i_2 \longrightarrow v_2$$

then $$i_1 + i_2 \longrightarrow v_1 + v_2 \qquad (2.2\text{-}1)$$

Homogeneity:

$$i \longrightarrow v$$

then $$ki \longrightarrow kv \qquad (2.2\text{-}2)$$

A device that does not satisfy either the superposition or the homogeneity principle is said to be nonlinear.

EXAMPLE 2.2-1 *A Linear Device*

Consider the element represented by the relationship between current and voltage as

$$v = Ri$$

Determine whether this device is linear.

Solution

The response to a current i_1 is

$$v_1 = Ri_1$$

The response to a current i_2 is

$$v_2 = Ri_2$$

The sum of these responses is

$$v_1 + v_2 = Ri_1 + Ri_2 = R(i_1 + i_2)$$

Since the sum of the responses to i_1 and i_2 is equal to the response to $i_1 + i_2$, the principle of superposition is satisfied. Next, consider the principle of homogeneity. Since

$$v_1 = Ri_1$$

we have for an excitation $i_2 = ki_1$

$$v_2 = Ri_2 = Rki_1$$

Therefore,

$$v_2 = kv_1$$

satisfies the principle of homogeneity. Because the element satisfies the properties of both superposition and homogeneity, it is linear.

EXAMPLE 2.2-2 *A Nonlinear Device*

Now let us consider an element represented by the relationship between current and voltage:

$$v = i^2$$

Determine whether this device is linear.

Solution

The response to a current i_1 is

$$v_1 = {i_1}^2$$

The response to a current i_2 is

$$v_2 = {i_2}^2$$

The sum of these responses is

$$v_1 + v_2 = {i_1}^2 + {i_2}^2$$

The response to $i_1 + i_2$ is

$$(i_1 + i_2)^2 = {i_1}^2 + 2i_1 i_2 + {i_2}^2$$

Since

$${i_1}^2 + {i_2}^2 \neq (i_1 + i_2)^2$$

the principle of superposition is not satisfied. Therefore, the device is nonlinear.

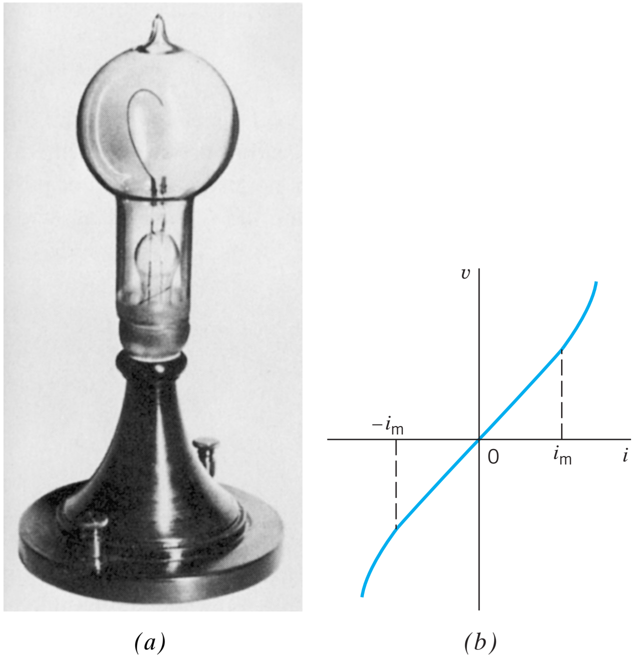


FIGURE 2.2-2
(*a*) An incandescent lamp. (*b*) Voltage–current relationship for an incandescent lamp. The lamp is linear within the range $-i_m < i < i_m$.

The distinguishing feature of an electric circuit component is that its behavior is described in terms of a voltage–current relation. The v–i characteristic may be obtained experimentally or from physical principles. Although no device is exactly linear for all values of current, we often assume a range of linear operation. Edison's first commercially available lamp is shown in Figure 2.2-2*a*. For example, consider the v–i characteristic of the incandescent lamp, shown in Figure 2.2-2*b*. The lamp is essentially linear for the range

$$-i_m < i < i_m$$

That is, i_m indicates the range of current for linearity.

In this book we will be concerned only with linear models of circuit components. In other words, only linear circuits will be considered. Of course, we recognize that these linear models are accurate representations for only some limited range of operation.

Exercise 2.2-1 Consider the circuit element shown in Figure E 2.2-1*a*. A plot of the element voltage, v, versus the element current, i, is shown in Figure E 2.2-1*b*. The plot is a straight line that passes through the origin and has a slope with value m. Consequently, v and i are related by

$$v = mi$$

Show that this device is linear.

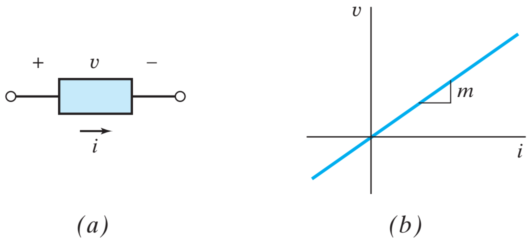


FIGURE E 2.2-1

P1: IML/OVY P2: IML/OVY QC: IML/OVY T1: IML
GTBL002-dorfc02 GTBL002-Dorf-v32 October 13, 2005 21:35

Exercise 2.2-2 Consider the circuit element shown in Figure E 2.2-2*a*. A plot of the element voltage, *v*, versus the element current, *i*, is shown in Figure E 2.2-2*b*. The plot is a straight line that has a *y*-intercept with value *b* and has a slope with value *m*. Consequently, *v* and *i* are related by

$$v = mi + b$$

Show that this device is not linear.

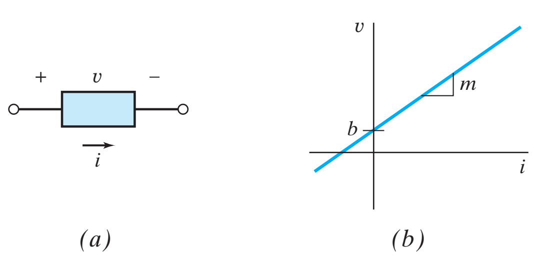


FIGURE E 2.2-2

2.3 Active and Passive Circuit Elements

We may classify circuit elements in two categories, *passive* and *active*, by determining whether they absorb energy or supply energy. An element is said to be passive if the total energy delivered to it from the rest of the circuit is always nonnegative (zero or positive). Then for a passive element, with the current flowing into the + terminal as shown in Figure 2.3-1*a*, this means that

$$w = \int_{-\infty}^{t} vi \, d\tau \geq 0 \tag{2.3-1}$$

for all values of *t*.

> A **passive element** absorbs energy.

An element is said to be *active* if it is capable of delivering energy. Thus, an active element violates Eq. 2.3-1 when it is represented by Figure 2.3-1*a*. In other words, an active element is one that is capable of generating energy. Active elements are potential sources of energy, whereas passive elements are sinks or absorbers of energy. Examples of active elements include batteries and generators. Consider the element shown in Figure 2.3-1*b*. Note that the current flows into the negative terminal and out of the positive terminal. This element is said to be active if

$$w = \int_{-\infty}^{t} vi \, d\tau \geq 0 \tag{2.3-2}$$

for at least one value of *t*.

> An **active element** is capable of supplying energy.

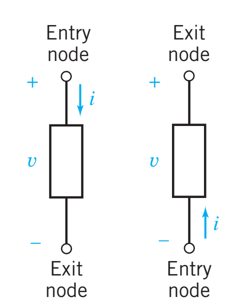


FIGURE 2.3-1
(*a*) The entry node of the current *i* is the positive node of the voltage *v*; (*b*) the entry node of the current *i* is the negative node of the voltage *v*. The current flows from the entry node to the exit node.

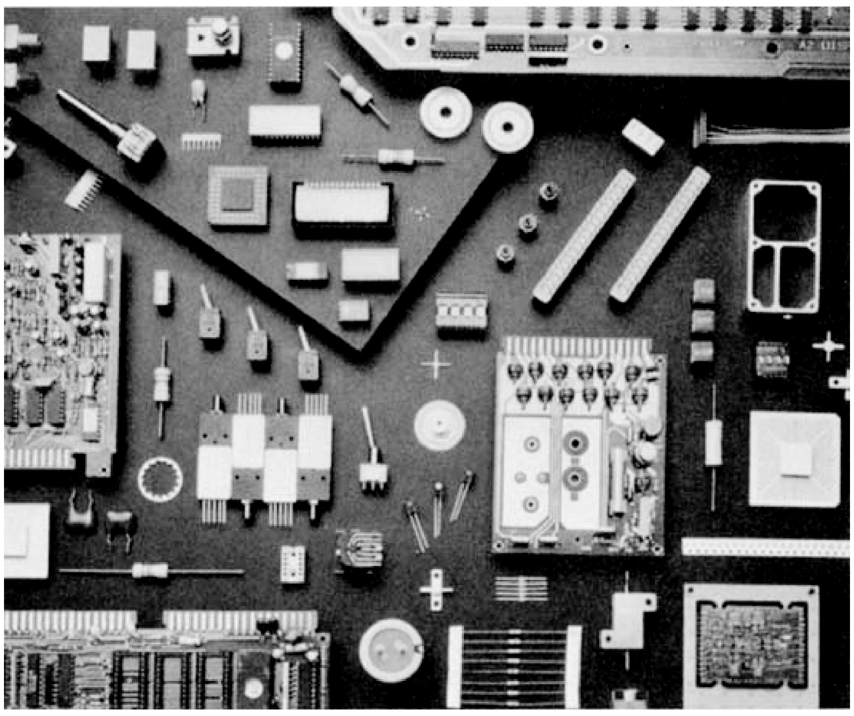


FIGURE 2.3-2 Collection of active and passive circuit elements used for an electrical circuit. Courtesy of Hewlett-Packard Co.

EXAMPLE 2.3-1 *An Active Circuit Element*

A circuit has an element represented by Figure 2.3-1*b* where the current is a constant 5 A and the voltage is a constant 6 V. Find the energy supplied over the time interval 0 to *T*.

Solution

Since the current enters the negative terminal, the energy *supplied by* the element is given by

$$w = \int_0^T (6)(5)d\tau = 30T \text{ J}$$

Thus, the device is a generator or an active element, in this case a dc battery.

A collection of circuit elements or components is shown in Figure 2.3-2. Useful circuits include both active and passive elements that are assembled into a circuit.

2.4 Resistors

The ability of a material to resist the flow of charge is called its *resistivity*, ρ. Materials that are good electrical insulators have a high value of resistivity. Materials that are good conductors of electric current have low values of resistivity. Resistivity values for selected materials are given in Table 2.4-1. Copper is commonly used for wires since it permits current to flow relatively unimpeded. Silicon is commonly used to provide resistance in semiconductor electric circuits. Polystyrene is used as an insulator.

Table 2.4-1 Resistivities of Selected Materials

MATERIAL	RESISTIVITY ρ (ohm·cm)
Polystyrene	1×10^{18}
Silicon	2.3×10^{5}
Carbon	4×10^{-3}
Aluminum	2.7×10^{-6}
Copper	1.7×10^{-6}

Resistance is the physical property of an element or device that impedes the flow of current; it is represented by the symbol R.

Georg Simon Ohm was able to show that the current in a circuit composed of a battery and a conducting wire of uniform cross section could be expressed as

$$i = \frac{Av}{\rho L} \tag{2.4-1}$$

where A is the cross-sectional area, ρ the resistivity, L the length, and v the voltage across the wire element. Ohm, who is shown in Figure 2.4-1, defined the constant resistance R as

$$R = \frac{\rho L}{A} \tag{2.4-2}$$

Ohm's law, which related the voltage and current, was published in 1827 as

$$v = Ri \tag{2.4-3}$$

The unit of resistance R was named the ohm in honor of Ohm and is usually abbreviated by the symbol Ω (capital omega), where 1 Ω = 1 V/A. The resistance of a 10-m length of common TV cable is 2 mΩ.

An element that has a resistance R is called a *resistor*. A resistor is represented by the two-terminal symbol shown in Figure 2.4-2. Ohm's law, Eq. 2.4-3, requires that the i-versus-v relationship be linear. As shown in Figure 2.4-3, a resistor may become nonlinear outside its normal rated range of operation. We will assume that a resistor is linear unless stated otherwise. Thus, we will use a linear model of the resistor as represented by Ohm's law.

In Figure 2.4-4 the element current and element voltage of a resistor are labeled. The relationship between the directions of this current and voltage is important. The voltage direction marks one resistor terminal + and the other −. The current i_a flows from the terminal marked + to the terminal marked −. This relationship between the current and voltage reference directions is a convention called the passive convention. Ohm's law states that when the element voltage

FIGURE 2.4-1 Georg Simon Ohm (1787–1854), who determined Ohm's law in 1827. The ohm was chosen as the unit of electrical resistance in his honor.

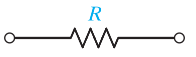


FIGURE 2.4-2 Symbol for a resistor having a resistance of R ohms.

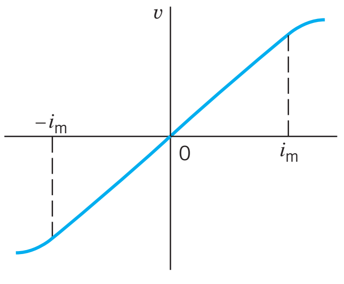


FIGURE 2.4-3 A resistor operating within its specified current range, $\pm i_m$, can be modeled by Ohm's law.

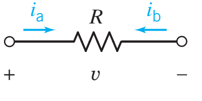


FIGURE 2.4-4 A resistor with element current and element voltage.

FIGURE 2.4-5
(*a*) Wirewound resistor with an adjustable center tap. (*b*) Wirewound resistor with a fixed tap. Courtesy of Dale Electronics.

and the element current adhere to the passive convention, then

$$v = Ri_a \tag{2.4-4}$$

Consider Figure 2.4-4. The element currents i_a and i_b are the same except for the assigned direction, so

$$i_a = -i_b$$

The element current i_a and the element voltage v adhere to the passive convention,

$$v = Ri_a$$

Replacing i_a by $-i_b$ gives

$$v = -Ri_b$$

There is a minus sign in this equation because the element current i_b and the element voltage v do not adhere to the passive convention. We must pay attention to the current direction so that we don't overlook this minus sign.

Ohm's law, Eq. 2.4-4, can also be written as

$$i = Gv \tag{2.4-5}$$

where G denotes the *conductance* in siemens (S) and is the reciprocal of R; that is, $G = 1/R$. Many engineers denote the units of conductance as mhos with the symbol ℧, which is an inverted omega (mho is *ohm* spelled backward). However, we will use SI units and retain siemens as the units for conductance.

Most discrete resistors fall into one of four basic categories: carbon composition, carbon film, metal film, or wirewound. Carbon composition resistors have been in use for nearly 100 years and are still popular. Carbon film resistors have supplanted carbon composition resistors for many general-purpose uses because of their lower cost and better tolerances. Two wirewound resistors are shown in Figure 2.4-5.

Thick-film resistors, as shown in Figure 2.4-6, are used in circuits because of their low cost and small size. General-purpose resistors are available in standard values for tolerances of 2, 5, 10, and 20 percent. Carbon composition resistors and some wirewounds have a color code

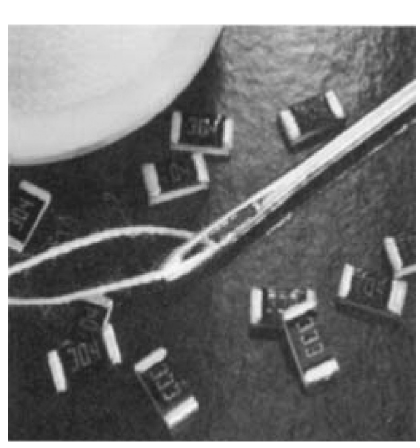


FIGURE 2.4-6
Small thick-film resistor chips used for miniaturized circuits. Courtesy of Corning Electronics.

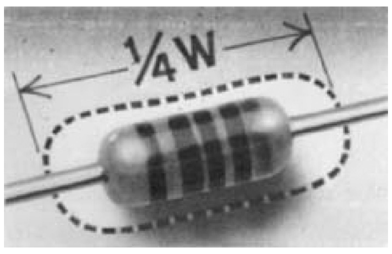


FIGURE 2.4-7
A 1/4-watt metal film resistor. The body of the resistor is 6 mm long. Courtesy of Dale Electronics.

with three to five bands. A color code is a system of standard colors adopted for identification of the resistance of resistors. Figure 2.4-7 shows a metal film resistor with its color bands. This is a 1/4-watt resistor, implying that it should be operated at or below 1/4 watt of power delivered to it. The normal range of resistors is from less than 1 ohm to 10 megohms. Typical values of some commercially available resistors are given in Appendix E.

The power delivered to a resistor (when the passive convention is used) is

$$p = vi = v\left(\frac{v}{R}\right) = \frac{v^2}{R} \tag{2.4-6}$$

Alternatively, since $v = iR$, we can write the equation for power as

$$p = vi = (iR)i = i^2 R \tag{2.4-7}$$

Thus, the power is expressed as a nonlinear function of the current i through the resistor or of the voltage v across it.

Recall the definition of a passive element as one for which the energy absorbed is always nonnegative. The equation for energy delivered to a resistor is

$$w = \int_{-\infty}^{t} p\,d\tau = \int_{-\infty}^{t} i^2 R\,d\tau \tag{2.4-8}$$

Since i^2 is always positive, the energy is always positive and the resistor is a passive element.

Resistance is a measure of an element's ability to dissipate power irreversibly.

EXAMPLE 2.4-1 *Power Dissipated by a Resistor*

Let us devise a model for a car battery when the lights are left on and the engine is off. We have all experienced or seen a car parked with its lights on. If we leave the car for a period, the battery will "run down" or "go dead." An auto battery is a 12-V constant-voltage source, and the lightbulb can be modeled by a resistor of 6 ohms. The circuit is shown in Figure 2.4-8. Let us find the current i, the power p, and the energy supplied by the battery for a four-hour period.

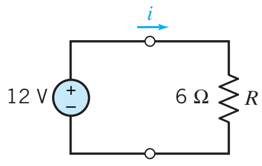


FIGURE 2.4-8
Model of a car battery and the headlight lamp.

Solution

According to Ohm's law, Eq. 2.4-4, we have

$$v = Ri$$

Since $v = 12$ V and $R = 6\Omega$, we have $i = 2$ A.

In order to find the power delivered by the battery, we use

$$p = vi = 12(2) = 24 \text{ W}$$

Finally, the energy delivered in the four-hour period is

$$w = \int_0^t p\,d\tau = 24t = 24(60 \times 60 \times 4) = 3.46 \times 10^5 \text{ J}$$

Since the battery has a finite amount of stored energy, it will deliver this energy and eventually be unable to deliver further energy without recharging. We then say the battery is run down or dead until recharged. A typical auto battery may store 10^6 J in a fully charged condition.

Exercise 2.4-1 Find the power absorbed by a 100-ohm resistor when it is connected directly across a constant 10-V source.

Answer: 1-W

Exercise 2.4-2 A voltage source $v = 10 \cos t$ V is connected across a resistor of 10 ohms. Find the power delivered to the resistor.

Answer: $10 \cos^2 t$ W

2.5 Independent Sources

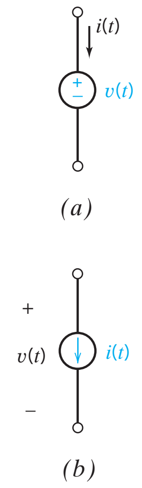


FIGURE 2.5-1
(*a*) Voltage source.
(*b*) Current source.

Some devices are intended to supply energy to a circuit. These devices are called sources. Sources are categorized as being one of two types: voltage sources and current sources. Figure 2.5-1*a* shows the symbol that is used to represent a voltage source. The voltage of a voltage source is specified, but the current is determined by the rest of the circuit. A voltage source is described by specifying the function $v(t)$, for example,

$$v(t) = 12 \cos 1000t \quad \text{or} \quad v(t) = 9 \quad \text{or} \quad v(t) = 12 - 2t$$

An active two-terminal element that supplies energy to a circuit is a *source* of energy. An *independent voltage source* provides a specified voltage independent of the current through it and is independent of any other circuit variable.

> A **source** is a voltage or current generator capable of supplying energy to a circuit.

An *independent current source* provides a current independent of the voltage across the source element and is independent of any other circuit variable. Thus, when we say a source is independent, we mean it is independent of any other voltage or current in the circuit.

> An **independent source** is a voltage or current generator not dependent on other circuit variables.

Suppose the voltage source is a battery and

$$v(t) = 9 \text{ volts}$$

The voltage of this battery is known to be 9 volts regardless of the circuit in which the battery is used. In contrast, the current of the voltage source is not known and depends on the circuit

P1: IML/OVY P2: IML/OVY QC: IML/OVY T1: IML
GTBL002-dorfc02 GTBL002-Dorf-v32 October 13, 2005 21:35

in which the source is used. The current could be 6 amps when the voltage source is connected to one circuit and 6 milliamps when the voltage source is connected to another circuit.

Figure 2.5-1*b* shows the symbol that is used to represent a current source. The current of a current source is specified, but the voltage is determined by the rest of the circuit. A current source is described by specifying the function $i(t)$, for example,

$$i(t) = 6 \sin 500t \quad \text{or} \quad i(t) = -0.25 \quad \text{or} \quad i(t) = t + 8$$

A current source specified by $i(t) = -0.25$ milliamps will have a current of -0.25 milliamps in any circuit in which it is used. The voltage across this current source will depend on the particular circuit.

The preceding paragraphs have ignored some complexities in order to give a simple description of the way sources work. The voltage across a 9-volt battery may not actually be 9 volts. This voltage depends on the age of the battery, the temperature, variations in manufacturing, and the battery current. It is useful to make a distinction between real sources, such as batteries, and the simple voltage and current sources described in the preceding paragraphs. It would be *ideal* if the real sources worked like these simple sources. Indeed, the word *ideal* is used to make this distinction. The simple sources described in the previous paragraph are called the *ideal voltage source* and the *ideal current source.*

> The voltage of an **ideal voltage source** is given to be a specified function, say $v(t)$. The current is determined by the rest of the circuit.
>
> The current of an **ideal current source** is given to be a specified function, say $i(t)$. The voltage is determined by the rest of the circuit.
>
> An **ideal source** is a voltage or a current generator independent of the current through the voltage source or the voltage across the current source.

EXAMPLE 2.5-1 *A Battery Modeled as a Voltage Source*

Consider the plight of the engineer who needs to analyze a circuit containing a 9-volt battery. Is it really necessary for this engineer to include the dependence of battery voltage on the age of the battery, the temperature, variations in manufacturing, and the battery current in this analysis? Hopefully not. We expect the battery to act enough like an ideal 9-volt voltage source that the differences can be ignored. In this case it is said that the battery is *modeled* as an ideal voltage source.

To be specific, consider a battery specified by the plot of voltage versus current shown in Figure 2.5-2*a*. This plot indicates that the battery voltage will be $v = 9$ volts when $i \le 10$ milliamps. As the current increases above 10 milliamps, the voltage decreases from 9 volts. When $i \le 10$ milliamps, the dependence of the battery voltage on the battery current can be ignored and the battery can be modeled as an independent voltage source.

Suppose a resistor is connected across the terminals of the battery as shown in Figure 2.5-2*b*. The battery current will be

$$i = \frac{v}{R} \tag{2.5-1}$$

The relationship between v and i shown in Figure 2.5-2*a* complicates this equation. This complication can be safely ignored when $i \le 10$ milliamps. When the battery is modeled as an ideal 9-volt voltage source, the voltage source current is given by

$$i = \frac{9}{R} \tag{2.5-2}$$

The distinction between these two equations is important. Eq. 2.5-1, involving the $v{-}i$ relationship shown in Figure 2.5-2*a*, is more accurate but also more complicated. Equation 2.5-2 is simpler but may be inaccurate.

Suppose that $R = 1000$ ohms. Equation 2.5-2 gives the current of the ideal voltage source:

$$i = \frac{9}{1000} = 9 \text{ mA} \tag{2.5-3}$$

Since this current is less than 10 milliamps, the ideal voltage source is a good model for the battery and it is reasonable to expect that the battery current is 9 milliamps.

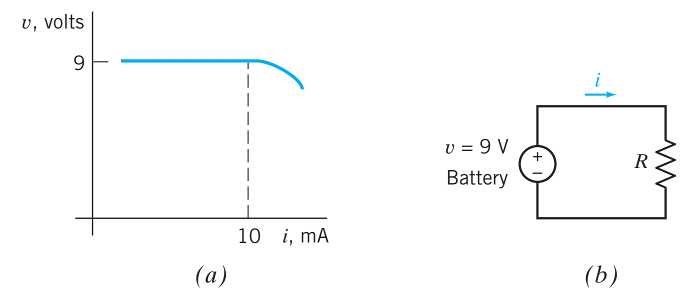


FIGURE 2.5-2 (*a*) A plot of battery voltage versus battery current. (*b*) The battery is modeled as an independent voltage source.

Suppose instead that $R = 600$ ohms. Once again, Eq. 2.5-2 gives the current of the ideal voltage source:

$$i = \frac{9}{600} = 15 \text{ mA} \tag{2.5-4}$$

Since this current is greater than 10 milliamps, the ideal voltage source is not a good model for the battery. In this case it is reasonable to expect that the battery current is different from the current for the ideal voltage source.

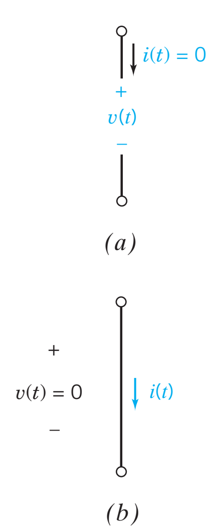


FIGURE 2.5-3
(*a*) Open circuit. (*b*) Short circuit.

Engineers frequently face a trade-off when selecting a model for a device. Simple models are easy to work with but may not be accurate. Accurate models are usually more complicated and harder to use. The conventional wisdom suggests that simple models be used first. The results obtained using the models must be checked to verify that use of these simple models is appropriate. More accurate models are used when necessary.

The short circuit and open circuit are special cases of ideal sources. A *short circuit* is an ideal voltage source having $v(t) = 0$. The current in a short circuit is determined by the rest of the circuit. An *open circuit* is an ideal current source having $i(t) = 0$. The voltage across an open circuit is determined by the rest of the circuit. Figure 2.5-3 shows the symbols used to represent the short circuit and the open circuit. Notice that the power absorbed by each of these devices is zero.

Open and short circuits can be added to a circuit without disturbing the branch currents and voltages of all the other devices in the circuit. Figure 2.6-3 shows how this can be done. Figure 2.6-3*a* shows an example circuit. In Figure 2.6-3*b* an open circuit and a short circuit have been added to this example circuit. The open circuit was connected between two nodes of the original circuit. In contrast, the short circuit was added by cutting a wire and inserting the short circuit. Adding open circuits and short circuits to a network in this way does not change the network.

Open circuits and short circuits can also be described as special cases of resistors. A resistor with resistance $R = 0$ ($G = \infty$) is a short circuit. A resistor with conductance $G = 0$ ($R = \infty$) is an open circuit.

2.6 Voltmeters and Ammeters

Measurements of dc current and voltage are made with direct-reading (analog) or digital meters, as shown in Figure 2.6-1. A direct-reading meter has an indicating pointer whose angular deflection depends on the magnitude of the variable it is measuring. A digital meter displays a set of digits indicating the measured variable value.

To measure a voltage or current, a meter is connected to a circuit using terminals called probes. These probes are color coded to indicate the reference direction of the variable being measured. Frequently, meter probes are colored red and black. An ideal voltmeter measures the voltage from the red to the black probe. The red terminal is the positive terminal, and the black terminal is the negative terminal (see Figure 2.6-2*b*).

An ideal ammeter measures the current flowing through its terminals, as shown in Figure 2.6-2*a* and has zero voltage, v_m, across its terminals. An ideal voltmeter measures the voltage across its terminals, as shown in Figure 2.6-2*b,* and has terminal current, i_m, equal to zero. Practical measuring instruments only approximate the ideal conditions. For a practical ammeter the voltage across its terminals is usually negligibly small. Similarly, the current into a voltmeter is usually negligible.

Ideal voltmeters act like open circuits, and ideal ammeters act like short circuits. In other words, the model of an ideal voltmeter is an open circuit, and the model of an ideal ammeter is a short circuit. Consider the circuit of Figure 2.6-3*a* and then add an open circuit with a

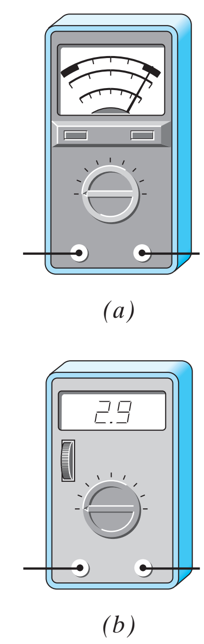


FIGURE 2.6-1 (*a*) A direct-reading (analog) meter. (*b*) A digital meter.

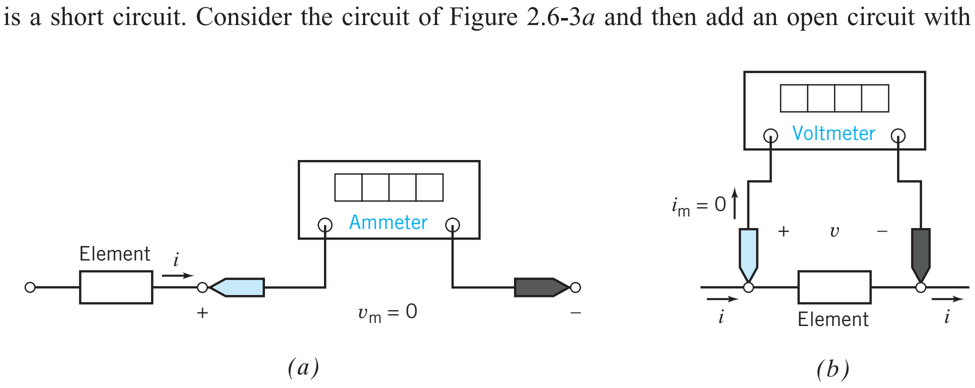
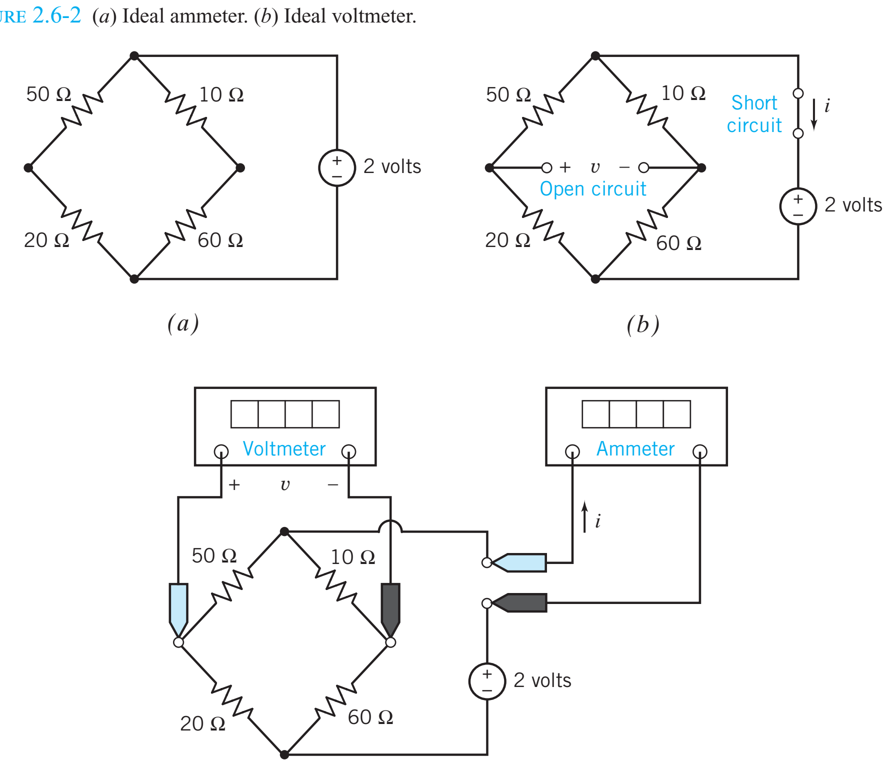


FIGURE 2.6-2 (*a*) Ideal ammeter. (*b*) Ideal voltmeter.



(*c*)

FIGURE 2.6-3 (*a*) An example circuit, (*b*) plus an open circuit and a short circuit. (*c*) The open circuit is replaced by a voltmeter, and the short circuit is replaced by an ammeter. All resistances are in ohms.

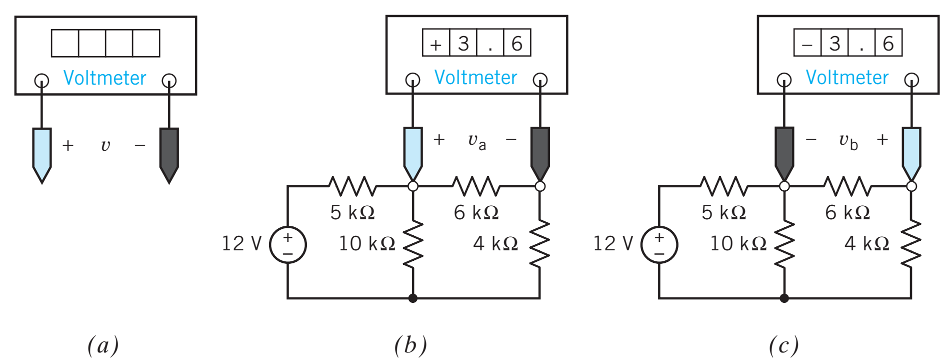


FIGURE 2.6-4 (*a*) The correspondence between the color-coded probes of the voltmeter and the reference direction of the measured voltage. In (*b*) the + sign of v_a is on the left, while in (*c*) the + sign of v_b is on the right.

voltage *v* and a short circuit with a current *i* as shown in Figure 2.6-3*b*. In Figure 2.6-3*c* the open circuit has been replaced by a voltmeter, and the short circuit has been replaced by an ammeter. The voltmeter will measure the voltage labeled *v* in Figure 2.6-3*b* while the ammeter will measure the current labeled *i*. Notice that Figure 2.6-3*c* could be obtained from Figure 2.6-3*a* by adding a voltmeter and an ammeter. Ideally, adding the voltmeter and ammeter in this way does not disturb the circuit. One more interpretation of Figure 2.6-3 is useful. Figure 2.6-3*b* could be formed from Figure 2.6-3*c* by replacing the voltmeter and the ammeter by their (ideal) models.

The reference direction is an important part of an element voltage or element current. Figures 2.6-4 and 2.6-5 show that attention must be paid to reference directions when measuring an element voltage or element current. Figure 2.6-4*a* shows a voltmeter. Voltmeters have two color-coded probes. This color coding indicates the reference direction of the voltage being measured. In Figures 2.6-4*b* and 2.6-4*c* the voltmeter is used to measure the voltage across the 6-kΩ resistor. When the voltmeter is connected to the circuit as shown in Figure 2.6-4*b*, the voltmeter measures v_a, with + on the left, at the red probe. When the voltmeter probes are interchanged as shown in Figure 2.6-4*c*, the voltmeter measures v_b, with + on the right, again at the red probe. Note $v_b = -v_a$.

Figure 2.6-5*a* shows an ammeter. Ammeters have two color-coded probes. This color coding indicates the reference direction of the current being measured. In Figures 2.6-5*b,c* the ammeter is used to measure the current in the 6-kΩ resistor. When the ammeter is connected to the circuit as shown in Figure 2.6-5*b*, the ammeter measures i_a, directed from the red probe toward the

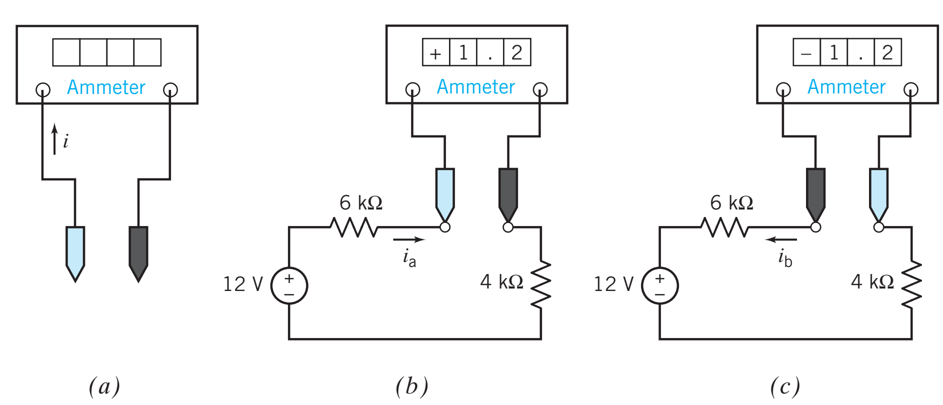


FIGURE 2.6-5 (*a*) The correspondence between the color-coded probes of the ammeter and the reference direction of the measured current. In (*b*) the current i_a is directed to the right, while in (*c*) the current i_b is directed to the left.

black probe. When the ammeter probes are interchanged as shown in Figure 2.6-5*c,* the ammeter measures i_b, again directed from the red probe toward the black probe. Note $i_b = -i_a$.

2.7 Dependent Sources

Dependent sources model the situation in which the voltage or current of one circuit element is proportional to the voltage or current of the second circuit element. (In contrast, a resistor is a circuit element in which the voltage of the element is proportional to the current in the *same* element.) Dependent sources are used to model electronic devices such as transistors and amplifiers. For example, the output voltage of an amplifier is proportional to the input voltage of that amplifier, so an amplifier can be modeled as a dependent source.

Figure 2.7-1*a* shows a circuit that includes a dependent source. The diamond symbol represents a dependent source. The plus and minus signs inside the diamond identify the dependent source as a voltage source and indicate the reference polarity of the element voltage. The label "5*i*" represents the voltage of this dependent source. This voltage is a product of two factors, 5 and *i.* The second factor, *i,* indicates that the voltage of this dependent source is controlled by the current, *i,* in the 18-Ω resistor. The first factor, 5, is the gain of this dependent source. The gain of this dependent source is the ratio of the controlled voltage, 5*i,* to the controlling current, *i.* This gain has units of V/A or Ω. Because this dependent source is a voltage source and because a current controls the voltage, the dependent source is called a current-controlled voltage source (CCVS).

Figure 2.7-1*b* shows the circuit from Figure 2.7-1*a* using a different point of view. In Figure 2.7-1*b* a short circuit has been inserted in series with the 18-Ω resistor. Now we think of the controlling current *i* as the current in a short circuit rather than the current in the 18-Ω resistor itself. In this way, we can always treat the controlling current of a dependent source as the current in a short circuit. We will use this second point of view to categorize dependent sources in this section.

Figure 2.7-1*c* shows a circuit that includes a dependent source, represented by the diamond symbol. The arrow inside the diamond identifies the dependent source as a current source

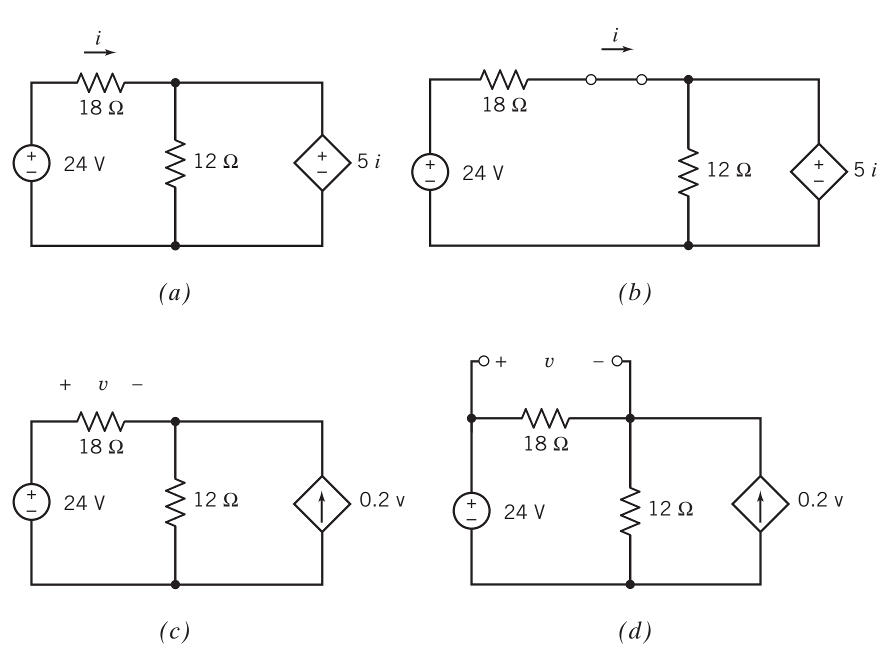


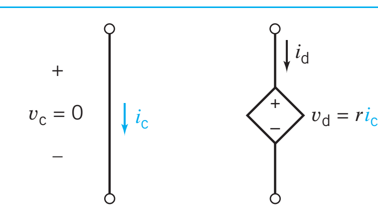
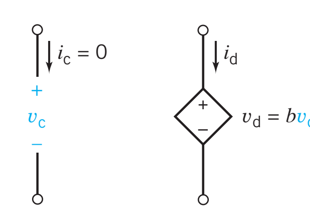
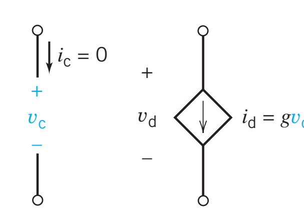
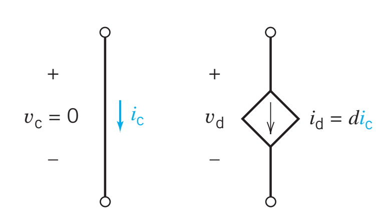
FIGURE 2.7-1 The controlling current of a dependent source shown as (*a*) the current in an element and as (*b*) the current in a short circuit in series with that element. The controlling voltage of a dependent source shown as (*c*) the voltage across an element and as (*d*) the voltage across an open circuit in parallel with that element.

and indicates the reference direction of the element current. The label "0.2*v*" represents the current of this dependent source. This current is a product of two factors, 0.2 and *v*. The second factor, *v*, indicates that the current of this dependent source is controlled by the voltage, *v*, across the 18-Ω resistor. The first factor, 0.2, is the gain of this dependent source. The gain of this dependent source is the ratio of the controlled current, 0.2*v*, to the controlling voltage, *v*. This gain has units of A/V. Because this dependent source is a current source and because a voltage controls the current, the dependent source is called a voltage-controlled current source (VCCS).

Figure 2.7-1*d* shows the circuit from Figure 2.7-1*c* using a different point of view. In Figure 2.7-1*d* an open circuit has been added in parallel with the 18-Ω resistor. Now we think of the controlling voltage *v* as the voltage across an open circuit, rather than the voltage across the 18-Ω resistor itself. In this way, we can always treat the controlling voltage of a dependent source as the voltage across an open circuit.

We are now ready to categorize dependent source. Each dependent source consists of two parts: the controlling part and the controlled part. The controlling part is either an open circuit or a short circuit. The controlled part is either a voltage source or a current source. There are four types of dependent source that correspond to the four ways of choosing a controlling part and a controlled part. These four dependent sources are called the voltage-controlled voltage source (VCVS), current-controlled voltage source (CCVS), voltage-controlled current source (VCCS), and current-controlled current source (CCCS). The symbols that represent dependent sources are shown in Table 2.7-1.

Table 2.7-1 Dependent Sources

DESCRIPTION	SYMBOL
Current-Controlled Voltage Source (CCVS) *r* is the gain of the CCVS. *r* has units of volts/ampere.	
Voltage-Controlled Voltage Source (VCVS) *b* is the gain of the VCVS. *b* has units of volts/volt.	
Voltage-Controlled Current Source (VCCS) *g* is the gain of the VCCS. *g* has units of amperes/volt.	
Current-Controlled Current Source (CCCS) *d* is the gain of the CCCS. *d* has units of amperes/ampere.	

Consider the CCVS shown in Table 2.7-1. The controlling element is a short circuit. The element current and voltage of the controlling element are denoted as i_c and v_c. The voltage across a short circuit is zero, so $v_c = 0$. The short-circuit current, i_c, is the controlling signal of this dependent source. The controlled element is a voltage source. The element current and voltage of the controlled element are denoted as i_d and v_d. The voltage v_d is controlled by i_c:

$$v_d = r i_c$$

The constant r is called the gain of the CCVS. The current i_d, like the current in any voltage source, is determined by the rest of the circuit.

Next consider the VCVS shown in Table 2.7-1. The controlling element is an open-circuit. The current in an open circuit is zero, so $i_c = 0$. The open-circuit voltage, v_c, is the controlling signal of this dependent source. The controlled element is a voltage source. The voltage v_d is controlled by v_c:

$$v_d = b v_c$$

The constant b is called the gain of the VCVS. The current i_d is determined by the rest of the circuit.

The controlling element of the VCCS shown in Table 2.7-1 is an open circuit. The current in this open circuit is $i_c = 0$. The open-circuit voltage, v_c, is the controlling signal of this dependent source. The controlled element is a current source. The current i_d is controlled by v_c:

$$i_d = g v_c$$

The constant g is called the gain of the VCCS. The voltage v_d, like the voltage across any current source, is determined by the rest of the circuit.

The controlling element of the CCCS shown in Table 2.7-1 is a short circuit. The voltage across this open circuit is $v_c = 0$. The short-circuit current, i_c, is the controlling signal of this dependent source. The controlled element is a current source. The current i_d is controlled by i_c:

$$i_d = d i_c$$

The constant d is called the gain of the CCCS. The voltage v_d, like the voltage across any current source, is determined by the rest of the circuit.

Figure 2.7-2 illustrates the use of dependent sources to model electronic devices. In certain circumstances, the behavior of the transistor shown in Figure 2.7-2*a* can be represented using

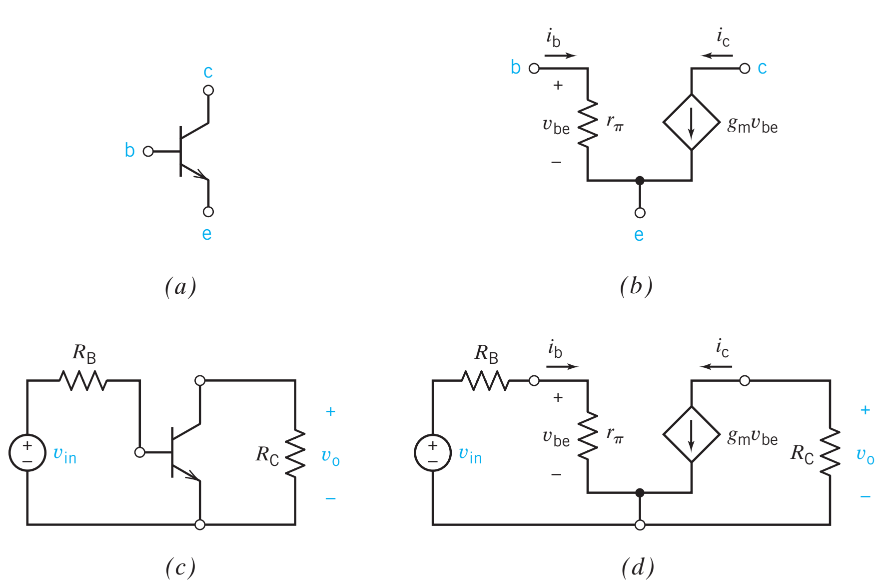


FIGURE 2.7-2 (*a*) A symbol for a transistor. (*b*) A model of the transistor. (*c*) A transistor amplifier. (*d*) A model of the transistor amplifier.

the model shown in Figure 2.7-2*b*. This model consists of a dependent source and a resistor. The controlling element of the dependent source is an open circuit connected across the resistor. The controlling voltage is v_{be}. The gain of the dependent source is g_m. The dependent source is used in this model to represent a property of the transistor, namely, that the current i_c is proportional to the voltage v_{be}, that is,

$$i_c = g_m v_{be}$$

where g_m has units of amperes/volt. Figures 2.7-2*c,d* illustrate the utility of this model. Figure 2.7-2*d* is obtained from Figure 2.7-2*c* by replacing the transistor by the transistor model.

EXAMPLE 2.7-1 *Power and Dependent Sources*

Determine the power absorbed by the VCVS in Figure 2.7-3.

Solution

The VCVS consists of an open circuit and a controlled-voltage source. There is no current in the open circuit, so no power is absorbed by the open circuit.

The voltage, v_c, across the open circuit is the controlling signal of the VCVS. The voltmeter measures v_c to be

$$v_c = 2 \text{ V}$$

The voltage of the controlled voltage source is

$$v_d = 2\, v_c = 4 \text{ V}$$

The ammeter measures the current in the controlled voltage source to be

$$i_d = 1.5 \text{ A}$$

The element current, i_d, and voltage, v_d, adhere to the passive convention. Therefore

$$p = i_d v_d = (1.5)(4) = 6 \text{ W}$$

is the power absorbed by the VCVS.

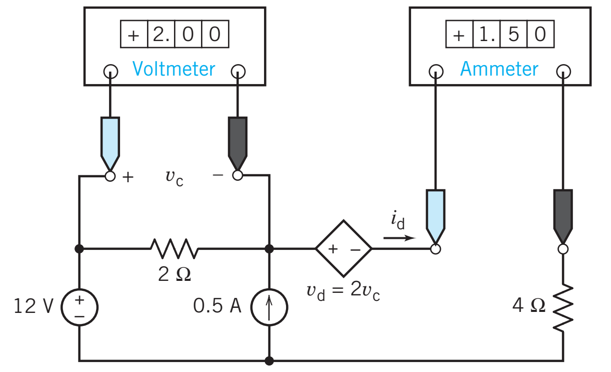


FIGURE 2.7-3 A circuit containing a VCVS. The meters indicate that the voltage of the controlling element is $v_c = 2.0$ volts and that the current of the controlled element is $i_d = 1.5$ amperes.

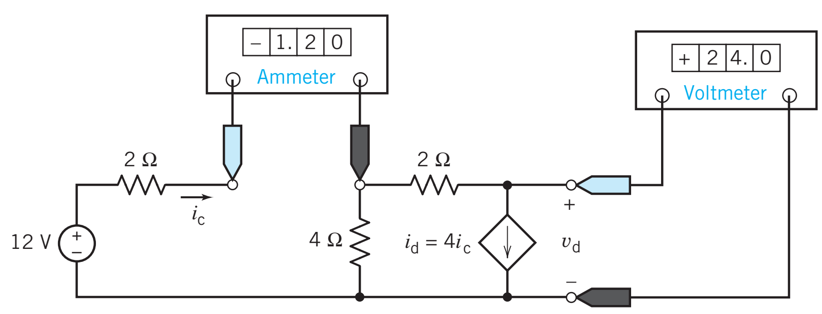


FIGURE E 2.7-1 A circuit containing a CCCS. The meters indicate that the current of the controlling element is $i_c = -1.2$ amperes and that the voltage of the controlled element is $v_d = 24$ volts.

Exercise 2.7-1 Find the power absorbed by the CCCS in Figure E 2.7-1.

Hint: The controlling element of this dependent source is a short circuit. The voltage across a short circuit is zero. Hence, the power absorbed by the controlling element is zero. How much power is absorbed by the controlled element?

Answer: −115.2 watts are absorbed by the CCCS. (The CCCS delivers +115.2 watts to the rest of the circuit.)

2.8 Transducers

Transducers are devices that convert physical quantities to electrical quantities. This section describes two transducers: potentiometers and temperature sensors. Potentiometers convert position to resistance, and temperature sensors convert temperature to current.

Figure 2.8-1*a* shows the symbol for the potentiometer. The potentiometer is a resistor having a third contact, called the wiper, that slides along the resistor. Two parameters, R_p and *a*, are needed to describe the potentiometer. The parameter R_p specifies the potentiometer resistance ($R_p > 0$). The parameter a represents the wiper position and takes values in the range $0 \le a \le 1$. The values $a = 0$ and $a = 1$ correspond to the extreme positions of the wiper.

Figure 2.8-1*b* shows a model for the potentiometer that consists of two resistors. The resistances of these resistors depend on the potentiometer parameters R_p and *a*.

Frequently, the position of the wiper corresponds to the angular position of a shaft connected to the potentiometer. Suppose θ is the angle in degrees and $0 \le \theta \le 360$. Then

$$a = \frac{\theta}{360}$$

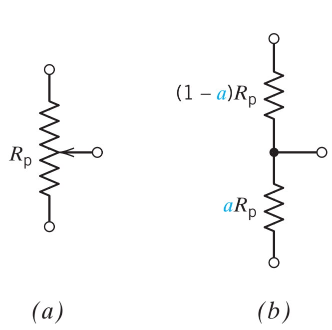


FIGURE 2.8-1
(*a*) The symbol and (*b*) a model for the potentiometer.

EXAMPLE 2.8-1 *Potentiometer Circuit*

Figure 2.8-2*a* shows a circuit in which the voltage measured by the meter gives an indication of the angular position of the shaft. In Figure 2.8-2*b* the current source, the potentiometer, and the voltmeter have been replaced by models of these devices. Analysis of Figure 2.8-2*b* yields

$$v_m = R_p I a = \frac{R_p I}{360}\theta$$

Solving for the angle gives

$$\theta = \frac{360}{R_p I} v_m$$

Suppose $R_p = 10\,k\Omega$ and $I = 1$ mA. An angle of 163° would cause an output of $v_m = 4.53$ V. A meter reading of 7.83 V would indicate that $\theta = 282°$.

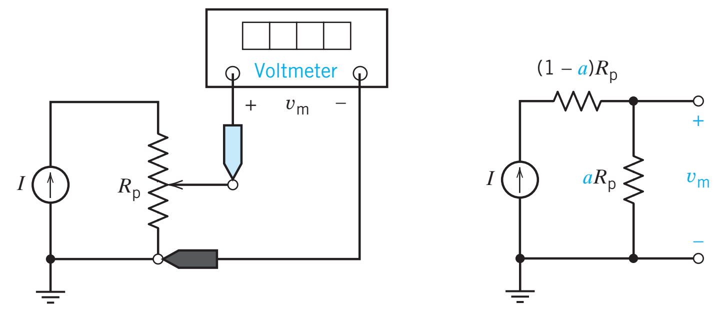


FIGURE 2.8-2 (*a*) A circuit containing a potentiometer. (*b*) An equivalent circuit containing a model of the potentiometer.

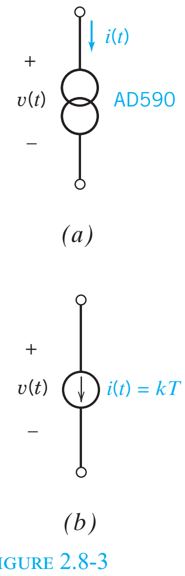


FIGURE 2.8-3 (*a*) The symbol and (*b*) a model for the temperature sensor.

Temperature sensors, such as the AD590 manufactured by Analog Devices, are current sources having current proportional to absolute temperature. Figure 2.8-3*a* shows the symbol used to represent the temperature sensor. Figure 2.8-3*b* shows the circuit model of the temperature sensor. In order for the temperature sensor to operate properly, the branch voltage v must satisfy the condition

$$4 \text{ volts} \le v \le 30 \text{ volts}$$

When this condition is satisfied, the current, i, in microamps, is numerically equal to the temperature T, in degrees Kelvin. The phrase "numerically equal" indicates that the current and temperature have the same value but different units. This relationship can be expressed as

$$i = k \cdot T$$

where $k = 1\frac{\mu A}{°K}$, a constant associated with the sensor.

Exercise 2.8-1 For the potentiometer circuit of Figure 2.8-2, calculate the meter voltage, v_m, when $\theta = 45°$, $R_p = 20\,k\Omega$, and $I = 2$ mA.

Answer: $v_m = 5$ V

Exercise 2.8-2 The voltage and current of an AD590 temperature sensor of Figure 2.8-3 are 10 V and 280 μA, respectively. Determine the measured temperature.

Answer: $T = 280°\text{K}$, or approximately 6.85°C

2.9 Switches

Switches have two distinct states: open and closed. Ideally, a switch acts as a short circuit when it is closed and as an open circuit when it is open.

Figures 2.9-1 and 2.9-2 show several types of switches. In each case, the time when the switch changes state is indicated. Consider first the single-pole, single-throw (SPST) switches shown in Figure 2.9-1. The switch in Figure 2.9-1*a* is initially open. This switch changes state, becoming closed, at time $t = 0$ s. When this switch is modeled as an ideal switch, it is treated like an open circuit when $t < 0$ s and like a short circuit when $t > 0$ s. The ideal switch changes state instantaneously. The switch in Figure 2.9-1*b* is initially closed. This switch changes state, becoming open, at time $t = 0$ s.

Next, consider the single-pole, double-throw (SPDT) switch shown in Figure 2.9-2*a.* This SPDT switch acts like two SPST switches, one between terminals c and a, another between terminals c and b. Before $t = 0$ s, the switch between c and a is closed and the switch between c and b is open. At $t = 0$ s both switches change state; that is, the switch between a and c opens, and the switch between c and b closes. Once again, the ideal switches are modeled as open circuits when they are open and as short circuits when they are closed.

In some applications, it makes a difference whether the switch between c and b closes before, or after, the switch between c and a opens. Different symbols are used to represent these two types of single-pole, double-throw switch. The break-before-make switch is manufactured so that the switch between c and b closes after the switch between c and a opens. The symbol for the break-before-make switch is shown in Figure 2.9-2*a.* The make-before-break switch is manufactured so that the switch between c and b closes before the switch between c and a opens. The symbol for the make-before-break switch is shown in Figure 2.9-2*b.* Remember: the switch transition from terminal a to terminal b is assumed to take place instantaneously. This instantaneous transition is an accurate model when the actual make-before-break transition is very fast compared to the circuit time response.

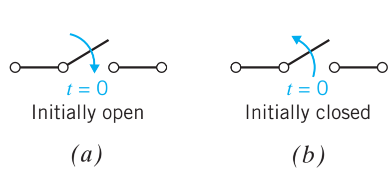


FIGURE 2.9-1
SPST switches. (*a*) Initially open and (*b*) initially closed.

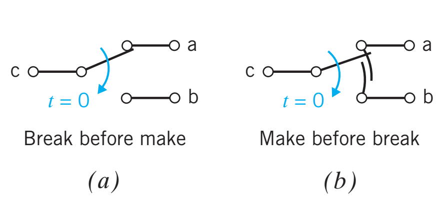


FIGURE 2.9-2
SPDT switches. (*a*) Break before make and (*b*) make before break.

P1: IML/OVY P2: IML/OVY QC: IML/OVY T1: IML
GTBL002-dorfc02 GTBL002-Dorf-v32 October 21, 2005 21:17

EXAMPLE 2.9-1 *Switches*

Figure 2.9-3 illustrates the use of open and short circuits for modeling ideal switches. In Figure 2.9-3*a* a circuit containing three switches is shown. In Figure 2.9-3*b* the circuit is shown as it would be modeled before $t = 0$ s. The two single-pole, single-throw switches change state at time $t = 0$ s. Figure 2.9-3*c* shows the circuit as it would be modeled when the time is between 0 s and 2 s. The single-pole, double-throw switch changes state at time $t = 2$ s. Figure 2.9-3*d* shows the circuit as it would be modeled after 2 s.

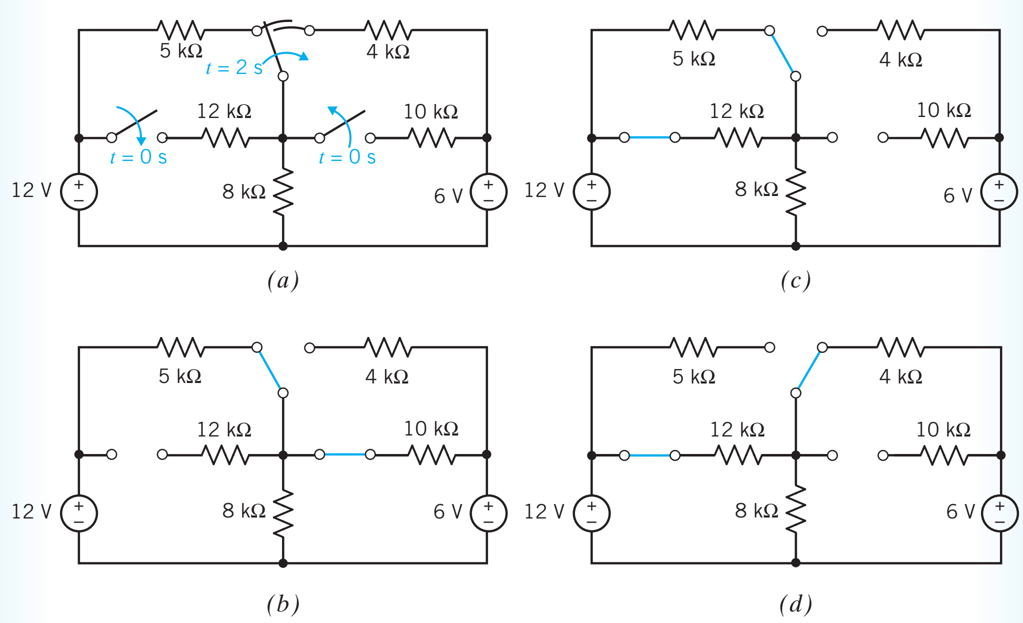


FIGURE 2.9-3 (*a*) A circuit containing several switches. (*b*) The equivalent circuit for $t \leq 0$ s. (*c*) The equivalent circuit for $0 < t < 2$ s. (*d*) The equivalent circuit for $t > 2$ s.

Exercise 2.9-1 What is the value of the current i in Figure E 2.9-1 at time $t = 4$ s?

Answer: $i = 0$ amperes at $t = 4$ s (both switches are open).

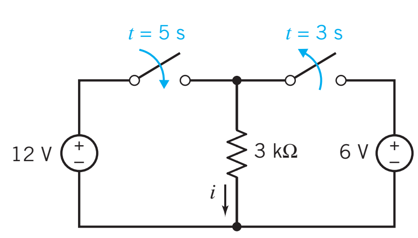


FIGURE E 2.9-1
A circuit with two SPST switches.

Exercise 2.9-2 What is the value of the voltage v in Figure E 2.9-2 at time $t = 4$ s? At $t = 6$ s?

Answer: $v = 6$ volts at $t = 4$ s, and $v = 0$ volts at $t = 6$ s.

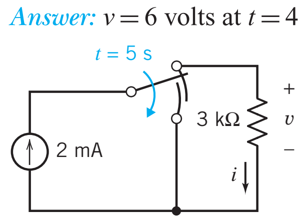


FIGURE E 2.9-2
A circuit with a make-before-break SPDT switch.

2.10 How Can We Check . . . ?

Engineers are frequently called upon to check that a solution to a problem is indeed correct. For example, proposed solutions to design problems must be checked to confirm that all of the specifications have been satisfied. In addition, computer output must be reviewed to guard against data-entry errors, and claims made by vendors must be examined critically.

Engineering students are also asked to check the correctness of their work. For example, occasionally just a little time remains at the end of an exam. It is useful to be able to quickly identify those solutions that need more work.

The following example illustrates techniques useful for checking the solutions of the sort of problem discussed in this chapter.

EXAMPLE 2.10-1 *How Can We Check Voltage and Current Values?*

The meters in the circuit of Figure 2.10-1 indicate that $v_1 = -4$ V, $v_2 = 8$ V and that $i = 1$ A. **How can we check** that the values of v_1, v_2, and i have been measured correctly? Let's check the values of v_1, v_2, and i in two ways:

(a) Verify that the given values satisfy Ohm's law for both resistors.

(b) Verify that the power supplied by the voltage source is equal to the power absorbed by the resistors.

Solution

(a) Consider the 8-Ω resistor. The current i flows through this resistor from top to bottom. Thus the current i and the voltage v_2 adhere to the passive convention. Therefore, Ohm's law requires that $v_2 = 8i$. The values $v_2 = 8$ V and $i = 1$ A satisfy this equation.

Next, consider the 4-Ω resistor. The current i flows through this resistor from left to right. Thus the current i and the voltage v_1 do not adhere to the passive convention. Therefore, Ohm's law requires that $v_1 = 4(-i)$. The values $v_1 = -4$ V and $i = 1$ A satisfy this equation.

Thus, Ohm's law is satisfied.

(b) The current i flows through the voltage source from bottom to top. Thus the current i and the voltage 12 V do not adhere to the passive convention. Therefore, $12i = 12(1) = 12$ W is the power supplied by the voltage source. The power absorbed by the 4-Ω resistor is $4i^2 = 4(1^2) = 4$ W, and the power absorbed by the 8-Ω resistor is $8i^2 = 8(1^2) = 8$ W. The power supplied by the voltage source is indeed equal to the power absorbed by the resistors.

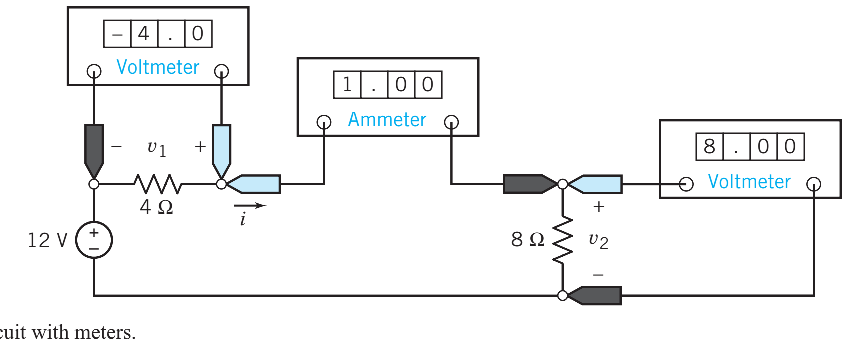


FIGURE 2.10-1 A circuit with meters.

2.11 Design Example

Temperature Sensor

Currents can be measured easily using ammeters. A temperature sensor, such as Analog Devices' AD590, can be used to measure temperature by converting temperature to current. Figure 2.11-1 shows a symbol used to represent a temperature sensor. In order for this sensor to operate properly, the voltage v must satisfy the condition

$$4 \text{ volts } \leq v \leq 30 \text{ volts}$$

When this condition is satisfied, the current i, in μA, is numerically equal to the temperature T, in °K. The phrase *numerically equal* indicates that the two variables have the same value but different units.

$$i = k \cdot T \quad \text{where} \quad k = 1 \frac{\mu \text{A}}{^\circ\text{K}}$$

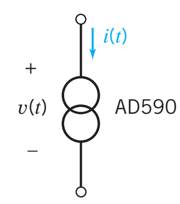


FIGURE 2.11-1
A temperature sensor.

The goal is to design a circuit using the AD590 to measure the temperature of a container of water. In addition to the AD590 and an ammeter, several power supplies and an assortment of standard 2 percent resistors are available. The power supplies are voltage sources. Power supplies having voltages of 10, 12, 15, 18, or 24 volts are available.

Describe the Situation and the Assumptions

In order for the temperature transducer to operate properly, its element voltage must be between 4 volts and 30 volts. The power supplies and resistors will be used to establish this voltage. An ammeter will be used to measure the current in the temperature transducer.

The circuit must be able to measure temperatures in the range from 0°C to 100°C since water is a liquid at these temperatures. Recall that the temperature in °C is equal to the temperature in °K minus 273°.

State the Goal

Use the power supplies and resistors to cause the voltage, v, of the temperature transducer to be between 4 volts and 30 volts.

Use an ammeter to measure the current, i, in the temperature transducer.

Generate a Plan

Model the power supply as an ideal voltage source and the temperature transducer as an ideal current source. The circuit shown in Figure 2.11-2*a* causes the voltage across the

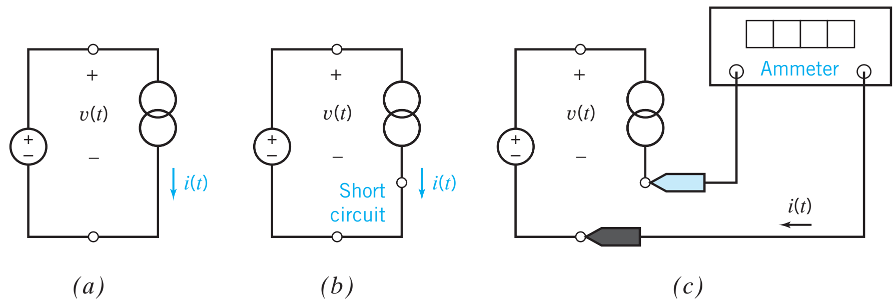


FIGURE 2.11-2 (*a*) Measuring temperature with a temperature sensor. (*b*) Adding a short circuit. (*c*) Replacing the short circuit by an ammeter.

temperature transducer to be equal to the power supply voltage. Since all of the available power supplies have voltages between 4 volts and 30 volts, any one of the power supplies can be used. Notice that the resistors are not needed.

In Figure 2.11-2*b* a short circuit has been added in a way that does not disturb the network. In Figure 2.11-2*c* this short circuit has been replaced with an (ideal) ammeter. Since the ammeter will measure the current in the temperature transducer, the ammeter reading will be numerically equal to the temperature in °K.

Although any of the available power supplies is adequate to meet the specifications, there may still be an advantage to choosing a particular power supply. For example, it is reasonable to choose the power supply that causes the transducer to absorb as little power as possible.

Act on the Plan

The power absorbed by the transducer is

$$p = v \cdot i$$

where v is the power supply voltage. Choosing v as small as possible, 10 volts in this case, makes the power absorbed by the temperature transducer as small as possible. Figure 2.11-3*a* shows the final design. Figure 2.11-3*b* shows a graph that can be used to find the temperature corresponding to any ammeter current.

Verify the Proposed Solution

Let's try an example. Suppose the temperature of the water is 80.6°F. This temperature is equal to 27°C or 300°K. The current in the temperature sensor will be

$$i = \left(1 \frac{\mu\text{A}}{^\circ\text{K}}\right) 300^\circ\text{K} = 300\ \mu\text{A}$$

Next, suppose that the ammeter in Figure 2.11-3*a* reads 300 μA. A sensor current of 300 μA corresponds to a temperature of

$$T = \frac{300\mu\text{A}}{1 \frac{\mu\text{A}}{^\circ\text{K}}} = 300^\circ\text{K} = 27^\circ\text{C} = 80.6^\circ\text{F}$$

The graph in Figure 2.11-3*b* indicates that a sensor current of 300 μA does correspond to a temperature of 27°C.

This example shows that the circuit is working properly.

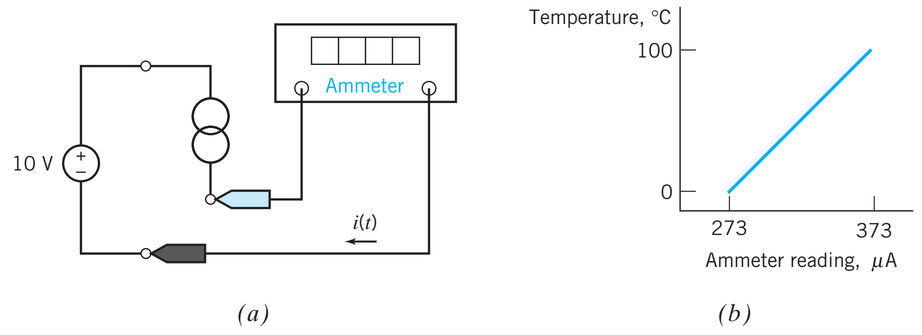


FIGURE 2.11-3 (*a*) Final design of a circuit that measures temperature with a temperature sensor. (*b*) Graph of temperature versus ammeter current.

2.12 Summary

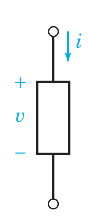
- The engineer uses models, called circuit elements, to represent the devices that make up a circuit. In this book, we consider only linear elements or linear models of devices. A device is linear if it satisfies the properties of both superposition and homogeneity.
- The relationship between the reference directions of the current and voltage of a circuit element is important. The voltage polarity marks one terminal + and the other −. The element voltage and current adhere to the passive convention if the current is directed from the terminal marked + to the terminal marked −.
- Resistors are widely used as circuit elements. When the resistor voltage and current adhere to the passive convention, resistors obey Ohm's law; the voltage across the terminals of the resistor is related to the current into the positive terminal as $v = Ri$. The power delivered to a resistance is $p = i^2R = v^2/R$ watts.
- An independent source provides a current or a voltage independent of other circuit variables. The voltage of an independent voltage source is specified, but the current is not. Conversely, the current of an independent current source is specified while the voltage is not. The voltages of independent voltage sources and currents of independent current sources are frequently used as the inputs to electric circuits.
- A dependent source provides a current (or a voltage) that is dependent on another variable elsewhere in the circuit. The constitutive equations of dependent sources are summarized in Table 2.7-1.
- The **short circuit** and **open circuit** are special cases of independent sources. A **short circuit** is an ideal voltage source having $v(t) = 0$. The current in a short circuit is determined by the rest of the circuit. An **open circuit** is an ideal current source having $i(t) = 0$. The voltage across an open circuit is determined by the rest of the circuit. Open circuits and short circuits can also be described as special cases of resistors. A resistor with resistance $R = 0$ ($G = \infty$) is a short circuit. A resistor with conductance $G = 0$ ($R = \infty$) is an open circuit.
- An ideal ammeter measures the current flowing through its terminals and has zero voltage across its terminals. An ideal voltmeter measures the voltage across its terminals and has terminal current equal to zero. Ideal voltmeters act like open circuits, and ideal ammeters act like short circuits.
- Transducers are devices that convert physical quantities, such as rotational position, to an electrical quantity such as voltage. In this chapter we describe two transducers: potentiometers and temperature sensors.
- Switches are widely used in circuits to connect and disconnect elements and circuits. They can also be used to create discontinuous voltages or currents.

Problems

Section 2.2 Engineering and Linear Models

P 2.2-1 An element has voltage v and current i as shown in Figure P 2.2-1*a*. Values of the current i and corresponding voltage v have been tabulated as shown in Figure P 2.2-1*b*. Determine if the element is linear.

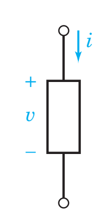
Hint: Plot the data. We expect the data points to lie on a straight line. Obtain a linear model of the element by representing that straight line by an equation.



v, V	i, A
−3	−3
−4	−2
0	0
12	2
32	4
60	6

FIGURE P 2.2-1 (*a*) (*b*)

P 2.2-2 A linear element has voltage v and current i as shown in Figure P 2.2-2*a*. Values of the current i and corresponding voltage v have been tabulated as shown in Figure P 2.2-2*b*. Represent the element by an equation that expresses v as a function of i. This equation is a model of the element. (a) Verify that the model is linear. (b) Use the model to predict the value of v corresponding to a current of $i = 40$ mA. (c) Use the model to predict the value of i corresponding to a voltage of $v = 4$ V.



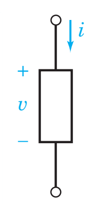
v, V	i, A
−3.6	−30
2.4	20
6.0	50

FIGURE P 2.2-2 (*a*) (*b*)

P 2.2-3 A linear element has voltage v and current i as shown in Figure P 2.2-3*a*. Values of the current i and corresponding voltage v have been tabulated as shown in Figure P 2.2-3*b*. Represent the element by an equation that expresses v as a function of i. This equation is a model of the element. (a) Verify that the model is linear. (b) Use the model to predict the value of v corresponding to a current of $i = 4$ mA. (c) Use the model to predict the value of i corresponding to a voltage of $v = 12$ V.

Hint: Plot the data. We expect the data points to lie on a straight line. Obtain a linear model of the element by representing that straight line by an equation.

P1: IML/OVY P2: IML/OVY QC: IML/OVY T1: IML
GTBL002-dorfc02 GTBL002-Dorf-v32 October 13, 2005 21:35



v, V	i, mA
3.078	12
5.13	20
12.825	50

FIGURE P 2.2-3 (a) (b)

P 2.2-4 An element is represented by the relation between current and voltage as

$$v = 3i + 5$$

Determine whether the element is linear.

P 2.2-5 The circuit shown in Figure P 2.2-5 consists of a current source, a resistor, and element A. Consider three cases.

(a) When element A is a 40-Ω resistor, described by $i = v / 40$, then the circuit is represented by

$$0.4 = \frac{v}{10} + \frac{v}{40}$$

Determine the values of v and i. Notice that the above equation has a unique solution.

(b) When element A is a nonlinear resistor described by $i = v^2 / 2$, then the circuit is represented by

$$0.4 = \frac{v}{10} + \frac{v^2}{2}$$

Determine the values of v and i. In this case there are two solutions of the above equation. Nonlinear circuits exhibit more complicated behavior than linear circuits.

(c) When element A is a nonlinear resistor described by $i = 0.8 + \frac{v^2}{2}$, then the circuit is described by

$$0.4 = \frac{v}{10} + 0.8 + \frac{v^2}{2}$$

Show that this equation has no solution. This result usually indicates a modeling problem. At least one of the three elements in the circuit has not been modeled accurately.

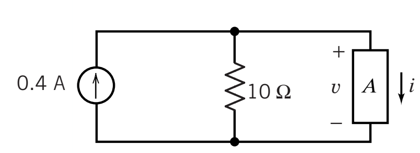


FIGURE P 2.2-5

Section 2.4 Resistors

P 2.4-1 A current source and a resistor are connected in series in the circuit shown in Figure P 2.4-1. Elements connected in series have the same current, so $i = i_s$ in this circuit. Suppose that $i_s = 3$ A and $R = 7$ Ω. Calculate the voltage v across the resistor and the power absorbed by the resistor.
Answer: $v = 21$ V and the resistor absorbs 63 W.

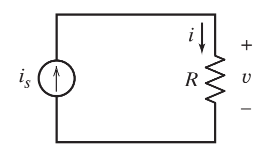


FIGURE P 2.4-1

P 2.4-2 A current source and a resistor are connected in series in the circuit shown in Figure P 2.4-1. Elements connected in series have the same current, so $i = i_s$ in this circuit. Suppose that $i = 3$ mA and $v = 24$ V. Calculate the resistance R and the power absorbed by the resistor.
Answer: $R = 8$ kΩ and the resistor absorbs 72 mW.

P 2.4-3 A voltage source and a resistor are connected in parallel in the circuit shown in Figure P 2.4-3. Elements connected in parallel have the same voltage, so $v = v_s$ in this circuit. Suppose that $v_s = 10$ V and $R = 5$ Ω. Calculate the current i in the resistor and the power absorbed by the resistor.
Answer: $i = 2$ A and the resistor absorbs 20 W.

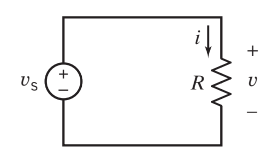


FIGURE P 2.4-3

P 2.4-4 A voltage source and a resistor are connected in parallel in the circuit shown in Figure P 2.4-3. Elements connected in parallel have the same voltage, so $v = v_s$ in this circuit. Suppose that $v_s = 24$ V and $i = 2$ A. Calculate the resistance R and the power absorbed by the resistor.
Answer: $R = 12$ Ω and the resistor absorbs 48 W.

P 2.4-5 A voltage source and two resistors are connected in parallel in the circuit shown in Figure P 2.4-5. Elements connected in parallel have the same voltage, so $v_1 = v_s$ and $v_2 = v_s$ in this circuit. Suppose that $v_s = 150$ V, $R_1 = 50$ Ω, and $R_2 = 25$ Ω. Calculate the current in each resistor and the power absorbed by each resistor.
Hint: Notice the reference directions of the resistor currents.
Answer: $i_1 = 3$ A and $i_2 = -6$ A. R_1 absorbs 450 W and R_2 absorbs 900 W.

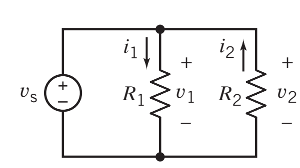


FIGURE P 2.4-5

P 2.4-6 A current source and two resistors are connected in series in the circuit shown in Figure P 2.4-6. Elements connected in series have the same current, so $i_1 = i_s$ and $i_2 = i_s$ in this circuit. Suppose that $i_s = 2$ A, $R_1 = 4$ Ω, and $R_2 = 8$ Ω. Calculate the voltage across each resistor and the power absorbed by each resistor.
Hint: Notice the reference directions of the resistor voltages.
Answer: $v_1 = -8$ V and $v_2 = 16$ V. R_1 absorbs 16 W and R_2 absorbs 32 W.

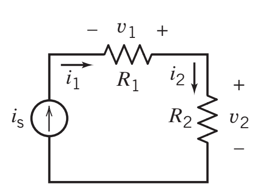


FIGURE P 2.4-6

P 2.4-7 An electric heater is connected to a constant 250-V source and absorbs 1000 W. Subsequently, this heater is connected to a constant 210-V source. What power does it absorb from the 210-V source? What is the resistance of the heater?
Hint: Model the electric heater as a resistor.

P 2.4-8 The portable lighting equipment for a mine is located 100 meters from its dc supply source. The mine lights use a total of 5 kW and operate at 120 V dc. Determine the required cross-sectional area of the copper wires used to connect the source to the mine lights if we require that the power lost in the copper wires be less than or equal to 5 percent of the power required by the mine lights.
Hint: Model both the lighting equipment and the wire as resistors.

***P 2.4-9** The resistance of a practical resistor depends on the nominal resistance and the resistance tolerance as follows:

$$R_{\text{nom}}\left(1-\frac{t}{100}\right) \le R \le R_{\text{nom}}\left(1+\frac{t}{100}\right)$$

where R_{nom} is the nominal resistance and t is the resistance tolerance expressed as a percentage. For example, a 100-Ω, 2 percent resistor will have a resistance given by

$$98\ \Omega \le R \le 102\ \Omega$$

The circuit shown in Figure P 2.4-9 has one input, v_s, and one output, v_o. The gain of this circuit is given by

$$\text{gain} = \frac{v_o}{v_s} = \frac{R_2}{R_1 + R_2}$$

Determine the range of possible values of the gain when R_1 is the resistance of a 100-Ω, 2 percent resistor and R_2 is the resistance of a 400-Ω, 5 percent resistor. Express the gain in terms of a nominal gain and a gain tolerance.

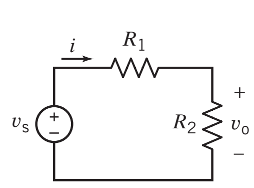
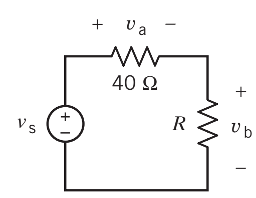


FIGURE P 2.4-9

P 2.4-10 The voltage source shown in Figure P 2.4-10 is an adjustable dc voltage source. In other words, the voltage v_s is a constant voltage, but the value of that constant can be adjusted. The tabulated data were collected as follows. The voltage, v_s, was set to some value, and the voltages across the resistor, v_a and v_b, were measured and recorded. Next, the value of v_s was changed, and the voltages across the resistors were measured again and recorded. This procedure was repeated several times. (The values of v_s were not recorded.) Determine the value of the resistance, R.



v_a, V	v_b, V
11.75	7.05
7.5	4.5
5.625	3.375
10	6
4.375	2.625

FIGURE P 2.4-10

Section 2.5 Independent Sources

P 2.5-1 A current source and a voltage source are connected in parallel with a resistor as shown in Figure P 2.5-1. All of the elements connected in parallel have the same voltage, v_s in this circuit. Suppose that $v_s = 15$ V, $i_s = 3$ A, and $R = 5$ Ω. (a) Calculate the current i in the resistor and the power absorbed by the resistor. (b) Change the current source current to $i_s = 5$ A and recalculate the current, i, in the resistor and the power absorbed by the resistor.
Answer: $i = 3$ A and the resistor absorbs 45 W both when $i_s = 3$ A and when $i_s = 5$ A.

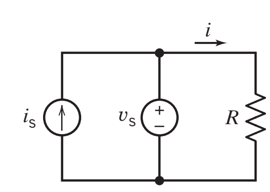


FIGURE P 2.5-1

P 2.5-2 A current source and a voltage source are connected in series with a resistor as shown in Figure P 2.5-2. All of the elements connected in series have the same current, i_s, in this circuit. Suppose that $v_s = 10$ V, $i_s = 2$ A, and $R = 5$ Ω. (a) Calculate the voltage v across the resistor and the power absorbed by the resistor. (b) Change the voltage source voltage to $v_s = 5$ V and recalculate the voltage, v, across the resistor and the power absorbed by the resistor.
Answer: $v = 10$ V and the resistor absorbs 20 W both when $v_s = 10$ V and when $v_s = 5$ V.

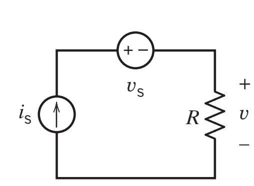


FIGURE P 2.5-2

P 2.5-3 The current source and voltage source in the circuit shown in Figure P 2.5-3 are connected in parallel so that they both have the same voltage, v_s. The current source and voltage source are also connected in series so that they both have the same current, i_s. Suppose that $v_s = 12$ V and $i_s = 3$ A. Calculate the power supplied by each source.

Answer: The voltage source supplies −36 W, and the current source supplies 36 W.

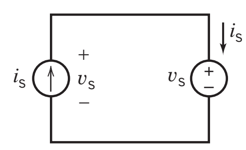


FIGURE P 2.5-3

P 2.5-4 The current source and voltage source in the circuit shown in Figure P 2.5-4 are connected in parallel so that they both have the same voltage, v_s. The current source and voltage source are also connected in series so that they both have the same current, i_s. Suppose that $v_s = 12$ V and $i_s = 3$ A. Calculate the power supplied by each source.

Answer: The voltage source supplies 36 W, and the current source supplies −36 W.

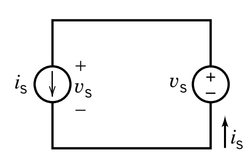


FIGURE P 2.5-4

P 2.5-5

(a) Find the power supplied by the voltage source shown in Figure P 2.5-5 when for $t \geq 0$ we have

$$v = 2 \cos t \text{ V}$$

and

$$i = 10 \cos t \text{ mA}$$

(b) Determine the energy supplied by this voltage source for the period $0 \leq t \leq 1$ s.

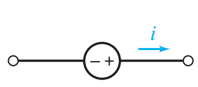


FIGURE P 2.5-5

P 2.5-6 Figure P 2.5-6 shows a battery connected to a load. The load in Figure P 2.5-6 might represent automobile headlights, a digital camera, or a cell phone. The energy supplied by the battery to load is given by

$$w = \int_{t_1}^{t_2} vi \, dt$$

When the battery voltage is constant and the load resistance is fixed, then the battery current will be constant and

$$w = vi(t_2 - t_1)$$

The capacity of a battery is the product of the battery current and time required to discharge the battery. Consequently, the energy stored in a battery is equal to the product of the battery voltage and the battery capacity. The capacity is usually given with the units of Ampere-hours (Ah). A new 12-V battery having a capacity of 800 mAh is connected to a load that draws a current of 50 mA. (a) How long will it take for the load to discharge the battery? (b) How much energy will be supplied to the load during the time required to discharge the battery?

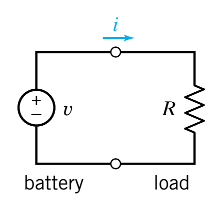


FIGURE P 2.5-6

Section 2.6 Voltmeters and Ammeters

P 2.6-1 For the circuit of Figure P 2.6-1:

(a) What is the value of the resistance R?

(b) How much power is delivered by the voltage source?

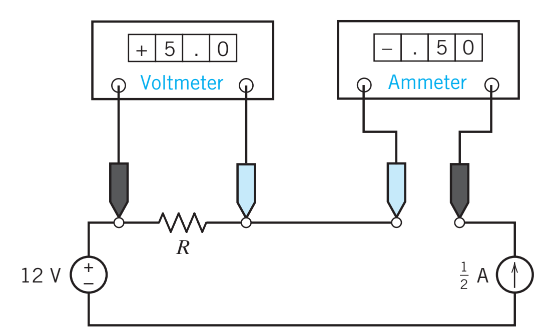


FIGURE P 2.6-1

P 2.6-2 The current source in Figure P 2.6-2 supplies 40 W. What values do the meters in Figure P 2.6-2 read?

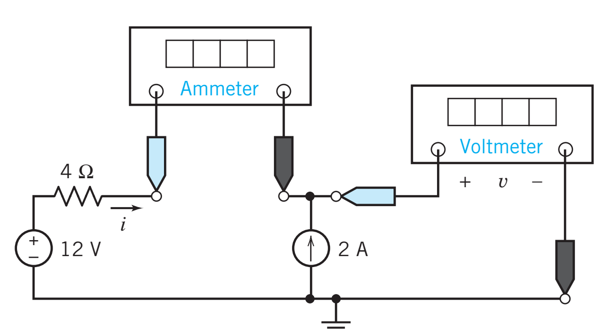


FIGURE P 2.6-2

P 2.6-3 An ideal voltmeter is modeled as an open circuit. A more realistic model of a voltmeter is a large resistance. Figure P 2.6-3*a* shows a circuit with a voltmeter that measures the voltage v_m. In Figure P 2.6-3*b* the voltmeter is replaced by the model of an ideal voltmeter, an open circuit. Ideally, there is no current in the 100-Ω resistor and the voltmeter measures $v_{mi} = 12$ V, the ideal value of v_m. In Figure P 2.6-3*c* the voltmeter is modeled by the resistance R_m. Now the voltage measured by the voltmeter is

$$v_m = \left(\frac{R_m}{R_m + 100}\right)12$$

As $R_m \to \infty$, the voltmeter becomes an ideal voltmeter and $v_m \to v_{mi} = 12$ V . When $R_m < \infty$, the voltmeter is not ideal and $v_m < v_{mi}$. The difference between v_m and v_{mi} is a measurement error caused by the fact that the voltmeter is not ideal.

(a) Express the measurement error that occurs when $R_m = 900$ Ω as a percent of v_{mi}.

(b) Determine the minimum value of R_m required to ensure that the measurement error is smaller than 2 percent of v_{mi}.

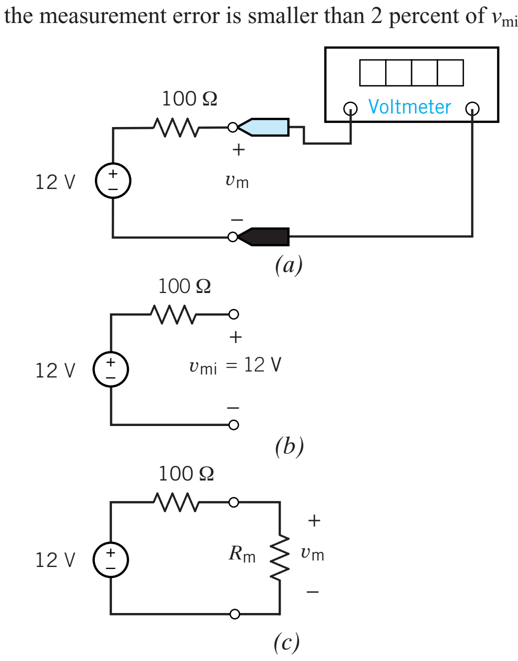


FIGURE P 2.6-3

P 2.6-4 An ideal ammeter is modeled as a short circuit. A more realistic model of an ammeter is a small resistance. Figure P 2.6-4*a* shows a circuit with an ammeter that measures the current i_m. In Figure P 2.6-4*b* the ammeter is replaced by the model of an ideal ammeter, a short circuit. Ideally, there is no voltage across the 1-kΩ resistor and the ammeter measures $i_{mi} = 2$ A, the ideal value of i_m. In Figure P 2.6-4*c* the ammeter is modeled by the resistance R_m. Now the current measured by the ammeter is

$$i_m = \left(\frac{1000}{1000 + R_m}\right)2$$

As $R_m \to 0$, the ammeter becomes an ideal ammeter and $i_m \to i_{mi} = 2$ A. When $R_m > 0$, the ammeter is not ideal and $i_m < i_{mi}$. The difference between i_m and i_{mi} is a measurement error caused by the fact that the ammeter is not ideal.

(a) Express the measurement error that occurs when $R_m = 10$ Ω as a percent of i_{mi}.

(b) Determine the maximum value of R_m required to ensure that the measurement error is smaller than 5 percent.

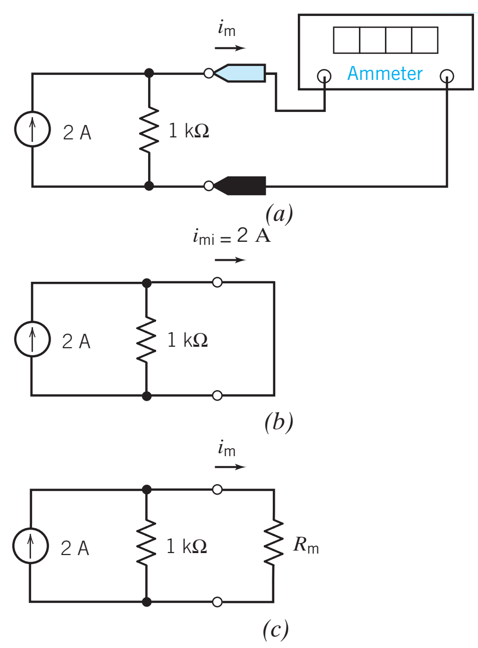


FIGURE P 2.6-4

P 2.6-5 The voltmeter in Figure P 2.6-5*a* measures the voltage across the current source. Figure P 2.6-5*b* shows the circuit after removing the voltmeter and labeling the voltage measured by the voltmeter as v_m. Also, the other element voltages and currents are labeled in Figure P 2.6-5*b*.

Given that

$$12 = v_R + v_m \quad \text{and} \quad -i_R = i_s = 2 \text{ A}$$

and

$$v_R = 25 i_R$$

(a) Determine the value of the voltage measured by the meter.

(b) Determine the power supplied by each element.

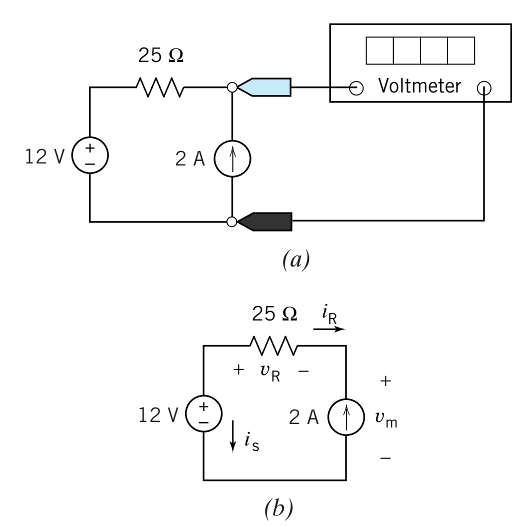


FIGURE P 2.6-5

P 2.6-6 The ammeter in Figure P 2.6-6*a* measures the current in the voltage source. Figure P 2.6-6*b* shows the circuit after removing the ammeter and labeling the current measured by the ammeter as i_m. Also, the other element voltages and currents are labeled in Figure P 2.6-6*b*.

Given that

$$2 + i_m = i_R \text{ and } v_R = v_s = 12 \text{ V}$$

and

$$v_R = 25 i_R$$

(a) Determine the value of the current measured by the meter.

(b) Determine the power supplied by each element.

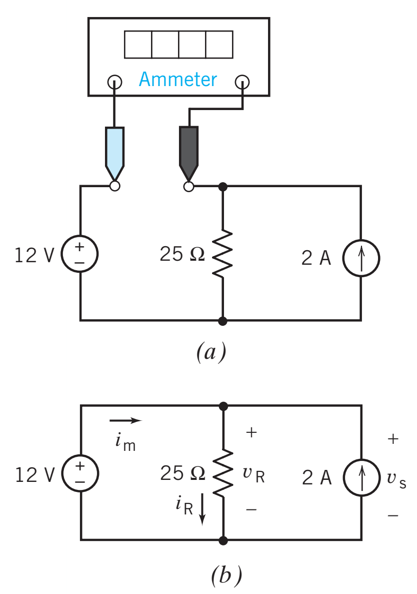


FIGURE P 2.6-6

Section 2.7 Dependent Sources

P 2.7-1 The ammeter in the circuit shown in Figure P 2.7-1 indicates that $i_a = 2$ A, and the voltmeter indicates that $v_b = 8$ V. Determine the value of *r*, the gain of the CCVS.

Answer: $r = 4$ V/A

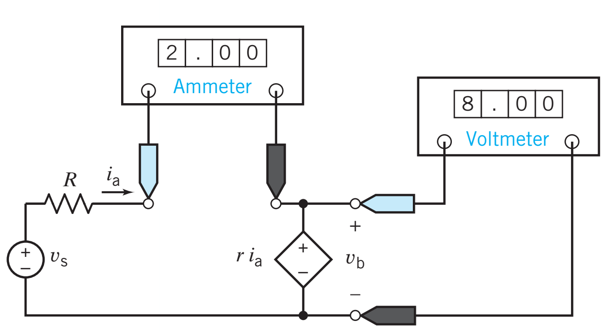


FIGURE P 2.7-1

P 2.7-2 The ammeter in the circuit shown in Figure P 2.7-2 indicates that $i_a = 2$ A, and the voltmeter indicates that $v_b = 8$ V. Determine the value of *g*, the gain of the VCCS.

Answer: $g = 0.25$ A/V

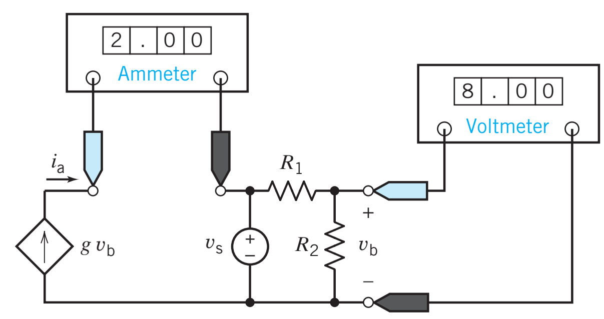


FIGURE P 2.7-2

P 2.7-3 The ammeters in the circuit shown in Figure P 2.7-3 indicate that $i_a = 32$ A and $i_b = 8$ A. Determine the value of *d*, the gain of the CCCS.

Answer: $d = 4$ A/A

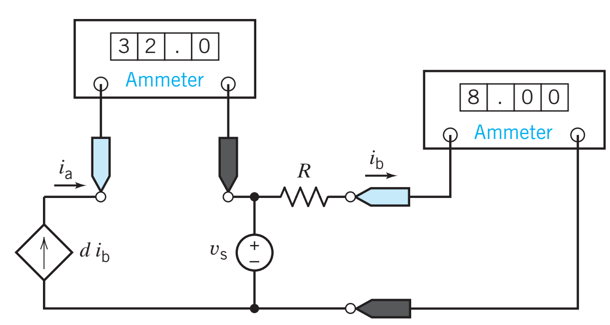


FIGURE P 2.7-3

P 2.7-4 The voltmeters in the circuit shown in Figure P 2.7-4 indicate that $v_a = 2$ V and $v_b = 8$ V. Determine the value of *b*, the gain of the VCVS.

Answer: $b = 4$ V/V

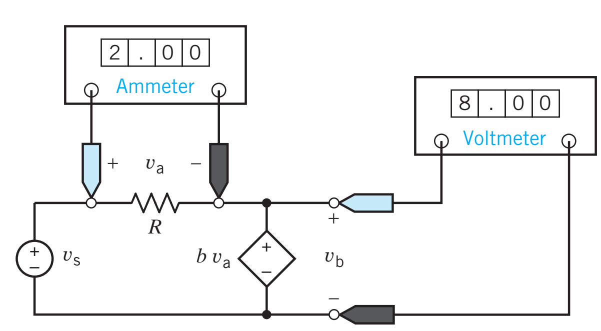


FIGURE P 2.7-4

P 2.7-5 The values of the current and voltage of each circuit element are shown in Figure P 2.7-5. Determine the values of the resistance, *R*, and of the gain of the dependent source, *A*.

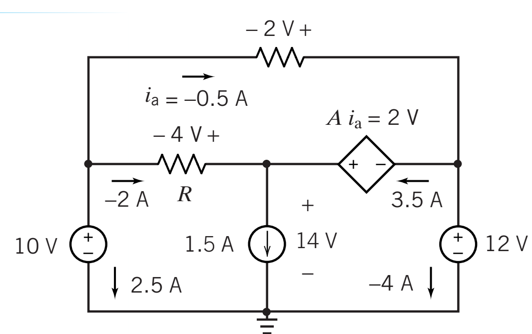


FIGURE P 2.7-5

P 2.7-6 Find the power supplied by the VCCS in Figure P 2.7-6.

Answer: 17.6 watts are supplied by the VCCS. (−17.6 watts are absorbed by the VCCS.)

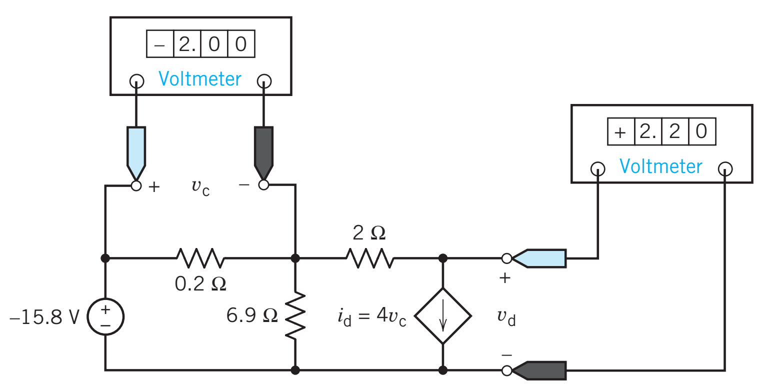


FIGURE P 2.7-6

P 2.7-7 Find the power absorbed by the CCVS in Figure P 2.7-7.

Answer: 4.375 watts are delivered to the CCVS.

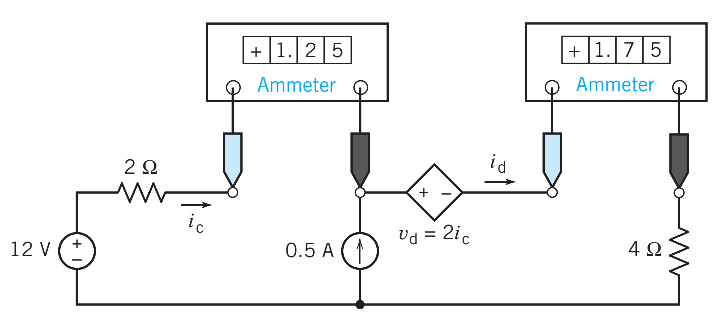


FIGURE P 2.7-7

Section 2.8 Transducers

P 2.8-1 For the potentiometer circuit of Figure 2.8-2, the current source current and potentiometer resistance are 1.1 mA and 100 kΩ, respectively. Calculate the required angle, θ, so that the measured voltage is 23 V.

P 2.8-2 An AD590 sensor has an associated constant $k = 1\frac{\mu A}{^\circ K}$. The sensor has a voltage $v = 20$ V; and the measured current, $i(t)$, as shown in Figure 2.8-3, is $4\,\mu A < i < 13\,\mu A$ in a laboratory setting. Find the range of measured temperature.

Section 2.9 Switches

P 2.9-1 Determine the current, i, at $t = 1$ s and at $t = 4$ s for the circuit of Figure P 2.9-1.

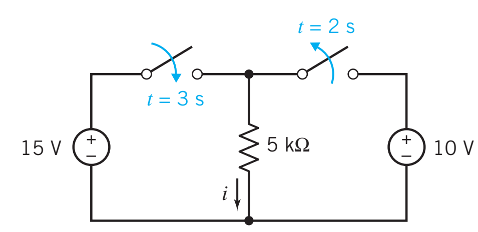


FIGURE P 2.9-1

P 2.9-2 Determine the voltage, v, at $t = 1$ s and at $t = 4$ s for the circuit shown in Figure P 2.9-2.

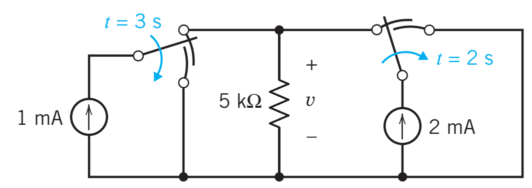


FIGURE P 2.9-2

P 2.9-3 Ideally an open switch is modeled as an open circuit and a closed switch is modeled as a closed circuit. More realistically, an open switch is modeled as a large resistance and a closed switch is modeled as a small resistance.

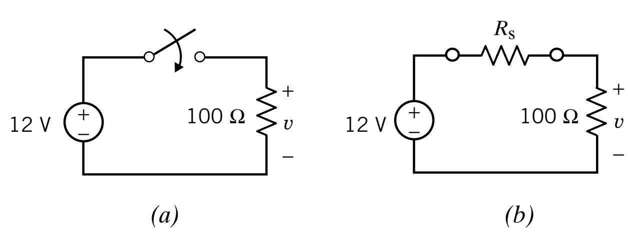


FIGURE P 2.9-3

Figure P 2.9-3*a* shows a circuit with a switch. In Figure P 2.9-3*b* the switch has been replaced with a resistance. In Figure P 2.9-3*b* the voltage *v* is given by

$$v = \left(\frac{100}{R_s + 100}\right)12$$

Determine the value of *v* for each of the following cases.

(a) The switch is closed and $R_s = 0$ (a short circuit).
(b) The switch is closed and $R_s = 5\ \Omega$.
(c) The switch is open and $R_s = \infty$ (an open circuit).
(d) The switch is open and $R_s = 10\ \text{k}\Omega$.

Section 2-10 How Can We Check . . . ?

P 2.10-1 The circuit shown in Figure P 2.10-1 is used to test the CCVS. Your lab partner claims that this measurement shows that the gain of the CCVS is −20 V/A instead of +20 V/A. Do you agree? Justify your answer.

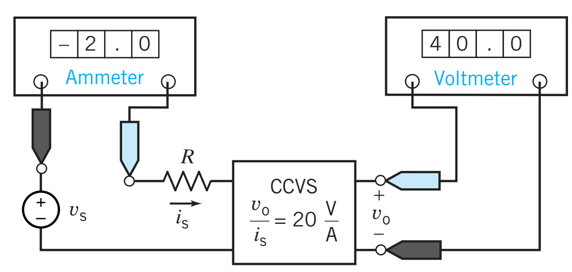


FIGURE P 2.10-1

P 2.10-2 The circuit of Figure P 2.10-2 is used to measure the current in the resistor. Once this current is known, the resistance can be calculated as $R = \frac{v_s}{i}$. The circuit is constructed using a voltage source with $v_s = 12$ V and a 25-Ω, 1/2-W resistor. After a puff of smoke and an unpleasant smell, the ammeter indicates that $i = 0$ A. The resistor must be bad. You have more 25-Ω, 1/2-W resistors. Should you try another resistor? Justify your answer.

Hint: 1/2-W resistors are able to safely dissipate one 1/2 W of power. These resistors may fail if required to dissipate more than 1/2 watt of power.

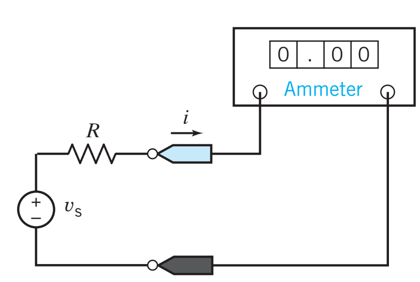


FIGURE P 2.10-2

Design Problems

DP 2-1 Specify the resistance *R* in Figure DP 2-1 so that both of the following conditions are satisfied:

1. $i > 40$ mA.
2. The power absorbed by the resistor is less than 0.5 W.

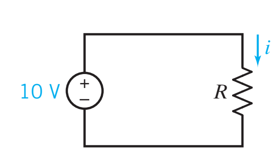


FIGURE DP 2-1

DP 2-2 Specify the resistance R in Figure DP 2-2 so that both of the following conditions are satisfied:

1. $v > 40$ V.
2. The power absorbed by the resistor is less than 15 W.

Hint: There is no guarantee that specifications can always be satisfied.

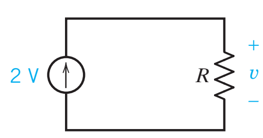


FIGURE DP 2-2

DP 2-3 Resistors are given a power rating. For example, resistors are available with ratings of 1/8 W, 1/4 W, 1/2 W, and 1 W. A 1/2-W resistor is able to safely dissipate 1/2 W of power, indefinitely. Resistors with larger power ratings are more expensive and bulkier than resistors with lower power ratings. Good engineering practice requires that resistor power ratings be specified to be as large as, but not larger than, necessary.

Consider the circuit shown in Figure DP 2-3. The values of the resistances are

$$R_1 = 1000\ \Omega,\ R_2 = 2000\ \Omega,\ \text{and}\ R_3 = 4000\ \Omega$$

The value of the current source current is

$$i_s = 30\ \text{mA}$$

Specify the power rating for each resistor.

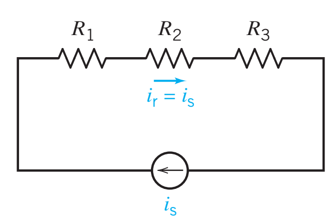


FIGURE DP 2-3