CHAPTER 3

Resistive Circuits

In This Chapter:

3.1 Introduction

In this chapter we will do the following:

- Write equations using Kirchhoff's laws.
  Not surprisingly, the behavior of an electric circuit is determined both by the types of elements that comprise the circuit and by the way those elements are connected together. The constitutive equations describe the elements themselves, and Kirchhoff's laws describe the way the elements are connected to each other to form the circuit.
- Analyze simple electric circuits using only Kirchhoff's laws and the constitutive equations of the circuit elements.
- Analyze two very common circuit configurations: series resistors and parallel resistors.
  We will see that series resistors act like a "voltage divider" and parallel resistors act like a "current divider." Also, series resistors and parallel resistors provide our first examples of an "equivalent circuit." Figure 3.1-1 illustrates this important concept. Here a circuit has been partitioned into two parts, A and B. Replacing B by an equivalent circuit, B_{eq}, does not change the current or voltage of any circuit element in part A. It is in this sense that B_{eq} is equivalent to B. We will see how to obtain an equivalent circuit when part B consists either of series resistors or of parallel resistors.
- Determine equivalent circuits for series voltage sources and parallel current sources.
- Determine the equivalent resistance of a resistive circuit.

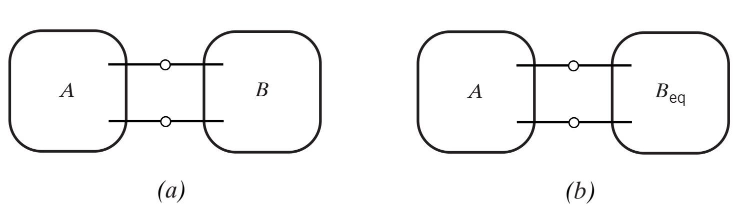

FIGURE 3.1-1 Replacing B by an equivalent circuit, B_{eq}, does not change the current or voltage of any circuit element in A.

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Often circuits consisting entirely of resistors can be reduced to a single equivalent resistor by repeatedly replacing series and/or parallel resistors by equivalent resistors.

3.2 Kirchhoff's Laws

An electric circuit consists of circuit elements that are connected together. The places where the elements are connected to each other are called nodes. Figure 3.2-1*a* shows an electric circuit that consists of six elements connected together at four nodes. It is common practice to draw electric circuits using straight lines and to position the elements horizontally or vertically as shown in Figure 3.2-1*b*.

The circuit is shown again in Figure 3.2-1*c*, this time emphasizing the nodes. Notice that redrawing the circuit using straight lines and horizontal and vertical elements has changed the way that the nodes are represented. In Figure 3.2-1*a,* nodes are represented as points. In Figures 3.2-1*b,c,* nodes are represented using both points and straight-line segments.

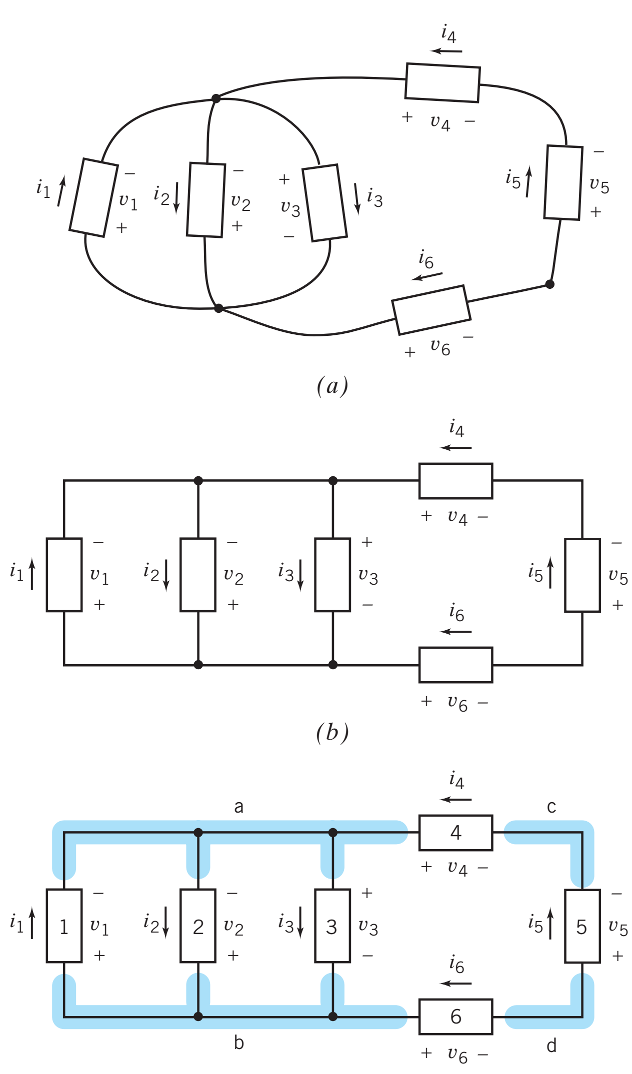

FIGURE 3.2-1
(*a*) An electric circuit. (*b*) The same circuit, redrawn using straight lines and horizontal and vertical elements. (*c*) The circuit after labeling the nodes and elements.

The same circuit can be drawn in several different ways. One drawing of a circuit might look much different from another drawing of the same circuit. How can we tell when two circuit drawings represent the same circuit? Informally, we say that two circuit drawings represent the same circuit if corresponding elements are connected to corresponding nodes. More formally, we say that circuit drawings A and B represent the same circuit when the following three conditions are met.

1. There is a one-to-one correspondence between the nodes of drawing A and the nodes of drawing B. (A one-to-one correspondence is a matching. In this one-to-one correspondence, each node in drawing A is matched to exactly one node of drawing B, and vice versa. The position of the nodes is not important.)
2. There is a one-to-one correspondence between the elements of drawing A and the elements of drawing B.
3. Corresponding elements are connected to corresponding nodes.

EXAMPLE 3.2-1 *Different Drawings of the Same Circuit*

Figure 3.2-2 shows four circuit drawings. Which of these drawings, if any, represent the same circuit as the circuit drawing in Figure 3.2-1*c*?

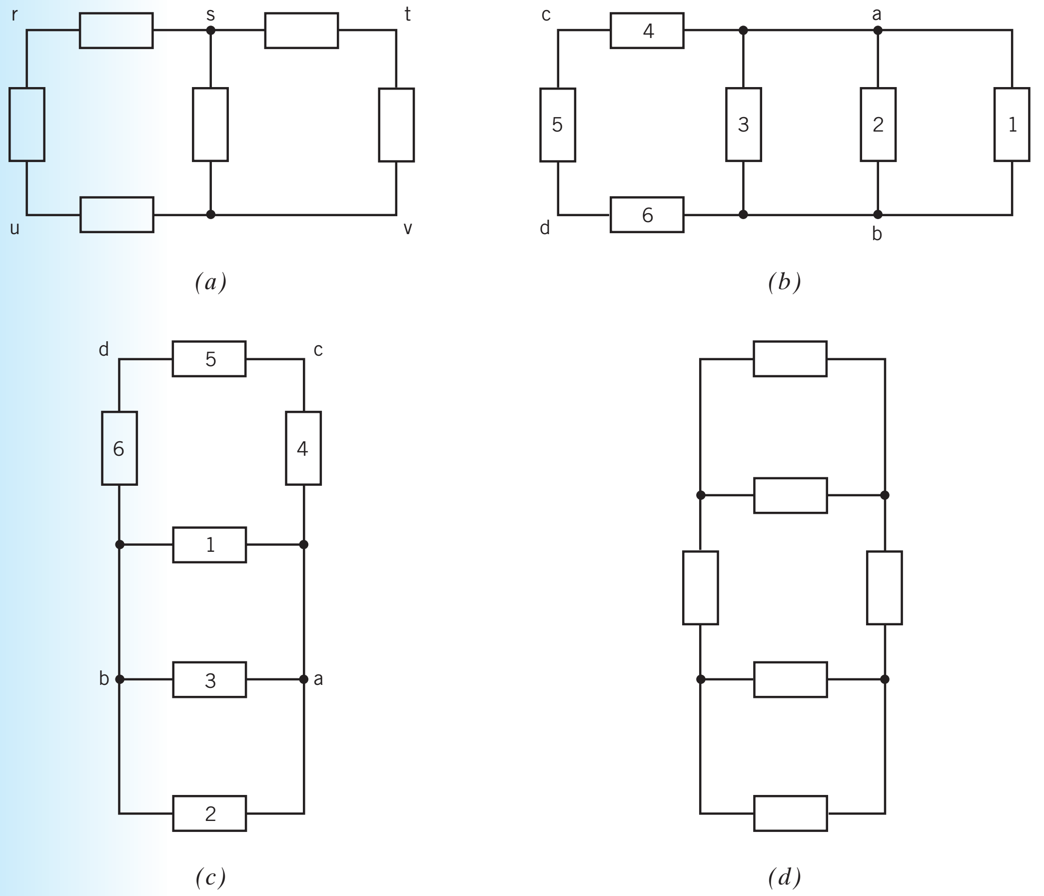

FIGURE 3.2-2 Four circuit drawings.

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Solution

The circuit drawing shown in Figure 3.2-2*a* has five nodes, labeled r, s, t, u, and v. The circuit drawing in Figure 3.2-1*c* has four nodes. Since the two drawings have different numbers of nodes, there cannot be a one-to-one correspondence between the nodes of the two drawings. Hence these drawings represent different circuits.

The circuit drawing shown in Figure 3.2-2*b* has four nodes and six elements, the same numbers of nodes and elements as the circuit drawing in Figure 3.2-1*c*. The nodes in Figure 3.2-2*b* have been labeled in the same way as the corresponding nodes in Figure 3.2-1*c*. For example, node c in Figure 3.2-2*b* corresponds to node c in Figure 3.2-1*c*. The elements in Figure 3.2-2*b* have been labeled in the same way as the corresponding elements in Figure 3.2-1*c*. For example, element 5 in Figure 3.2-2*b* corresponds to element 5 in Figure 3.2-1*c*. Corresponding elements are indeed connected to corresponding nodes. For example, element 2 is connected to nodes a and b, both in Figure 3.2-2*b* and in Figure 3.2-1*c*. Consequently, Figure 3.2-2*b* and Figure 3.2-1*c* represent the same circuit.

The circuit drawing shown in Figure 3.2-2*c* has four nodes and six elements, the same numbers of nodes and elements as the circuit drawing in Figure 3.2-1*c*. The nodes and elements in Figure 3.2-2*c* have been labeled in the same way as the corresponding nodes and elements in Figure 3.2-1*c*. Corresponding elements are indeed connected to corresponding nodes. Therefore Figure 3.2-2*c* and Figure 3.2-1*c* represent the same circuit.

The circuit drawing shown in Figure 3.2-2*d* has four nodes and six elements, the same numbers of nodes and elements as the circuit drawing in Figure 3.2-1*c*. However, the nodes and elements of Figure 3.2-2*d* cannot be labeled so that corresponding elements of Figure 3.2-1*c* are connected to corresponding nodes. (For example, in Figure 3.2-1*c* three elements are connected between the same pair of nodes, a and b. That does not happen in Figure 3.2-2*d*.) Consequently, Figure 3.2-2*d* and Figure 3.2-1*c* represent different circuits.

FIGURE 3.2-3 Gustav Robert Kirchhoff (1824–1887). Kirchhoff stated two laws in 1847 regarding the current and voltage in an electrical circuit. Courtesy of the Smithsonian Institution.

In 1847, Gustav Robert Kirchhoff, a professor at the University of Berlin, formulated two important laws that provide the foundation for analysis of electric circuits. These laws are referred to as *Kirchhoff's current law* (KCL) and *Kirchhoff's voltage law* (KVL) in his honor. Kirchhoff's laws are a consequence of conservation of charge and conservation of energy. Gustav Robert Kirchhoff is pictured in Figure 3.2-3.

Kirchhoff's current law states that the algebraic sum of the currents entering any node is identically zero for all instants of time.

Kirchhoff's current law (KCL): The algebraic sum of the currents into a node at any instant is zero.

The phrase *algebraic sum* indicates that we must take reference directions into account as we add up the currents of elements connected to a particular node. One way to take reference directions into account is to use a plus sign when the current is directed away from the node and a minus sign when the current is directed toward the node. For example, consider the circuit shown in Figure 3.2-1*c*. Four elements of this circuit—elements 1, 2, 3, and 4—are connected to node a. By Kirchhoff's current law, the algebraic sum of the element currents i_1, i_2, i_3, and i_4 must be zero. Currents i_2 and i_3 are directed away from node a, so we will use a plus sign for i_2 and i_3. In contrast, currents i_1 and i_4 are directed toward node a, so we will use a minus sign for i_1 and i_4. The KCL equation for node a of Figure 3.2-1*c* is

$$-i_1 + i_2 + i_3 - i_4 = 0 \tag{3.2-1}$$

An alternate way of obtaining the algebraic sum of the currents into a node is to set the sum of all the currents directed away from the node equal to the sum of all the currents directed toward that node. Using this technique, we find that the KCL equation for node a of Figure 3.2-1*c* is

$$i_2 + i_3 = i_1 + i_4 \tag{3.2-2}$$

Clearly, Eqs. 3.2-1 and 3.2-2 are equivalent.

Similarly, the Kirchhoff's current law equation for node b of Figure 3.2-1*c* is

$$i_1 = i_2 + i_3 + i_6$$

Before we can state Kirchhoff's voltage law, we need the definition of a loop. A *loop* is a closed path through a circuit that does not encounter any intermediate node more than once. For example, starting at node a in Figure 3.2-1*c,* we can move through element 4 to node c, then through element 5 to node d, through element 6 to node b, and finally through element 3 back to node a. We have a closed path, and we did not encounter any of the intermediate nodes—b, c, or d—more than once. Consequently, elements 3, 4, 5, and 6 comprise a loop. Similarly, elements 1, 4, 5, and 6 comprise a loop of the circuit shown in Figure 3.2-1*c*. Elements 1 and 3 comprise yet another loop of this circuit. The circuit has three other loops: elements 1 and 2, elements 2 and 3, and elements 2, 4, 5, and 6.

We are now ready to state Kirchhoff's voltage law.

Kirchhoff's voltage law (KVL): The algebraic sum of the voltages around any loop in a circuit is identically zero for all time.

The phrase *algebraic sum* indicates that we must take polarity into account as we add up the voltages of elements that comprise a loop. One way to take polarity into account is to move around the loop in the clockwise direction while observing the polarities of the element voltages. We write the voltage with a plus sign when we encounter the + of the voltage polarity before the −. In contrast, we write the voltage with a minus sign when we encounter the − of the voltage polarity before the +. For example, consider the circuit shown in Figure 3.2-1*c*. Elements 3, 4, 5, and 6 comprise a loop of the circuit. By Kirchhoff's voltage law, the algebraic sum of the element voltages v_3, v_4, v_5, and v_6 must be zero. As we move around the loop in the clockwise direction, we encounter the + of v_4 before the −, the − of v_5 before the +, the − of v_6 before the +, and the − of v_3 before the +. Consequently, we use a minus sign for v_3, v_5, and v_6 and a plus sign for v_4. The KCL equation for this loop of Figure 3.2-1*c* is

$$v_4 - v_5 - v_6 - v_3 = 0$$

Similarly, the Kirchhoff's voltage law equation for the loop consisting of elements 1, 4, 5, and 6 is

$$v_4 - v_5 - v_6 + v_1 = 0$$

The Kirchhoff's voltage law equation for the loop consisting of elements 1 and 2 is

$$-v_2 + v_1 = 0$$

EXAMPLE 3.2-2 *Kirchhoff's Laws*

INTERACTIVE EXAMPLE

Consider the circuit shown in Figure 3.2-4*a*. Determine the power supplied by element *C* and the power received by element *D*.

Solution

Figure 3.2-4*a* provides a value for the current in element *C* but not for the voltage, *v,* across element *C*. The voltage and current of element *C* given in Figure 3.2-4*a* adhere to the passive convention, so the product of this voltage and current is the power *received* by element *C*. Figure 3.2-4*a* provides a value for the voltage across element *D* but not for the current, *i,* in element *D*. The voltage and current of element *D* given in Figure 3.2-4*a* do not adhere to the passive convention, so the product of this voltage and current is the power *supplied* by element *D*.

We need to determine the voltage, *v,* across element *C* and the current, *i,* in element *D*. We will use Kirchhoff's laws to determine values of *v* and *i*. First, we identify and label the nodes of the circuit as shown in Figure 3.2-4*b*.

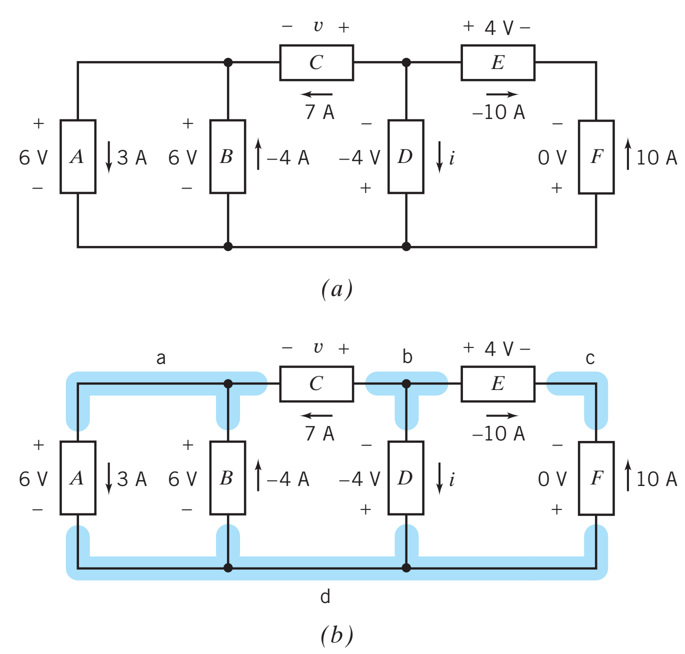

FIGURE 3.2-4 (*a*) The circuit considered in Example 3.2-2 and (*b*) the circuit redrawn to emphasize the nodes.

Apply Kirchhoff's voltage law (KVL) to the loop consisting of elements *C, D,* and *B* to get

$$-v - (-4) - 6 = 0 \quad \Rightarrow \quad v = -2\ \text{V}$$

The value of the current in element *C* in Figure 3.2-4*b* is 7 A. The voltage and current of element *C* given in Figure 3.2-4*b* adhere to the passive convention, so

$$p_C = v(7) = (-2)(7) = -14\ \text{W}$$

is the power *received* by element *C*. Therefore element *C supplies* 14 W.

Next, apply Kirchhoff's current law (KCL) at node b to get

$$7 + (-10) + i = 0 \quad \Rightarrow \quad i = 3\ \text{A}$$

The value of the voltage across element *D* in Figure 3.2-4*b* is −4 V. The voltage and current of element *D* given in Figure 3.2-4*b* do not adhere to the passive convention, so the power *supplied* by element *F* is given by

$$p_D = (-4)\,i = (-4)(3) = -12\ \text{W}$$

Therefore, element *D receives* 12 W.

EXAMPLE 3.2-3 *Ohm's and Kirchhoff's Laws*

Consider the circuit shown in Figure 3.2-5. Notice that the passive convention was used to assign reference directions to the resistor voltages and currents. This anticipates using Ohm's law. Find each current and each voltage when $R_1 = 8\ \Omega$, $v_2 = -10$ V, $i_3 = 2$ A, and $R_3 = 1\ \Omega$. Also, determine the resistance R_2.

Solution

The sum of the currents entering node a is

$$i_1 - i_2 - i_3 = 0$$

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Using Ohm's law for R_3, we find that

$$v_3 = R_3 i_3 = 1(2) = 2\ \text{V}$$

Kirchhoff's voltage law for the bottom loop incorporating v_1, v_3, and the 10-V source is

$$-10 + v_1 + v_3 = 0$$

Therefore,

$$v_1 = 10 - v_3 = 8\ \text{V}$$

Ohm's law for the resistor R_1 is

$$v_1 = R_1 i_1$$

or

$$i_1 = v_1/R_1 = 8/8 = 1\ \text{A}$$

Since we have now found $i_1 = 1$ A and $i_3 = 2$ A as originally stated, then

$$i_2 = i_1 - i_3 = 1 - 2 = -1\ \text{A}$$

We can now find the resistance R_2 from

$$v_2 = R_2 i_2$$

or

$$R_2 = v_2/i_2 = -10/-1 = 10\ \Omega$$

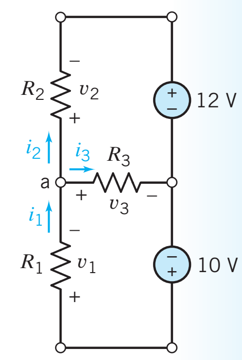

FIGURE 3.2-5 Circuit with two constant-voltage sources.

EXAMPLE 3.2-4 *Ohm's and Kirchhoff's Laws*

INTERACTIVE EXAMPLE

Determine the value of the current, in amps, measured by the ammeter in Figure 3.2-6*a*.

Solution

An ideal ammeter is equivalent to a short circuit. The current measured by the ammeter is the current in the short circuit. Figure 3.2-6*b* shows the circuit after replacing the ammeter by the equivalent short circuit.

The circuit has been redrawn in Figure 3.2-7 to label the nodes of the circuit. This circuit consists of a voltage source, a dependent current source, two resistors, and two short circuits. One of the short circuits is the controlling element of the CCCS, and the other short circuit is a model of the ammeter.

Applying KCL twice, once at node d and again at node a, shows that the current in the voltage source and the current in the 4-Ω resistor are both equal to i_a. These currents are labeled in Figure 3.2-7. Applying KCL again, at node c, shows that the current in the 2-Ω resistor is equal to i_m. This current is labeled in Figure 3.2-7.

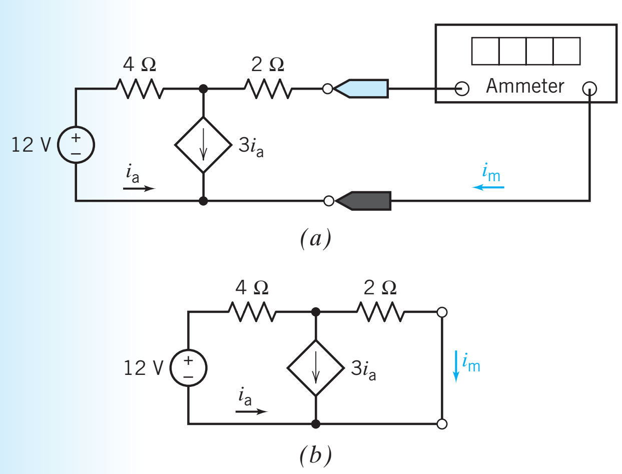

FIGURE 3.2-6 (*a*) A circuit with dependent source and an ammeter. (*b*) The equivalent circuit after replacing the ammeter by a short circuit.

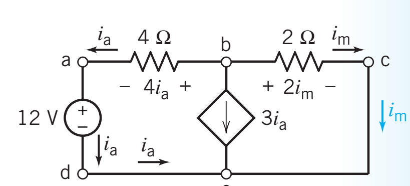

FIGURE 3.2-7 The circuit of Figure 3.2-6 after labeling the nodes and some element currents and voltages.

Next, Ohm's law tells us that the voltage across the 4-Ω resistor is equal to 4 i_a and that the voltage across the 2-Ω resistor is equal to 2 i_m. Both of these voltages are labeled in Figure 3.2-7.

Applying KCL at node b gives

$$-i_a - 3i_a - i_m = 0$$

Applying KVL to closed path a-b-c-e-d-a gives

$$0 = -4i_a + 2i_m - 12 = -4\left(-\frac{1}{4}i_m\right) + 2i_m - 12 = 3i_m - 12$$

Finally, solving this equation gives

$$i_m = 4\text{ A}$$

EXAMPLE 3.2-5 *Ohm's and Kirchhoff's Laws*

INTERACTIVE EXAMPLE

Determine the value of the voltage, in volts, measured by the voltmeter in Figure 3.2-8*a*.

Solution

An ideal voltmeter is equivalent to an open circuit. The voltage measured by the voltmeter is the voltage across the open circuit. Figure 3.2-8*b* shows the circuit after replacing the voltmeter by the equivalent open circuit.

The circuit has been redrawn in Figure 3.2-9 to label the nodes of the circuit. This circuit consists of a voltage source, a dependent voltage source, two resistors, a short circuit, and an open circuit. The short circuit is the controlling element of the CCVS, and the open circuit is a model of the voltmeter.

Applying KCL twice, once at node d and again at node a, shows that the current in the voltage source and the current in the 4-Ω resistor are both equal to i_a. These currents are labeled in Figure 3.2-9. Applying KCL again, at node c, shows that the current in the 5-Ω resistor is equal to the current in the open circuit, that is, zero. This current is labeled in Figure 3.2-9. Ohm's law tells us that the voltage across the 5-Ω resistor is also equal to zero. Next, applying KVL to the closed path b-c-f-e-b gives $v_m = 3\ i_a$.

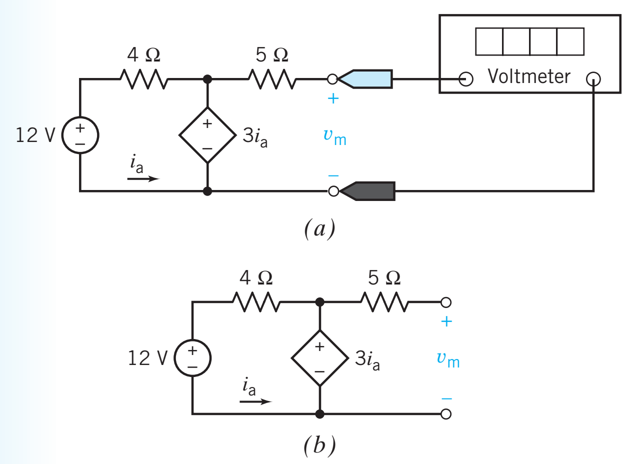

FIGURE 3.2-8 (*a*) A circuit with dependent source and a voltmeter. (*b*) The equivalent circuit after replacing the voltmeter by a open circuit.

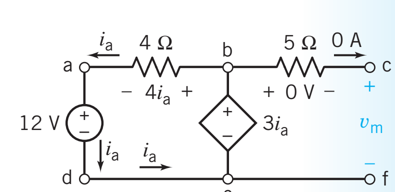

FIGURE 3.2-9 The circuit of Figure 3.2-8*b* after labeling the nodes and some element currents and voltages.

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Applying KVL to the closed path a-b-e-d-a gives

$$-4i_a + 3i_a - 12 = 0$$

so

$$i_a = -12 \text{ A}$$

Finally

$$v_m = 3i_a = 3(-12) = -36 \text{ V}$$

Exercise 3.2-1 Determine the values of i_3, i_4, i_6, v_2, v_4, and v_6 in Figure E 3.2-1.

Answer: $i_3 = -3$ A, $i_4 = 3$ A, $i_6 = 4$ A, $v_2 = -3$ V, $v_4 = -6$ V, $v_6 = 6$ V

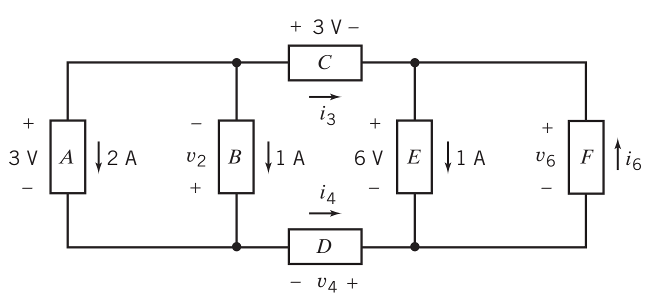

FIGURE E 3.2-1

3.3 Series Resistors and Voltage Division

Let us consider a single-loop circuit, as shown in Figure 3.3-1. In anticipation of using Ohm's law, the passive convention has been used to assign reference directions to resistor voltages and currents.

The connection of resistors in Figure 3.3-1 is said to be a *series* connection since all the elements carry the same current. To identify a pair of series elements, we look for two elements connected to a single node that has no other elements connected to it. Notice, for example, that resistors R_1 and R_2 are both connected to node b and that no other circuit elements are connected to node b. Consequently, $i_1 = i_2$, so both resistors have the same current. A similar argument shows that resistors R_2 and R_3 are also connected in series. Noticing that R_2 is connected in series with both R_1 and R_3, we say that all three resistors are connected in series. The order of series resistors is not important. For example, the voltages and currents of the three resistors in Figure 3.3-1 will not change if we interchange the positions R_2 and R_3.

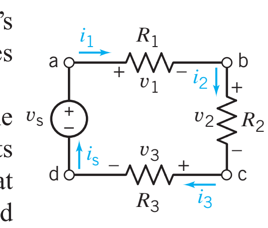

FIGURE 3.3-1 Single-loop circuit with a voltage source v_s.

Using KCL at each node of the circuit in Figure 3.3-1, we obtain

$$\text{a: } i_s = i_1$$
$$\text{b: } i_1 = i_2$$
$$\text{c: } i_2 = i_3$$
$$\text{d: } i_3 = i_s$$

Consequently

$$i_s = i_1 = i_2 = i_3$$

In order to determine i_1, we use KVL around the loop to obtain

$$v_1 + v_2 + v_3 - v_s = 0$$

where, for example, v_1 is the voltage across the resistor R_1. Using Ohm's law for each resistor,

$$R_1 i_1 + R_2 i_2 + R_2 i_3 - v_s = 0 \quad \Rightarrow \quad R_1 i_1 + R_2 i_1 + R_2 i_1 = v_s$$

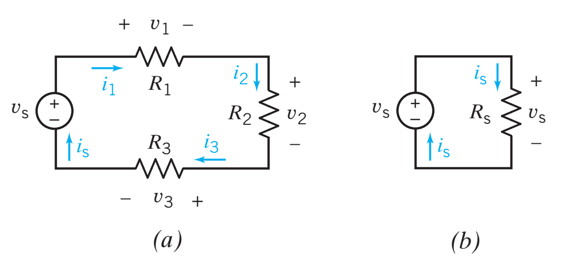

FIGURE 3.3-2

Solving for i_1, we have

$$i_1 = \frac{v_s}{R_1 + R_2 + R_3}$$

Thus, the voltage across the nth resistor R_n is v_n and can be obtained as

$$v_n = i_1 R_n = \frac{v_s R_n}{R_1 + R_2 + R_3}$$

For example, the voltage across resistor R_2 is

$$v_2 = \frac{R_2}{R_1 + R_2 + R_3} v_s$$

Thus, the voltage appearing across one of a series of resistors connected in series with a voltage source will be the ratio of its resistance to the total resistance. This circuit demonstrates the principle of *voltage division,* and the circuit is called a *voltage divider*.

In general, we may represent the voltage divider principle by the equation

$$v_n = \frac{R_n}{R_1 + R_2 + \cdots + R_N} v_s$$

where v_n is the voltage across the nth resistor of N resistors connected in series.

We can replace series resistors by an equivalent resistor. This is illustrated in Figure 3.3-2. The series resistors R_1, R_2, and R_3 in Figure 3.3-2*a* are replaced by a single, equivalent resistor R_s in Figure 3.3-2*b*. R_s is said to be equivalent to the series resistors R_1, R_2, and R_3 when replacing R_1, R_2, and R_3 by R_s does not change the current or voltage of any other element of the circuit. In this case, there is only one other element in the circuit, the voltage source. We must choose the value of the resistance R_s so that replacing R_1, R_2, and R_3 by R_s will not change the current of the voltage source. In Figure 3.3-2*a* we have

$$i_s = \frac{v_s}{R_1 + R_2 + R_3}$$

In Figure 3.3-2*b* we have

$$i_s = \frac{v_s}{R_s}$$

Since the voltage source current must be the same in both circuits, we require that

$$R_s = R_1 + R_2 + R_3$$

In general, the series connection of N resistors having resistances $R_1, R_2 \ldots R_N$ is equivalent to the single resistor having resistance

$$R_s = R_1 + R_2 + \cdots + R_N$$

Replacing series resistors by an equivalent resistor does not change the current or voltage of any other element of the circuit.

Next, let's calculate the power absorbed by the series resistors in Figure 3.3-2*a:*

$$p = i_s^{\,2} R_1 + i_s^{\,2} R_2 + i_s^{\,2} R_3$$

Doing a little algebra gives

$$p = i_s^{\,2}(R_1 + R_2 + R_3) = i_s^{\,2} R_s$$

which is equal to the power absorbed by the equivalent resistor in Figure 3.3-2*b*. We conclude that the power absorbed by series resistors is equal to the power absorbed by the equivalent resistor.

EXAMPLE 3.3-1 *Voltage Divider*

Let us consider the circuit shown in Figure 3.3-3 and determine the resistance R_2 required so that the voltage across R_2 will be 1/4 of the source voltage when $R_1 = 9\ \Omega$. Determine the current flowing when $v_s = 12$ V.

Solution

The voltage across resistor R_2 will be

$$v_2 = \frac{R_2}{R_1 + R_2} v_s$$

Since we desire $v_2/v_s = 1/4$, we have

$$\frac{R_2}{R_1 + R_2} = \frac{1}{4}$$

or

$$R_1 = 3R_2$$

Since $R_1 = 9\ \Omega$, we require that $R_2 = 3\ \Omega$. Using KVL around the loop, we have

$$-v_s + v_1 + v_2 = 0$$

or

$$v_s = iR_1 + iR_2$$

Therefore,

$$i = \frac{v_s}{R_1 + R_2} \qquad (3.3\text{-}1)$$

or

$$i = \frac{12}{12} = 1\ \text{A}$$

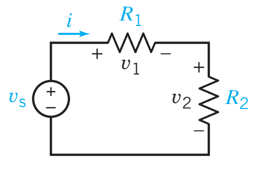

FIGURE 3.3-3
Voltage divider circuit with $R_1 = 9\ \Omega$.

EXAMPLE 3.3-2 *Series Resistors*

For the circuit of Figure 3.3-4*a,* find the current measured by the ammeter. Then show that the power absorbed by the two resistors is equal to that supplied by the source.

Solution

Figure 3.3-4*b* shows the circuit after the ideal ammeter has been replaced by the equivalent short circuit and a label has been added to indicate the current measured by the ammeter, i_m. Applying KVL gives

$$15 + 5i_m + 10i_m = 0$$

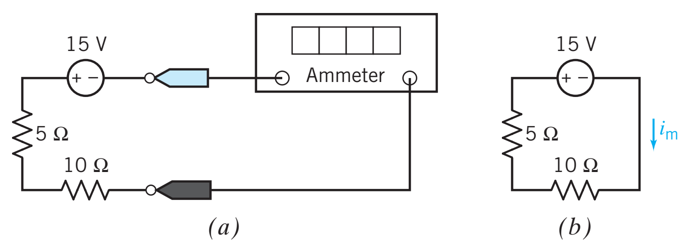

FIGURE 3.3-4 (*a*) A circuit containing series resistors. (*b*) The circuit after the ideal ammeter has been replaced by the equivalent short circuit and a label has been added to indicate the current measured by the ammeter, i_m.

The current measured by the ammeter is

$$i_m = -\frac{15}{5+10} = -1 \text{ A}$$

(Why is i_m negative? Why can't we just divide the source voltage by the equivalent resistance? Recall that when we use Ohm's law, the voltage and current must adhere to the passive convention. In this case, the current calculated by dividing the source voltage by the equivalent resistance does not have the same reference direction as i_m, and so we need a minus sign.)

The total power absorbed by the two resistors is

$$p_R = 5i_m^2 + 10i_m^2 = 15(1^2) = 15 \text{ W}$$

The power supplied by the source is

$$p_s = -v_s\, i_m = -15(-1) = 15 \text{ W}$$

Thus, the power supplied by the source is equal to that absorbed by the series connection of resistors.

EXAMPLE 3.3-3 *Voltage Divider Design*

The input to the voltage divider in Figure 3.3-5 is the voltage, v_s, of the voltage source. The output is the voltage, v_o, measured by the voltmeter. Design the voltage divider; that is, specify values of the resistances, R_1 and R_2, to satisfy both of these specifications

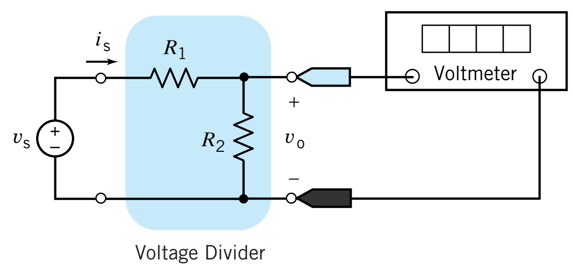

FIGURE 3.3-5
A voltage divider.

Specification 1: The input and output voltages are related by $v_o = 0.8\, v_s$.

Specification 2: The voltage source is required to supply no more than 1 mW of power when the input to the voltage divider is $v_s = 20$ V.

Solution

We'll examine each specification to see what it tells us about the resistor values.

Specification 1: The input and output voltages of the voltage divider are related by

$$v_o = \frac{R_2}{R_1 + R_2} v_s$$

So specification 1 requires

$$\frac{R_2}{R_1 + R_2} = 0.8 \quad \Rightarrow \quad R_2 = 4R_1$$

Specification 2: The power supplied by the voltage source is given by

$$p_s = i_s v_s = \left(\frac{v_s}{R_1 + R_2}\right) v_s = \frac{{v_s}^2}{R_1 + R_2}$$

So specification 2 requires

$$0.001 \geq \frac{20^2}{R_1 + R_2} \quad \Rightarrow \quad R_1 + R_2 \geq 400 \times 10^3 = 400\ \text{k}\Omega$$

Combining these results gives

$$5R_1 \geq 400\ \text{k}\Omega$$

The solution is not unique. One solution is

$$R_1 = 100\ \text{k}\Omega \ \text{ and } \ R_2 = 400\ \text{k}\Omega$$

Exercise 3.3-1 Determine the voltage measured by the voltmeter in the circuit shown in Figure E 3.3-1*a*.

Hint: Figure E 3.3-1*b* shows the circuit after the ideal voltmeter has been replaced by the equivalent open circuit and a label has been added to indicate the voltage measured by the voltmeter, v_m.

Answer: $v_m = 2$ V

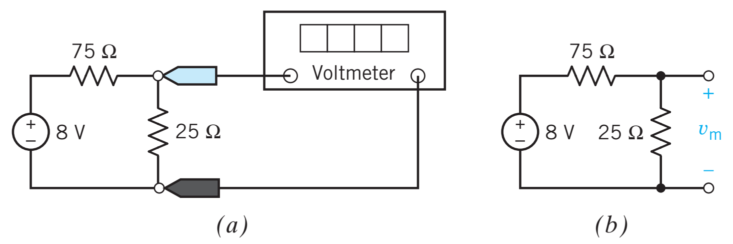

FIGURE E 3.3-1 (*a*) A voltage divider. (*b*) The voltage divider after the ideal voltmeter has been replaced by the equivalent open circuit and a label has been added to indicate the voltage measured by the voltmeter, v_m.

Exercise 3.3-2 Determine the voltage measured by the voltmeter in the circuit shown in Figure E 3.3-2*a*.

Hint: Figure E 3.3-2*b* shows the circuit after the ideal voltmeter has been replaced by the equivalent open circuit and a label has been added to indicate the voltage measured by the voltmeter, v_m.

Answer: $v_m = -2$ V

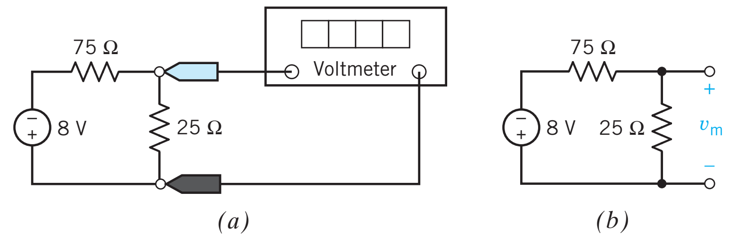

FIGURE E 3.3-2 (*a*) A voltage divider. (*b*) The voltage divider after the ideal voltmeter has been replaced by the equivalent open circuit and a label has been added to indicate the voltage measured by the voltmeter, v_m.

3.4 Parallel Resistors and Current Division

Circuit elements, such as resistors, are connected in *parallel* when the voltage across each element is identical. The resistors in Figure 3.4-1 are connected in *parallel*. Notice, for example, that resistors R_1 and R_2 are each connected to both node a and node b. Consequently, $v_1 = v_2$, so both resistors have the same voltage. A similar argument shows that resistors R_2 and R_3 are also connected in parallel. Noticing that R_2 is connected in parallel with both R_1 and R_3, we say that all three resistors are connected in parallel. The order of parallel resistors is not important. For example, the voltages and currents of the three resistors in Figure 3.4-1 will not change if we interchange the positions R_2 and R_3.

The defining characteristic of parallel elements is that they have the same voltage. To identify a pair of parallel elements, we look for two elements connected between the same pair of nodes.

Consider the circuit with two resistors and a current source shown in Figure 3.4-2. Note that both resistors are connected to terminals a and b and that the voltage v appears across each parallel element. In anticipation of using Ohm's law, the passive convention is used to assign reference directions to the resistor voltages and currents. We may write KCL at node a (or at node b) to obtain

$$i_s - i_1 - i_2 = 0$$

or

$$i_s = i_1 + i_2$$

However, from Ohm's law

$$i_1 = \frac{v}{R_1} \quad \text{and} \quad i_2 = \frac{v}{R_2}$$

Then

$$i_s = \frac{v}{R_1} + \frac{v}{R_2} \tag{3.4-1}$$

Recall that we defined conductance G as the inverse of resistance R. We may therefore rewrite Eq. 3.4-1 as

$$i_s = G_1 v + G_2 v = (G_1 + G_2)v \tag{3.4-2}$$

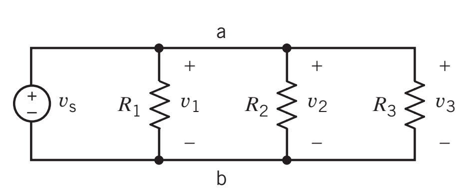

FIGURE 3.4-1 A circuit with parallel resistors.

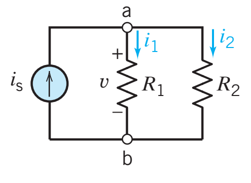

FIGURE 3.4-2 Parallel circuit with a current source.

Thus, the equivalent circuit for this parallel circuit is a conductance G_p, as shown in Figure 3.4-3, where

$$G_p = G_1 + G_2$$

The equivalent resistance for the two-resistor circuit is found from

$$G_p = \frac{1}{R_1} + \frac{1}{R_2}$$

Since $G_p = 1/R_p$, we have

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2}$$

or

$$R_p = \frac{R_1 R_2}{R_1 + R_2} \tag{3.4-3}$$

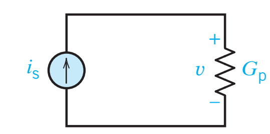

FIGURE 3.4-3
Equivalent circuit for a parallel circuit.

Note that the total conductance, G_p, increases as additional parallel elements are added and that the total resistance, R_p, declines as each resistor is added.

The circuit shown in Figure 3.4-2 is called a *current divider* circuit since it divides the source current. Note that

$$i_1 = G_1 v \tag{3.4-4}$$

Also, since $i_s = (G_1 + G_2)v$, we solve for v, obtaining

$$v = \frac{i_s}{G_1 + G_2} \tag{3.4-5}$$

Substituting v from Eq. 3.4-5 into Eq.3.4-4, we obtain

$$i_1 = \frac{G_1 i_s}{G_1 + G_2} \tag{3.4-6}$$

Similarly,

$$i_2 = \frac{G_2 i_s}{G_1 + G_2}$$

Note that we may use $G_2 = 1/R_2$ and $G_1 = 1/R_1$ to obtain the current i_2 in terms of two resistances as follows:

$$i_2 = \frac{R_1 i_s}{R_1 + R_2}$$

The current of the source divides between conductances G_1 and G_2 in proportion to their conductance values.

Let us consider the more general case of current division with a set of N parallel conductors as shown in Figure 3.4-4. The KCL gives

$$i_s = i_1 + i_2 + i_3 + \cdots + i_N \tag{3.4-7}$$

for which

$$i_n = G_n v \tag{3.4-8}$$

for $n = 1, \ldots, N$. We may write Eq. 3.4-7 as

$$i_s = (G_1 + G_2 + G_3 + \cdots + G_N)v \tag{3.4-9}$$

Therefore

$$i_s = v \sum_{n=1}^{N} G_n \tag{3.4-10}$$

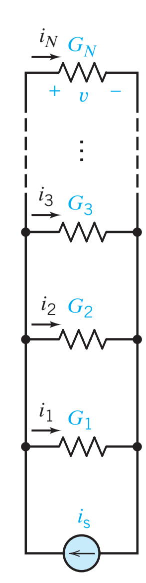

FIGURE 3.4-4
Set of N parallel conductances with a current source i_s.

Since $i_n = G_n v$, we may obtain v from Eq. 3.4-10 and substitute it in Eq. 3.4-8, obtaining

$$i_n = \frac{G_n i_s}{\sum_{n=1}^{N} G_n} \tag{3.4-11}$$

Recall that the equivalent circuit, Figure 3.4-3, has an equivalent conductance G_p such that

$$G_p = \sum_{n=1}^{N} G_n \tag{3.4-12}$$

Therefore

$$i_n = \frac{G_n i_s}{G_p} \tag{3.4-13}$$

which is the basic equation for the current divider with N conductances. Of course, Eq. 3.4-12 can be rewritten as

$$\frac{1}{R_p} = \sum_{n=1}^{N} \frac{1}{R_n} \tag{3.4-14}$$

EXAMPLE 3.4-1 *Parallel Resistors*

For the circuit in Figure 3.4-5 find (a) the current in each branch, (b) the equivalent circuit, and (c) the voltage v. The resistors are

$$R_1 = \frac{1}{2}\ \Omega, \quad R_2 = \frac{1}{4}\ \Omega, \quad R_3 = \frac{1}{8}\ \Omega$$

Solution

The current divider follows the equation

$$i_n = \frac{G_n i_s}{G_p}$$

so it is wise to find the equivalent circuit, as shown in Figure 3.4-6, with its equivalent conductance G_p. We have

$$G_p = \sum_{n=1}^{N} G_n = G_1 + G_2 + G_3 = 2 + 4 + 8 = 14\ \text{S}$$

Recall that the units for conductance are siemens (S). Then

$$i_1 = \frac{G_1 i_s}{G_p} = \frac{2}{14}(28) = 4\ \text{A}$$

Similarly,

$$i_2 = \frac{G_2 i_s}{G_p} = \frac{4(28)}{14} = 8\ \text{A}$$

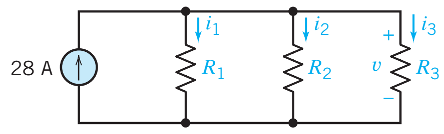

FIGURE 3.4-5 Parallel circuit for Example 3.4-1.

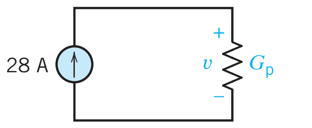

FIGURE 3.4-6 Equivalent circuit for the parallel circuit of Figure 3.4-5.

P1: IML/OVY P2: IML/OVY QC: IML/OVY T1: IML
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and

$$i_3 = \frac{G_3 i_s}{G_p} = 16 \text{ A}$$

Since $i_n = G_n v$, we have

$$v = \frac{i_1}{G_1} = \frac{4}{2} = 2 \text{ V}$$

EXAMPLE 3.4-2 *Parallel Resistors*

INTERACTIVE EXAMPLE

For the circuit of Figure 3.4-7*a,* find the voltage measured by the voltmeter. Then show that the power absorbed by the two resistors is equal to that supplied by the source.

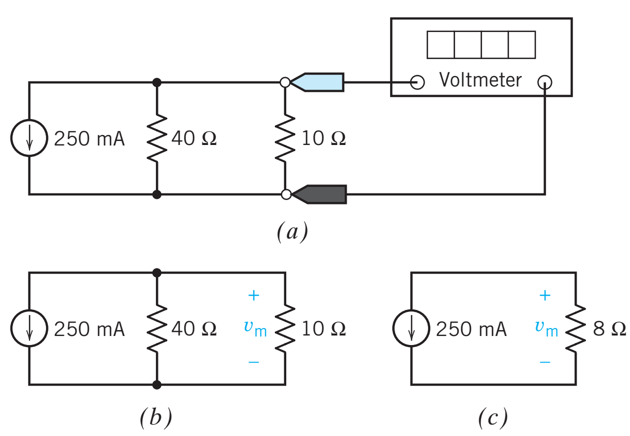

FIGURE 3.4-7 (*a*) A circuit containing parallel resistors. (*b*) The circuit after the ideal voltmeter has been replaced by the equivalent open circuit and a label has been added to indicate the voltage measured by the voltmeter, v_m. (*c*) The circuit after the parallel resistors have been replaced by an equivalent resistance.

Solution

Figure 3.4-7*b* shows the circuit after the ideal voltmeter has been replaced by the equivalent open circuit and a label has been added to indicate the voltage measured by the voltmeter, v_m. The two resistors are connected in parallel and can be replaced with a single equivalent resistor. The resistance of this equivalent resistor is calculated as

$$\frac{40 \cdot 10}{40 + 10} = 8\ \Omega$$

Figure 3.4-7*c* shows the circuit after the parallel resistors have been replaced by the equivalent resistor. The current in the equivalent resistor is 250 mA, directed upward. This current and the voltage v_m do not adhere to the passive convention. The current in the equivalent resistance can also be expressed as −250 mA, directed downward. This current and the voltage v_m do adhere to the passive convention. Ohm's law gives

$$v_m = 8(-0.25) = -2 \text{ V}$$

The voltage v_m in Figure 3.4-7*b* is equal to the voltage v_m in Figure 3.4-7*c*. This is a consequence of the equivalence of the 8-Ω resistor to the parallel combination of the 40-Ω and 10-Ω resistors. Looking at Figure 3.4-7*b,* we see that the power absorbed by the resistors is

$$p_R = \frac{v_m^2}{40} + \frac{v_m^2}{10} = \frac{2^2}{40} + \frac{2^2}{10} = 0.1 + 0.4 = 0.5 \text{ W}$$

The power supplied by the current source is

$$p_s = 2(0.25) = 0.5 \text{ W}$$

Thus, the power absorbed by the two resistors is equal to that supplied by the source.

EXAMPLE 3.4-3 *Current Divider Design*

The input to the current divider in Figure 3.4-8 is the current, i_s, of the current source. The output is the current, i_o, measured by the ammeter. Specify values of the resistances, R_1 and R_2, to satisfy both of these specifications:

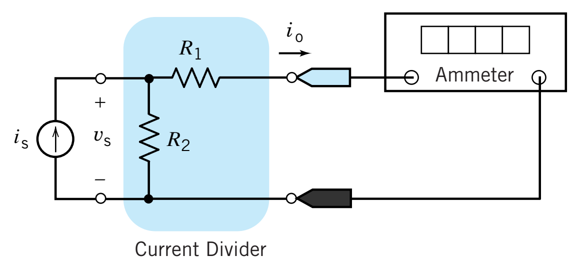

FIGURE 3.4-8

Specification 1: The input and output currents are related by $i_o = 0.8\, i_s$.

Specification 2: The current source is required to supply no more than 10 mW of power when the input to the current divider is $i_s = 2$ mA.

Solution

We'll examine each specification to see what it tells us about the resistor values.

Specification 1: The input and output currents of the current divider are related by

$$i_o = \frac{R_2}{R_1 + R_2} i_s$$

So specification 1 requires

$$\frac{R_2}{R_1 + R_2} = 0.8 \quad \Rightarrow \quad R_2 = 4R_1$$

Specification 2: The power supplied by the current source is given by

$$p_s = i_s v_s = i_s \left(i_s \left(\frac{R_1 R_2}{R_1 + R_2} \right) \right) = {i_s}^2 \left(\frac{R_1 R_2}{R_1 + R_2} \right)$$

So specification 2 requires

$$0.01 \geq (0.002)^2 \left(\frac{R_1 R_2}{R_1 + R_2} \right) \quad \Rightarrow \quad \frac{R_1 R_2}{R_1 + R_2} \leq 2500$$

Combining these results gives

$$\frac{R_1(4R_2)}{R_1 + 4R_2} \leq 2500 \Rightarrow \frac{4}{5} R_1 \leq 2500 \quad \Rightarrow \quad R_1 \leq 3125\,\Omega$$

The solution is not unique. One solution is

$$R_1 = 3 \text{ k}\Omega \quad \text{and} \quad R_2 = 12 \text{ k}\Omega$$

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Exercise 3.4-1 A resistor network consisting of parallel resistors is shown in a package used for printed circuit board electronics in Figure E 3.4-1*a*. This package is only 2 cm × 0.7 cm, and each resistor is 1 kΩ. The circuit is connected to use four resistors as shown in Figure E 3.4-1*b*. Find the equivalent circuit for this network. Determine the current in each resistor when $i_s = 1$ mA.

Answer: $R_p = 250\ \Omega$

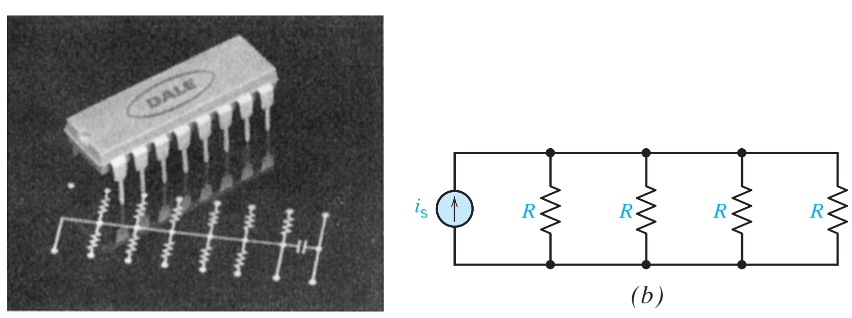


FIGURE E 3.4-1 (*a*) A parallel resistor network. Courtesy of Dale Electronics. (*b*) The connected circuit uses four resistors where $R = 1$ kΩ.

Exercise 3.4-2 Determine the current measured by the ammeter in the circuit shown in Figure E 3.4-2*a*.

Hint: Figure E 3.4-2*b* shows the circuit after the ideal ammeter has been replaced by the equivalent short circuit and a label has been added to indicate the current measured by the ammeter, i_m.

Answer: $i_m = -1$ A

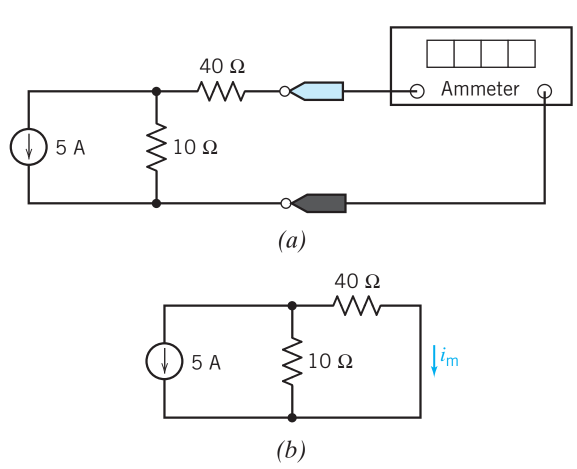


FIGURE E 3.4-2 (*a*) A current divider. (*b*) The current divider after the ideal ammeter has been replaced by the equivalent short circuit and a label has been added to indicate the current measured by the ammeter, i_m.

3.5 Series Voltage Sources and Parallel Current Sources

Voltage sources connected in series are equivalent to a single voltage source. The voltage of the equivalent voltage source is equal to the sum of voltages of the series voltage sources.

Consider the circuit shown in Figure 3.5-1*a*. Notice that the currents of both voltage sources are equal. Accordingly, define the current, i_s, to be

$$i_s = i_a = i_b \tag{3.5-1}$$

Next, define the voltage, v_s, to be

$$v_s = v_a + v_b \tag{3.5-2}$$

Using KCL, KVL, and Ohm's law, we can represent the circuit in Figure 3.5-1*a* by the equations

$$i_c = \frac{v_1}{R_1} + i_s \tag{3.5-3}$$

$$i_s = \frac{v_2}{R_2} + i_3 \tag{3.5-4}$$

$$v_c = v_1 \tag{3.5-5}$$

$$v_1 = v_s + v_2 \tag{3.5-6}$$

$$v_2 = i_3 R_3 \tag{3.5-7}$$

where $i_s = i_a = i_b$ and $v_s = v_a + v_b$. These same equations result from applying KCL, KVL, and Ohm's law to the circuit in Figure 3.5-1*b*. If $i_s = i_a = i_b$ and $v_s = v_a + v_b$, then the circuits shown in Figures 3.5-1*a* and 3.5-1*b* are equivalent because they are both represented by the same equations.

For example, suppose that $i_c = 4$ A, $R_1 = 2\ \Omega$, $R_2 = 6\ \Omega$, $R_3 = 3\ \Omega$, $v_a = 1$ V, and $v_b = 3$ V. The equations describing the circuit in Figure 3.5-1*a* become

$$4 = \frac{v_1}{2} + i_s \tag{3.5-8}$$

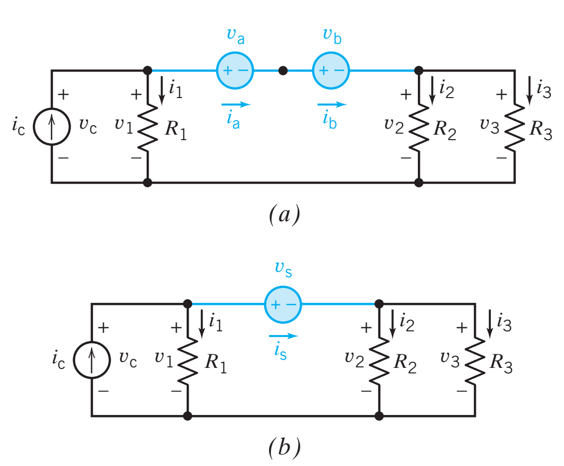


FIGURE 3.5-1
(*a*) A circuit containing voltage sources connected in series and (*b*) an equivalent circuit.

$$i_s = \frac{v_2}{6} + i_3 \tag{3.5-9}$$

$$v_c = v_1 \tag{3.5-10}$$

$$v_1 = 4 + v_2 \tag{3.5-11}$$

$$v_2 = 3i_3 \tag{3.5-12}$$

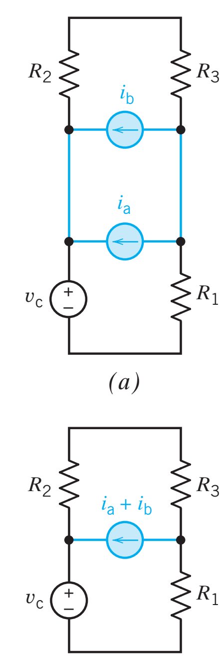


FIGURE 3.5-2 (*a*) A circuit containing parallel current sources and (*b*) an equivalent circuit.

The solution to this set of equations is $v_1 = 6$ V, $i_s = 1$ A, $i_3 = 0.66$ A, $v_2 = 2$ V, and $v_c = 6$ V. Eqs. 3.5-8 to 3.5-12 also describe the circuit in Figure 3.5-1*b*. Thus, $v_1 = 6$ V, $i_s = 1$ A, $i_3 = 0.66$ A, $v_2 = 2$ V, and $v_c = 6$ V in both circuits. Replacing series voltage sources by a single, equivalent voltage source does not change the voltage or current of other elements of the circuit.

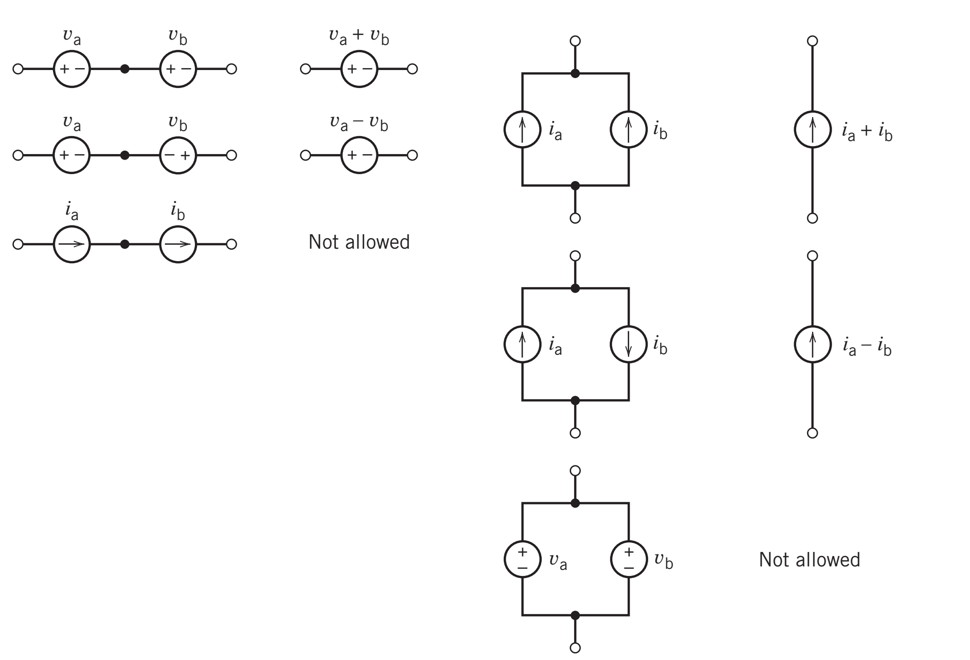
Figure 3.5-2*a* shows a circuit containing parallel current sources. The circuit in Figure 3.5-2*b* is obtained by replacing these parallel current sources by a single, equivalent current source. The current of the equivalent current source is equal to the sum of the currents of the parallel current sources.

We are not allowed to connect independent current sources in series. Series elements have the same current. This restriction prevents series current sources from being independent. Similarly, we are not allowed to connect independent voltage sources in parallel.

Table 3.5-1 summarizes the parallel and series connections of current and voltage sources.

Table 3.5-1 Parallel and Series Voltage and Current Sources

CIRCUIT	EQUIVALENT CIRCUIT	CIRCUIT	EQUIVALENT CIRCUIT



3.6 Circuit Analysis

In this section we consider the analysis of a circuit by replacing a set of resistors with an equivalent resistance, thus reducing the network to a form easily analyzed.

Consider the circuit shown in Figure 3.6-1. Note that it includes a set of resistors that is in series and another set of resistors that is in parallel. It is desired to find the output voltage v_o, so we wish to reduce the circuit to the equivalent circuit shown in Figure 3.6-2.

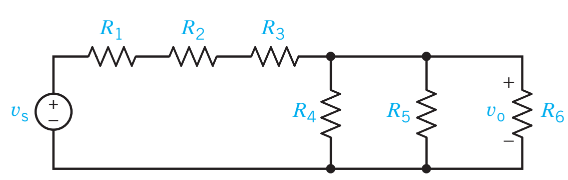


FIGURE 3.6-1 Circuit with a set of series resistors and a set of parallel resistors.

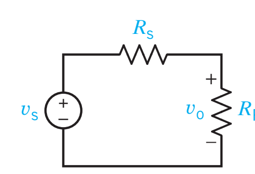


FIGURE 3.6-2 Equivalent circuit for the circuit of Figure 3.6-1.

We note that the equivalent series resistance is

$$R_s = R_1 + R_2 + R_3$$

and the equivalent parallel resistance is

$$R_p = \frac{1}{G_p}$$

where $$G_p = G_4 + G_5 + G_6$$

Then, using the voltage divider principle, with Figure 3.6-2, we have

$$v_o = \frac{R_p}{R_s + R_p} v_s$$

Replacing the series resistors by the equivalent resistor R_s did not change the current or voltage of any other circuit element. In particular, the voltage v_o did not change. Also, the voltage v_o across the equivalent resistor R_p is equal to the voltage across each of the parallel resistors. Consequently, the voltage v_o in Figure 3.6-2 is equal to the voltage v_o in Figure 3.6-1. We can analyze the simple circuit in Figure 3.6-2 to find the value of the voltage v_o and know that the voltage v_o in the more complicated circuit shown in Figure 3.6-1 has the same value.

EXAMPLE 3.6-1 *Series and Parallel Resistors*

Consider the circuit shown in Figure 3.6-3. Find the current i_1 when

$$R_4 = 2\,\Omega \quad \text{and} \quad R_2 = R_3 = 8\,\Omega$$

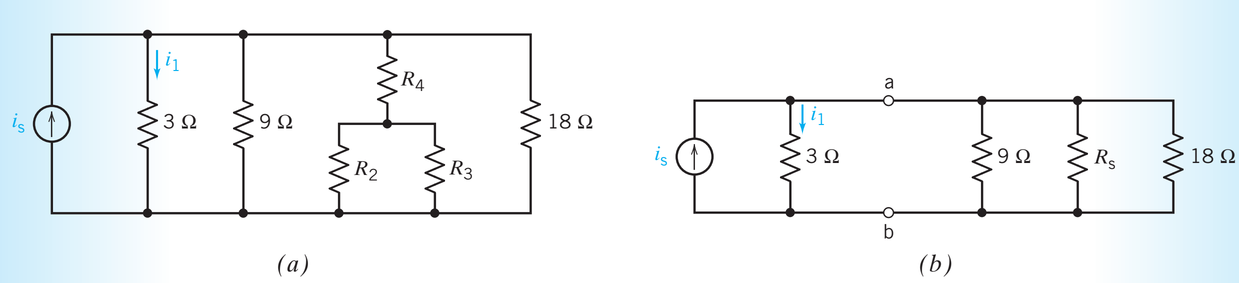


FIGURE 3.6-3 (*a*) Circuit for Example 3.6-1. (*b*) Partially reduced circuit for Example 3.6-1.

P1: IML/OVY P2: IML/OVY QC: IML/OVY T1: IML
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Solution

Since the objective is to find i_1, we will attempt to reduce the circuit so that the 3-Ω resistor is in parallel with one resistor and the current source i_s. Then we can use the current divider principle to obtain i_1. Since R_2 and R_3 are in parallel, we find an equivalent resistance as

$$R_{p1} = \frac{R_2 R_3}{R_2 + R_3} = 4\ \Omega$$

Then adding R_{p1} to R_4, we have a series equivalent resistor

$$R_s = R_4 + R_{p1} = 2 + 4 = 6\ \Omega$$

Now the R_s resistor is in parallel with three resistors as shown in Figure 3.6-3*b*. However, we wish to obtain the equivalent circuit as shown in Figure 3.6-4 so that we can find i_1. Therefore, we combine the 9-Ω resistor, the 18-Ω resistor, and R_s shown to the right of terminals a–b in Figure 3.6-3*b* into one parallel equivalent conductance G_{p2}. Thus, we find

$$G_{p2} = \frac{1}{9} + \frac{1}{18} + \frac{1}{R_s} = \frac{1}{9} + \frac{1}{18} + \frac{1}{6} = \frac{1}{3}\ \text{S}$$

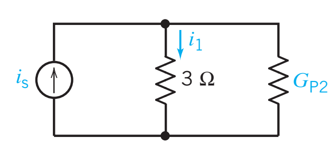

FIGURE 3.6-4
Equivalent circuit for Figure 3.6-3.

Then, using the current divider principle,

$$i_1 = \frac{G_1 i_s}{G_p}$$

where

$$G_p = G_1 + G_{p2} = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$$

Therefore,

$$i_1 = \frac{1/3}{2/3} i_s = \frac{1}{2} i_s$$

EXAMPLE 3.6-2 *Equivalent Resistance*

INTERACTIVE EXAMPLE

The circuit in Figure 3.6-5*a* contains an ohmmeter. An ohmmeter is an instrument that measures resistance in ohms. The ohmmeter will measure the equivalent resistance of the resistor circuit connected to its terminals. Determine the resistance measured by the ohmmeter in Figure 3.6-5*a*.

Solution

Working from left to right, the 30-Ω resistor is parallel to the 60-Ω resistor. The equivalent resistance is

$$\frac{60 \cdot 30}{60 + 30} = 20\ \Omega$$

In Figure 3.6-5*b* the parallel combination of the 30-Ω and 60-Ω resistors has been replaced with the equivalent 20-Ω resistor. Now the two 20-Ω resistors are in series. The equivalent resistance is

$$20 + 20 = 40\ \Omega$$

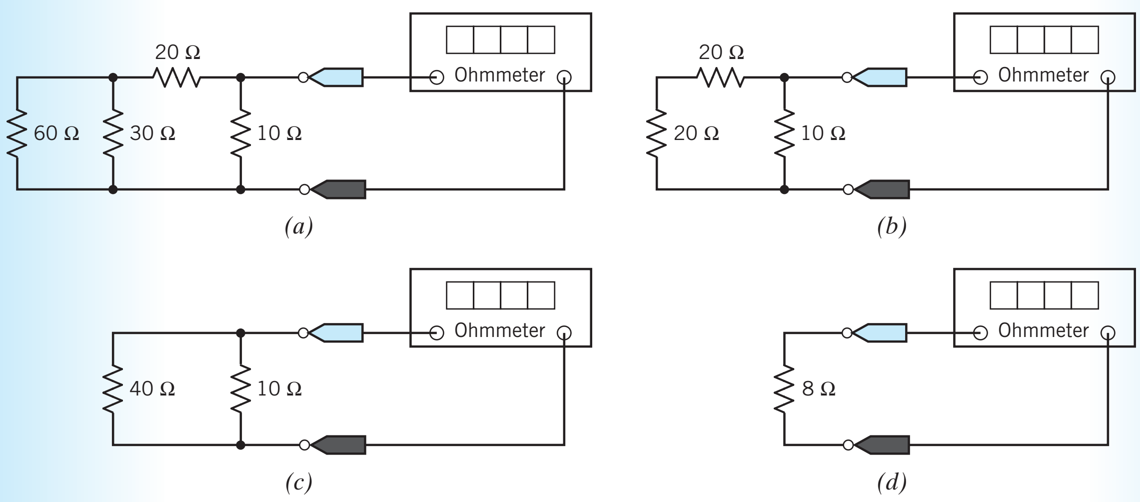

FIGURE 3.6-5

In Figure 3.6-5*c* the series combination of the two 20-Ω resistors has been replaced with the equivalent 40-Ω resistor. Now the 40-Ω resistor is parallel to the 10-Ω resistor. The equivalent resistance is

$$\frac{40 \cdot 10}{40 + 10} = 8\ \Omega$$

In Figure 3.6-5*d* the parallel combination of the 40-Ω and 10-Ω resistors has been replaced with the equivalent 8-Ω resistor. Thus, the ohmmeter measures a resistance equal to 8 Ω.

EXAMPLE 3.6-3 *Circuit Analysis Using Equivalent Resistances*

Determine the values of i_3, v_4, i_5, and v_6 in circuit shown in Figure 3.6-6.

Solution

The circuit shown in Figure 3.6-7 has been obtained from the circuit shown in Figure 3.6-6 by replacing series and parallel combinations of resistances by equivalent resistances. We can use this equivalent circuit to solve this problem in three steps:

1. Determine the values of the resistances R_1, R_2, and R_3 in Figure 3.6-7 that make the circuit in Figure 3.6-7 equivalent to the circuit in Figure 3.6-6.
2. Determine the values of v_1, v_2, and i in Figure 3.6-7.
3. Because the circuits are equivalent, the values of v_1, v_2, and i in Figure 3.6-6 are equal to the values of v_1, v_2, and i in Figure 3.6-7. Use voltage and current division to determine the values of i_3, v_4, i_5, and v_6 in Figure 3.6-6.

Step 1: Figure 3.6-8*a* shows the three resistors at the top of the circuit in Figure 3.6-6. We see that the 6-Ω resistor is connected in series with the 18-Ω resistor. In Figure 3.6-8*b* these series resistors have been replaced by the equivalent 24-Ω resistor. Now the 24-Ω resistor is connected in parallel with the 12-Ω resistor. Replacing series resistors by an equivalent resistance does not change the voltage or current in any other element of the circuit. In particular, v_1, the voltage across the 12-Ω resistor, does not change when the series resistors are replaced by the equivalent resistor. In contrast, v_4 is not an element voltage of the circuit shown in Figure 3.6-8*b*.

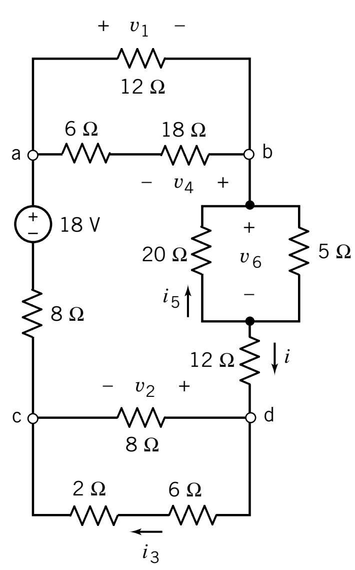

FIGURE 3.6-6 The circuit considered in Example 3.6-3.

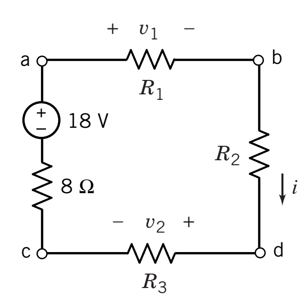

FIGURE 3.6-7 An equivalent circuit for the circuit in Figure 3.3-6.

In Figure 3.6-8*c*, the parallel resistors have been replaced by the equivalent 8-Ω resistor. The voltage across the equivalent resistor is equal to the voltage across each of the parallel resistors, v_1 in this case. In summary, the resistance R_1 in Figure 3.6-7 is given by

$$R_1 = 12 \parallel (6 + 18) = 8\,\Omega$$

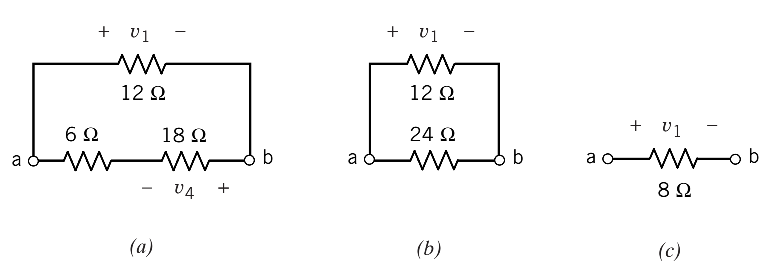

FIGURE 3.6-8

Similarly, the resistances R_2 and R_3 in Figure 3.6-7 are given by

$$R_2 = 12 + (20 \parallel 5) = 16\,\Omega$$
$$R_3 = 8 \parallel (2 + 6) = 4\,\Omega$$

Step 2: Apply KVL to the circuit of Figure 3.6-7 to get

$$R_1 i + R_2 i + R_3 i + 8i - 18 = 0 \quad \Rightarrow \quad i = \frac{18}{R_1 + R_2 + R_3 + 8} = \frac{18}{8 + 16 + 4 + 8} = 0.5\text{ A}$$

Next, Ohm's law gives

$$v_1 = R_1 i = 8(0.5) = 4\text{ V} \quad \text{and} \quad v_2 = R_3 i = 4(0.5) = 2\text{ V}$$

Step 3: The values of v_1, v_2, and i in Figure 3.6-6 are equal to the values of v_1, v_2, and i in Figure 3.6-7. Returning our attention to Figure 3.6-6, and paying attention to reference directions, we can determine the values of i_3, v_4, i_5, and v_6 using voltage division, current division, and Ohm's law:

$$i_3 = \frac{8}{8 + (2 + 6)} i = \frac{1}{2}(0.5) = 0.25 \text{ A}$$

$$v_4 = -\frac{18}{6 + 18} v_1 = -\frac{3}{4}(4) = -3 \text{ V}$$

$$i_5 = -\frac{5}{20 + 5} i = -\left(\frac{1}{5}\right)(0.5) = -0.1 \text{ A}$$

$$v_6 = (20 \parallel 5) i = 4(0.5) = 2 \text{ V}$$

In general, we may find the equivalent resistance (or conductance) for a portion of a circuit consisting only of resistors and then replace that portion of the circuit with the equivalent resistance. For example, consider the circuit shown in Figure 3.6-9. We use $R_{\text{eq x–y}}$ to denote the equivalent resistance seen looking into terminals x–y. We note that the equivalent resistance to the right of terminals c–d is

$$R_{\text{eq c–d}} = \frac{15(6 + 4)}{15 + (6 + 4)} = \frac{150}{25} = 6\,\Omega$$

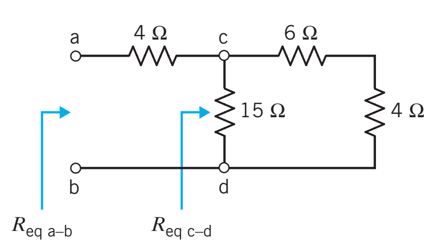

FIGURE 3.6-9
The equivalent resistance looking into terminals c–d is denoted as $R_{\text{eq c–d}}$.

Then the equivalent resistance of the circuit to the right of terminals a–b is

$$R_{\text{eq a–b}} = 4 + R_{\text{eq c–d}} = 4 + 6 = 10\,\Omega$$

Exercise 3.6-1 Determine the resistance measured by the ohmmeter in Figure E 3.6-1.

Answer: $\frac{(30 + 30) \cdot 30}{(30 + 30) + 30} + 30 = 50\,\Omega$

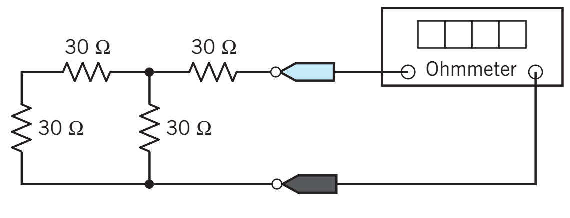

FIGURE E 3.6-1

3.7 Analyzing Resistive Circuits Using MATLAB

The computer program MATLAB is a tool for making mathematical calculations. In this section MATLAB is used to solve the equations encountered when analyzing a resistive circuit. Consider the resistive circuit shown in Figure 3.7-1*a*. The goal is to determine the value of the input voltage, V_s, required to cause the current I to be 1 A.

(Subscripts can't be used in the MATLAB input file. Thus V_s and R_p in Figure 3.7-1 become Vs and Rp in the MATLAB input file. We have been using lowercase letters to represent element voltages and currents, but in MATLAB examples we will use capital letters to represent currents and voltages to improve the readability of the MATLAB input file. For example, the input and output voltages are denoted as V_s and V_o rather than v_s and v_o.)

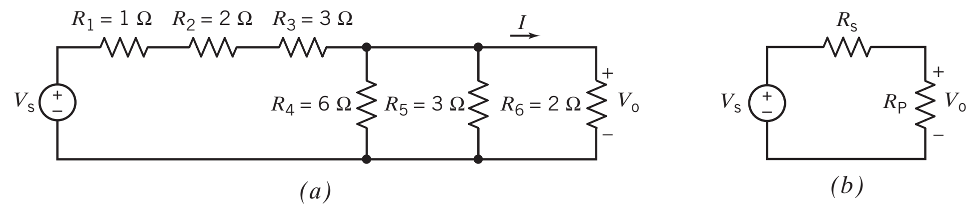

FIGURE 3.7-1 (*a*) A resistive circuit and (*b*) an equivalent circuit.

Resistors R_1, R_2, and R_3 are connected in series and can be replaced by an equivalent resistor, R_s, given by

$$R_s = R_1 + R_2 + R_3 \tag{3.7-1}$$

Resistors R_4, R_5, and R_6 are connected in parallel and can be replaced by an equivalent resistor, R_p, given by

$$R_p = \frac{1}{\dfrac{1}{R_4} + \dfrac{1}{R_5} + \dfrac{1}{R_6}} \tag{3.7-2}$$

Figure 3.7-1*b* shows the circuit after R_1, R_2, and R_3 are replaced by R_s and R_4, R_5, and R_6 are replaced by R_P. Applying voltage division to the circuit in Figure 3.7-1*b* gives

$$V_o = \frac{R_p}{R_s + R_p} V_s \tag{3.7-3}$$

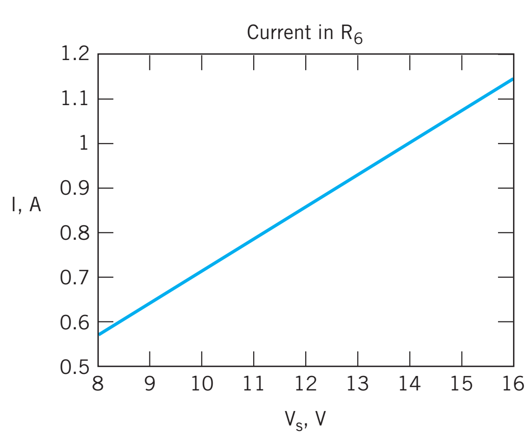

FIGURE 3.7-2 Plot of I versus V_s for the circuit shown in Figure 3.7-1.

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```
% Analyzing Resistive Circuits Using MATLAB - ch3ex.m
%— — — — — — — — — — — — — — — — — — — — — — — — — — — — — — — —
% Vary the input voltage from 8 to 16 volts in 0.1 volt steps.
%— — — — — — — — — — — — — — — — — — — — — — — — — — — — — — — —
Vs = 8:0.1:1:16;
%— — — — — — — — — — — — — — — — — — — — — — — — — — — — — — — —
%              Enter Values of the resistances.
%— — — — — — — — — — — — — — — — — — — — — — — — — — — — — — — —
R1 = 1; R2 = 2; R3 = 3;      % series resistors, ohms
R4 = 6; R5 = 3; R6 = 2;      % parallel resistors, ohms
%— — — — — — — — — — — — — — — — — — — — — — — — — — — — — — — —
%    Find the current, I, corresponding to each value of Vs.
%— — — — — — — — — — — — — — — — — — — — — — — — — — — — — — — —
Rs = R1 + R2 + R3;                     % Equation 3.7-1
Rp = 1 / (1/R4 +1/R5 +1/R6);           % Equation 3.7-2
for k = 1:length (Vs)
    Vo(k) = Vs(k) * Rp / (Rp + Rs);    % Equation 3.7-3
    I(k)    = Vo(k) / R6;              % Equation 3.7-4
end
%— — — — — — — — — — — — — — — — — — — — — — — — — — — — — — — —
%                  Plot I versus Vs
%— — — — — — — — — — — — — — — — — — — — — — — — — — — — — — — —
plot (Vs, I)
grid
xlabel (’Vs, V’), ylabel (’I,A’)
title (’Current in R6)
```

FIGURE 3.7-3 MATLAB input file used to obtain the plot of I versus V_s shown in Figure 3.7-2.

where V_o is the voltage across R_p in Figure 3.7-1*b* and is also the voltage across the parallel resistors in Figure 3.7-1*a*. Ohm's law indicates that the current in R_6 is given by

$$I = \frac{V_o}{R_6} \tag{3.7-4}$$

Figure 3.7-2 shows a plot of the output current I versus the input voltage V_s. This plot shows that I will be 1 A when $V_s = 14$ V. Figure 3.7-3 shows the MATLAB input file that was used to obtain Figure 3.7-2. The MATLAB program first causes V_s to vary over a range of voltages. Next, MATLAB calculates the value of I corresponding to each value of V_s using Eqs. 3.7-1 through 3.7-4. Finally, MATLAB plots the current I versus the voltage V_s.

3.8 How Can We Check . . . ?

Engineers are frequently called upon to check that a solution to a problem is indeed correct. For example, proposed solutions to design problems must be checked to confirm that all of the specifications have been satisfied. In addition, computer output must be reviewed to guard against data-entry errors, and claims made by vendors must be examined critically.

Engineering students are also asked to check the correctness of their work. For example, occasionally just a little time remains at the end of an exam. It is useful to be able to quickly identify those solutions that need more work.

The following example illustrates techniques useful for checking the solutions of the sort of problem discussed in this chapter.

P1: IML/OVY P2: IML/OVY QC: IML/OVY T1: IML
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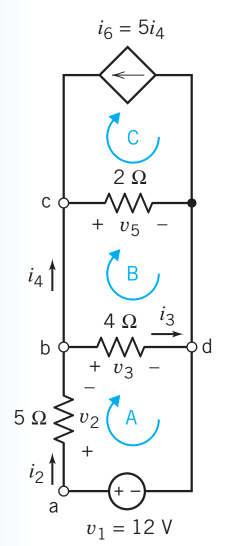
EXAMPLE 3.8-1 *How Can We Check Voltage and Current Values?*

The circuit shown in Figure 3.8-1*a* was analyzed by writing and solving a set of simultaneous equations:

$$12 = v_2 + 4i_3, \quad i_4 = \frac{v_2}{5} + i_3, \quad v_5 = 4i_3, \quad \text{and} \quad \frac{v_5}{2} = i_4 + 5i_4$$



(*a*)

```
v2 := 0   i3 := 0   i4 := 0   v5 := 0
Given
     12 ≈ v2 + 4 · i3          Apply KVL to loop A.
           v2
     i4 ≈ ---- + i3            Apply KCL at node b.
           5
     v5 ≈ 4 · i3               Apply KVL to loop B.
     v5
     --- ≈ i4 + 5 · i4         Apply KCL at node c.
      2
                              ⎡-60⎤
                              ⎢ 18⎥
   Find (v2,i3,i4,v5) =       ⎢  6⎥
                              ⎣ 72⎦
```

(*b*)

FIGURE 3.8-1 (*a*) An example circuit and (*b*) computer analysis using Mathcad.

The computer Mathcad (*Mathcad User's Guide*, 1991) was used to solve the equations as shown in Figure 3.8-1*b*. It was determined that

$$v_2 = -60\text{ V}, \quad i_3 = 18\text{ A}, \quad i_4 = 6\text{ A}, \quad \text{and} \quad v_5 = 72\text{ V}$$

How can we check that these currents and voltages are correct?

Solution

The current i_2 can be calculated from v_2, i_3, i_4, and v_5 in a couple of different ways. First, Ohm's law gives

$$i_2 = \frac{v_2}{5} = \frac{-60}{5} = -12\text{ A}$$

Next, applying KCL at node b gives

$$i_2 = i_3 + i_4 = 18 + 6 = 24\text{ A}$$

Clearly, i_2 cannot be both −12 and 24 A, so the values calculated for v_2, i_3, i_4, and v_5 cannot be correct. Checking the equations used to calculate v_2, i_3, i_4, and v_5, we find a sign error in the KCL equation corresponding to node b. This equation should be

$$i_4 = \frac{v_2}{5} - i_3$$

After making this correction, v_2, i_3, i_4, and v_5 are calculated to be

$$v_2 = 7.5\text{ V}, \quad i_3 = 1.125\text{ A}, \quad i_4 = 0.375\text{ A}, \quad v_5 = 4.5\text{ V}$$

Now

$$i_2 = \frac{v_2}{5} = \frac{7.5}{5} = 1.5$$

and

$$i_2 = i_3 + i_4 = 1.125 + 0.375 = 1.5$$

This checks as we expected.

As an additional check, consider v_3. First, Ohm's law gives

$$v_3 = 4i_3 = 4(1.125) = 4.5$$

Next, applying KVL to the loop consisting of the voltage source and the 4-Ω and 5-Ω resistors gives

$$v_3 = 12 - v_2 = 12 - 7.5 = 4.5$$

Finally, applying KVL to the loop consisting of the 2-Ω and 4-Ω resistors gives

$$v_3 = v_5 = 4.5$$

The results of these calculations agree with each other, indicating that

$$v_2 = 7.5\ \text{V}, \quad i_3 = 1.125\ \text{A}, \quad i_4 = 0.375\ \text{A}, \quad v_5 = 4.5\ \text{V}$$

are the correct values.

3.9 Design Example

Adjustable Voltage Source

A circuit is required to provide an adjustable voltage. The specifications for this circuit are:

1. It should be possible to adjust the voltage to any value between −5 V and +5 V. It should not be possible to accidentally obtain a voltage outside this range.
2. The load current will be negligible.
3. The circuit should use as little power as possible.

The available components are:

1. Potentiometers: resistance values of 10 kΩ, 20 kΩ, and 50 kΩ are in stock
2. A large assortment of standard 2 percent resistors having values between 10 Ω and 1 MΩ (see Appendix E)
3. Two power supplies (voltage sources): one 12-V supply and one −12-V supply, both rated at 100 mA (maximum)

Describe the Situation and the Assumptions

Figure 3.9-1 shows the situation. The voltage v is the adjustable voltage. The circuit that uses the output of the circuit being designed is frequently called the "load." In this case, the load current is negligible, so $i = 0$.

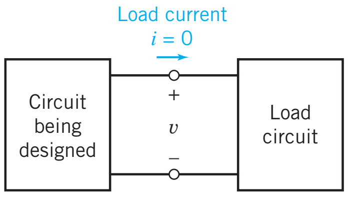


FIGURE 3.9-1 The circuit being designed provides an adjustable voltage, v, to the load circuit.

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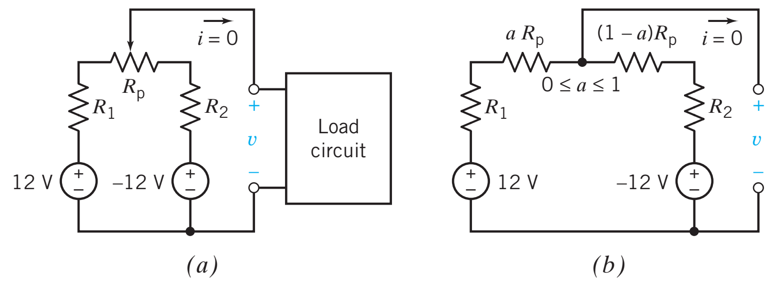


FIGURE 3.9-2 (*a*) A proposed circuit for producing the variable voltage, v, and (*b*) the equivalent circuit after the potentiometer is modeled.

State the Goal

A circuit providing the adjustable voltage

$$-5\text{V} \le v \le +5\text{V}$$

must be designed using the available components.

Generate a Plan

Make the following observations.

1. The adjustability of a potentiometer can be used to obtain an adjustable voltage v.
2. Both power supplies must be used so that the adjustable voltage can have both positive and negative values.
3. The terminals of the potentiometer cannot be connected directly to the power supplies because the voltage v is not allowed to be as large as 12 V or −12 V.

These observations suggest the circuit shown in Figure 3.9-2*a*. The circuit in Figure 3.9-2*b* is obtained by using the simplest model for each component in Figure 3.9-2*a*.

To complete the design, values need to be specified for R_1, R_2, and R_p. Then several results need to be checked and adjustments made, if necessary.

1. Can the voltage v be adjusted to any value in the range −5 V to +5V?
2. Are the voltage source currents less than 100 mA? This condition must be satisfied if the power supplies are to be modeled as ideal voltage sources.
3. Is it possible to reduce the power absorbed by R_1, R_2, and R_p?

Act on the Plan

It seems likely that R_1 and R_2 will have the same value, so let $R_1 = R_2 = R$. Then it is convenient to redraw Figure 3.9-2*b* as shown in Figure 3.9-3.

Applying KVL to the outside loop yields

$$-12 + Ri_a + aR_p i_a + (1-a)R_p i_a + Ri_a - 12 = 0$$

so

$$i_a = \frac{24}{2R + R_p}$$

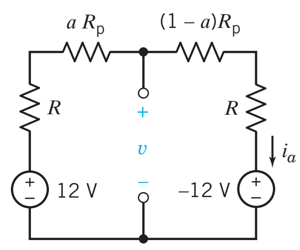


FIGURE 3.9-3
The circuit after setting $R_1 = R_2 = R$.

Next, applying KVL to the left loop gives

$$v = 12 - (R + aR_p)i_a$$

Substituting for i_a gives

$$v = 12 - \frac{24(R + aR_p)}{2R + R_p}$$

When $a = 0$, v must be 5 V, so

$$5 = 12 - \frac{24R}{2R + R_p}$$

Solving for R gives

$$R = 0.7R_p$$

Suppose the potentiometer resistance is selected to be $R_p = 20\,\text{k}\Omega$, the middle of the three available values. Then

$$R = 14\,\text{k}\Omega$$

Verify the Proposed Solution

As a check, notice that when $a = 1$

$$v = 12 - \left(\frac{14\,\text{k} + 20\,\text{k}}{28\,\text{k} + 20\,\text{k}}\right)24 = -5$$

as required. The specification that

$$-5\text{ V} \le v \le 5\text{ V}$$

has been satisfied. The power absorbed by the three resistances is

$$p = i_a^2(2R + R_p) = \frac{24^2}{2R + R_p}$$

so

$$p = 12\,\text{mW}$$

Notice that this power can be reduced by choosing R_p to be as large as possible, 50 k Ω in this case. Changing R_p to 50 kΩ requires a new value of R:

$$R = 0.7 \times R_p = 35\,\text{k}\Omega$$

Since

$$-5\text{ V} = 12 - \left(\frac{35\text{ k} + 50\text{ k}}{70\text{ k} + 50\text{ k}}\right)24 \le v \le 12 - \left(\frac{35\text{ k}}{70\text{ k} + 50\text{ k}}\right)24 = 5\text{ V}$$

the specification that

$$-5\text{ V} \le v \le 5\text{ V}$$

has been satisfied. The power absorbed by the three resistances is now

$$p = \frac{24^2}{50\text{ k} + 70\text{ k}} = 5\,\text{mW}$$

Finally, the power supply current is

$$i_a = \frac{24}{50\text{ k} + 70\text{ k}} = 0.2\,\text{mA}$$

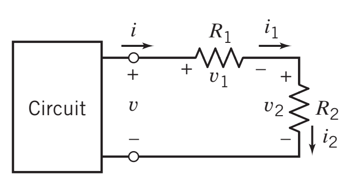
which is well below the 100 mA that the voltage sources are able to supply. The design is complete.

3.10 Summary

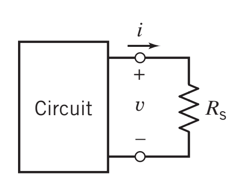
- Kirchhoff's current law (KCL) states that the algebraic sum of the currents entering a node is zero. Kirchhoff's voltage law (KVL) states that the algebraic sum of the voltages around a closed path (loop) is zero.
- Simple electric circuits can be analyzed using only Kirchhoff's laws and the constitutive equations of the circuit elements.
- Series resistors act like a "voltage divider," and parallel resistors act like a "current divider." The first two rows of Table 3.10-1 summarize the relevant equations.
- Series resistors are equivalent to a single "equivalent resistor." Similarly, parallel resistors are equivalent to a single "equivalent resistor." The first two rows of Table 3.10-1 summarize the relevant equations.
- Series voltage sources are equivalent to a single "equivalent voltage source." Similarly, parallel current sources are equivalent to a single "equivalent current." The last two rows of Table 3.10-1 summarize the relevant equations.
- Often circuits consisting entirely of resistors can be reduced to a single equivalent resistor by repeatedly replacing series and/or parallel resistors by equivalent resistors.

Table 3.10-1 Equivalent Circuits for Series and Parallel Elements

Series resistors

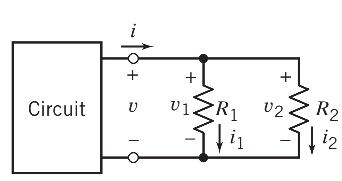


$i = i_1 = i_2,\ v_1 = \dfrac{R_1}{R_1 + R_2} v,$ and $v_2 = \dfrac{R_2}{R_1 + R_2}$

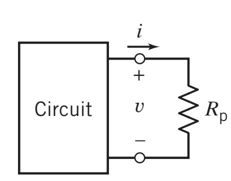


$R_s = R_1 + R_2$ and $v = R_s i$

Parallel resistors

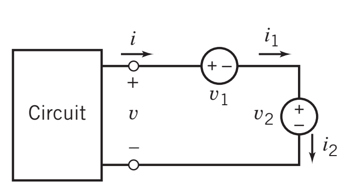


$v = v_1 = v_2,\ i_1 = \dfrac{R_2}{R_1 + R_2} i,$ and $i_2 = \dfrac{R_1}{R_1 + R_2}$

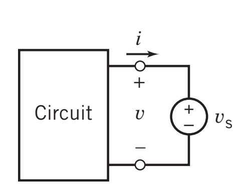


$R_p = \dfrac{R_1 R_2}{R_1 + R_2}$ and $v = R_p i$

Series voltage sources

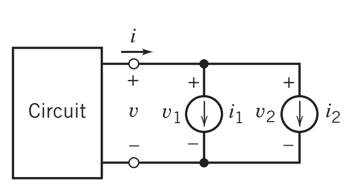


$i = i_1 = i_2$ and $v = v_1 + v_2$

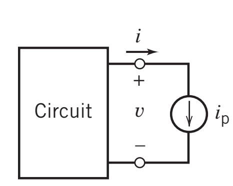


$v_s = v_1 + v_2$

Parallel current sources



$v = v_1 = v_2$ and $i = i_1 + i_2$



$i_p = i_1 + i_2$

Problems

Section 3.2 Kirchhoff's Laws

P 3.2-1 Consider the circuit shown in Figure P 3.2-1. Determine the values of the power supplied by branch B and the power supplied by branch F.

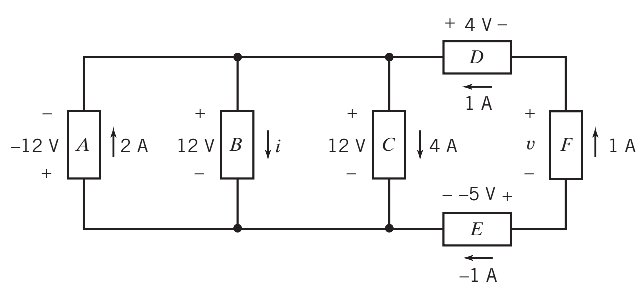


FIGURE P 3.2-1

P 3.2-2 Determine the values of i_2, i_4, v_2, v_3, and v_6 in Figure P 3.2-2.

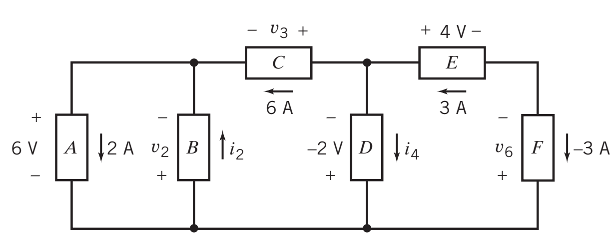


FIGURE P 3.2-2

P 3.2-3 Consider the circuit shown in Figure P 3.2-3.

(a) Suppose that $R_1 = 6\ \Omega$ and $R_2 = 3\ \Omega$. Find the current i and the voltage v.

(b) Suppose, instead, that $i = 1.5$ A and $v = 2$ V. Determine the resistances R_1 and R_2.

(c) Suppose, instead, that the voltage source supplies 24 W of power and that the current source supplies 9 W of power. Determine the current i, the voltage v, and the resistances R_1 and R_2.

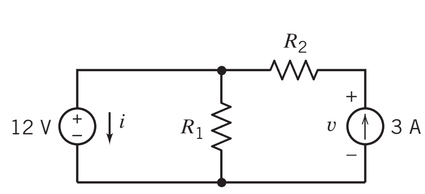


FIGURE P 3.2-3

P 3.2-4 Determine the power absorbed by each of the resistors in the circuit shown in Figure P 3.2-4.
Answer: The 4-Ω resistor absorbs 100 W, the 6-Ω resistor absorbs 24 W, and the 8-Ω resistor absorbs 72 W.

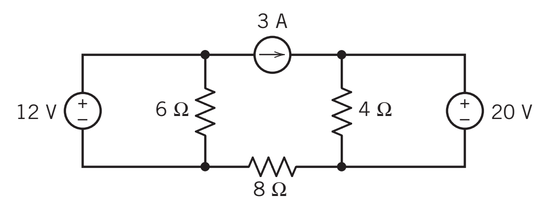


FIGURE P 3.2-4

P 3.2-5 Determine the power absorbed by each of the resistors in the circuit shown in Figure P 3.2-5.
Answer: The 4-Ω resistor absorbs 16 W, the 6-Ω resistor absorbs 24 W, and the 8-Ω resistor absorbs 8 W.

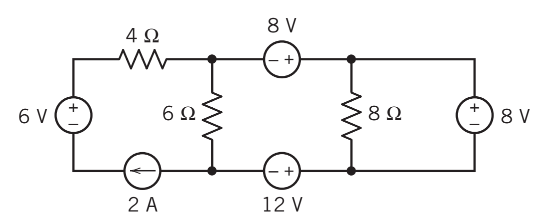


FIGURE P 3.2-5

P 3.2-6 Determine the power supplied by each current source in the circuit of Figure P 3.2-6.
Answer: The 2-mA current source supplies 6 mW and the 1-mA current source supplies −7 mW.

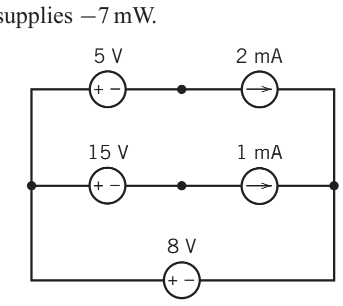


FIGURE P 3.2-6

P 3.2-7 Determine the power supplied by each voltage source in the circuit of Figure P 3.2-7.
Answer: The 2-V voltage source supplies 2 mW and the 3-V voltage source supplies −6 mW.

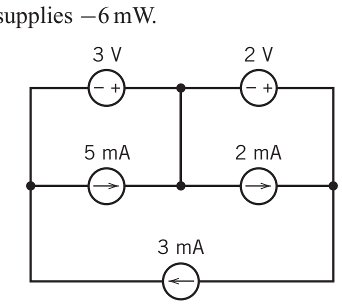


FIGURE P 3.2-7

P 3.2-8 What is the value of the resistance R in Figure P 3.2-8.?
Hint: Assume an ideal ammeter. An ideal ammeter is equivalent to a short circuit.
Answer: $R = 4\ \Omega$

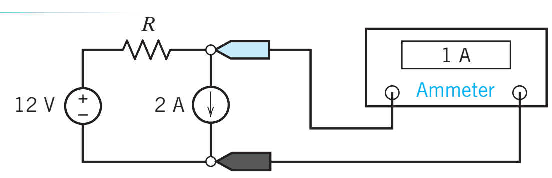


FIGURE P 3.2-8

P 3.2-9 The voltmeter in Figure P 3.2-9 measures the value of the voltage across the current source to be 56 V. What is the value of the resistance R?

Hint: Assume an ideal voltmeter. An ideal voltmeter is equivalent to an open circuit.

Answer: $R = 10\ \Omega$

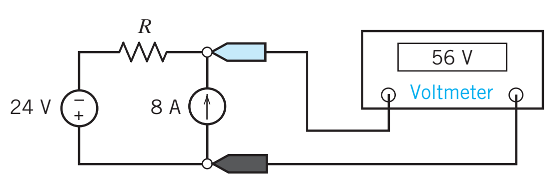


FIGURE P 3.2-9

P 3.2-10 Determine the values of the resistances R_1 and R_2 in Figure P 3.2-10.

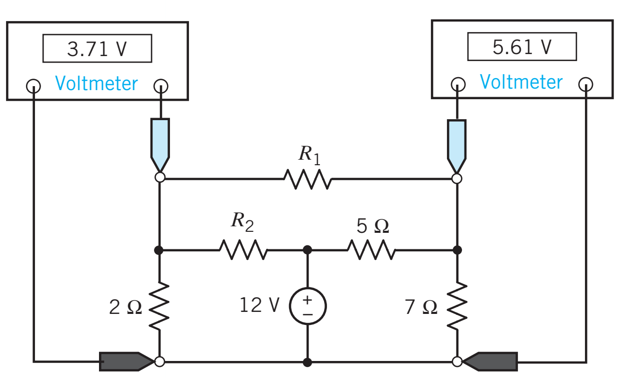


FIGURE P 3.2-10

P 3.2-11 The circuit shown in Figure P 3.2-11 consists of five voltage sources and four current sources. Express the power supplied by each source in terms of the voltage source voltages and the current source currents.

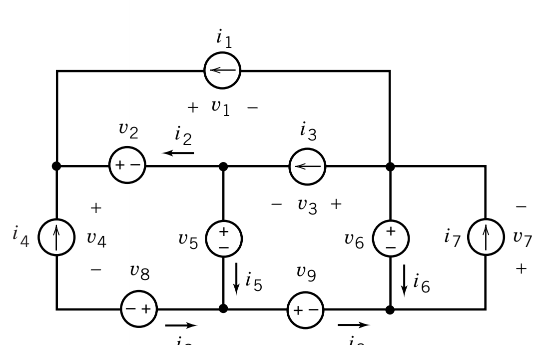


FIGURE P 3.2-11

P 3.2-12 Determine the power received by each of the resistors in the circuit shown in Figure P 3.2-12.

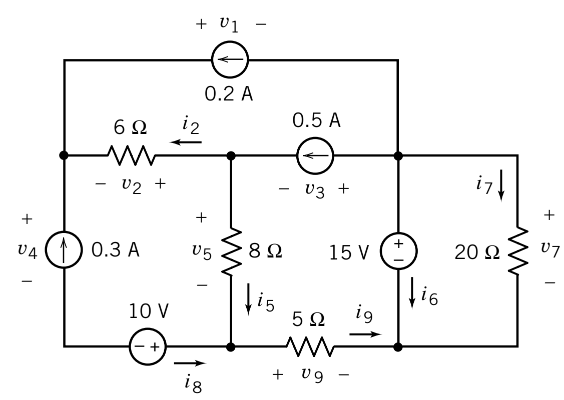


FIGURE P 3.2-12

P 3.2-13 Determine the voltage and current of each of the circuit elements in the circuit shown in Figure P 3.2-13.

Hint: You'll need to specify reference directions for the element voltages and currents. There is more than one way to do that, and your answers will depend on the reference directions that you choose.

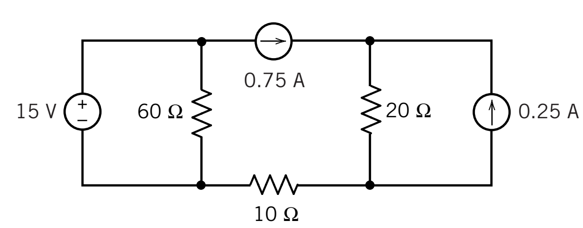


FIGURE P 3.2-13

P 3.2-14 Determine the voltage and current of each of the circuit elements in the circuit shown in Figure P 3.2-14.

Hint: You'll need to specify reference directions for the element voltages and currents. There is more than one way to do that, and your answers will depend on the reference directions that you choose.

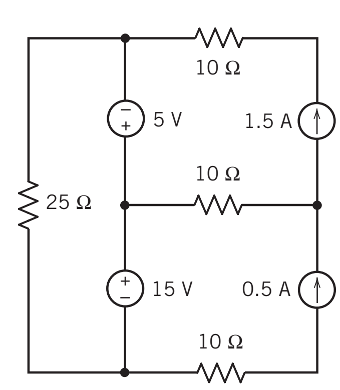


FIGURE P 3.2-14

P 3.2-15 Determine the value of the current that is measured by the meter in Figure P 3.2-15.

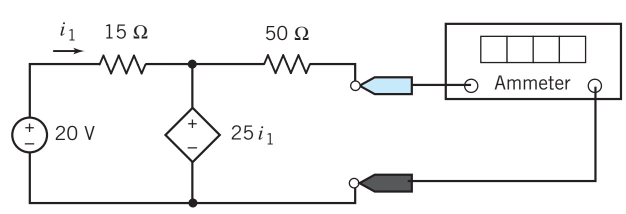


FIGURE P 3.2-15

P 3.2-16 Determine the value of the current that is measured by the meter in Figure P 3.2-16.

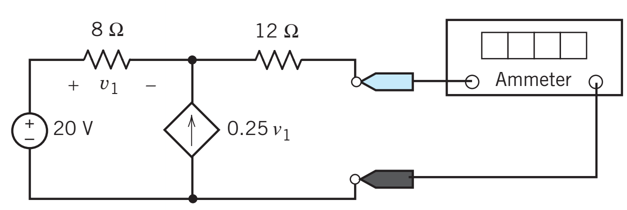


FIGURE P 3.2-16

P 3.2-17 Determine the value of the voltage that is measured by the meter in Figure P 3.2-17.

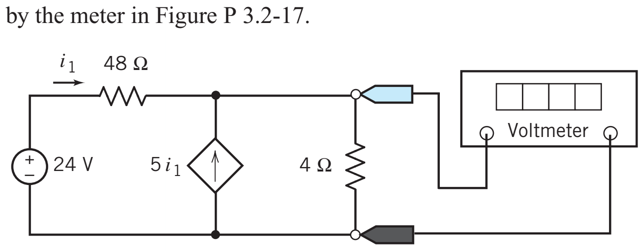


FIGURE P 3.2-17

P 3.2-18 Determine the value of the voltage that is measured by the meter in Figure P 3.2-18.

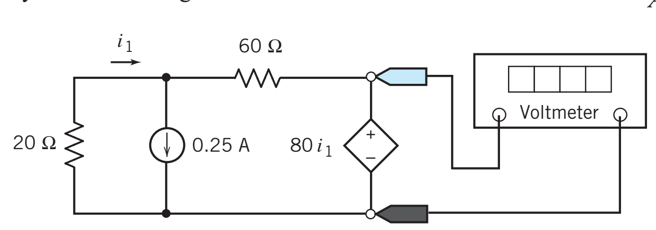


FIGURE P 3.2-18

P 3.2-19 The voltage source in Figure P 3.2-19 supplies 4.8 W of power. The current source supplies 3.6 W. Determine the values of the resistances, R_1 and R_2.

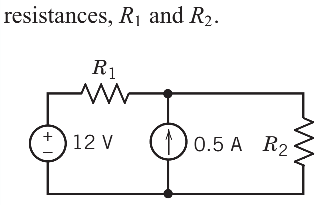


FIGURE P 3.2-19

P 3.2-20 Determine the current i in Figure P 3.2-20.
Answer: $i = 4$ A

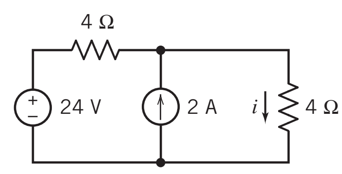


FIGURE P 3.2-20

P 3.2-21 Determine the value of the current i_m in Figure P 3.2-21*a*.
Hint: Apply KVL to the closed path a-b-d-c-a in Figure P 3.2-21*b* to determine v_a. Then apply KCL at node b to find i_m.
Answer: $i_m = 9$ A.

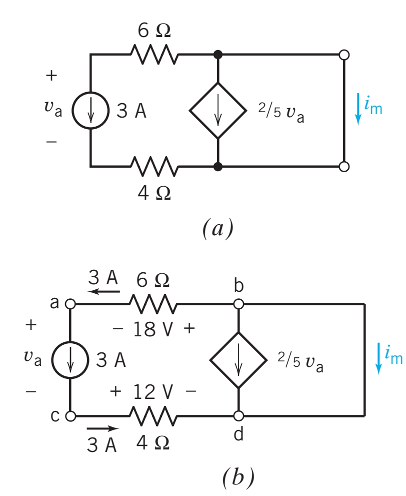


FIGURE P 3.2-21 (*a*) A circuit containing a VCCS. (*b*) The circuit after labeling the nodes and some element currents and voltages.

P 3.2-22 Determine the value of the voltage v_m in Figure P 3.2-22*a*.
Hint: Apply KVL to the closed path a-b-d-c-a in Figure P 3.2-22*b* to determine v_a.
Answer: $v_m = 24$ V

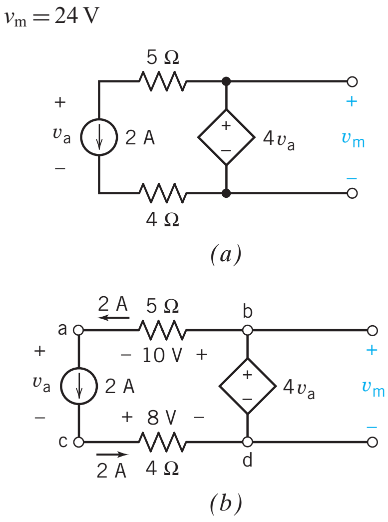


FIGURE P 3.2-22 (*a*) A circuit containing a VCVS. (*b*) The circuit after labeling the nodes and some element currents and voltages.

Section 3.3 Series Resistors and Voltage Division

P 3.3-1 Use voltage division to determine the voltages v_1, v_2, v_3, and v_4 in the circuit shown in Figure P 3.3-1.

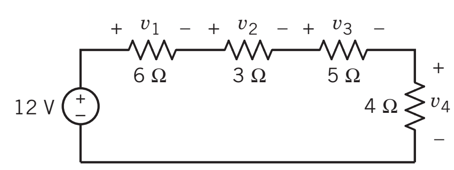


FIGURE P 3.3-1

P 3.3-2 Consider the circuits shown in Figure P 3.3-2.

(a) Determine the value of the resistance R in Figure P 3.3-2b that makes the circuit in Figure P 3.3-2b equivalent to the circuit in Figure P 3.3-2a.

(b) Determine the current i in Figure P 3.3-2b. Because the circuits are equivalent, the current i in Figure P 3.3-2a is equal to the current i in Figure P 3.3-2b.

(c) Determine the power supplied by the voltage source.

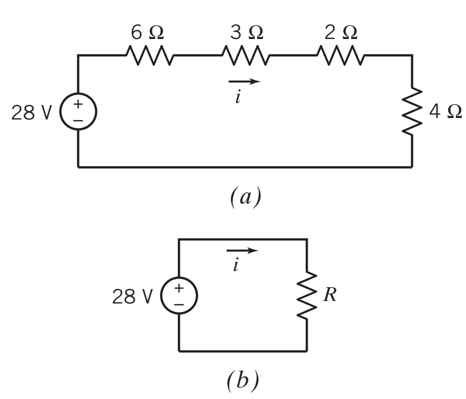


FIGURE P 3.3-2

P 3.3-3 The ideal voltmeter in the circuit shown in Figure P 3.3-3 measures the voltage v.

(a) Suppose $R_2 = 100\ \Omega$. Determine the value of R_1.

(b) Suppose, instead, $R_1 = 100\ \Omega$. Determine the value of R_2.

(c) Suppose, instead, that the voltage source supplies 1.2 W of power. Determine the values of both R_1 and R_2.

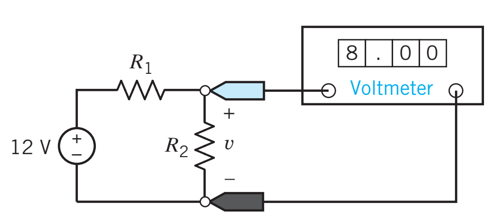


FIGURE P 3.3-3

P 3.3-4 Determine the voltage v in the circuit shown in Figure P 3.3-4.

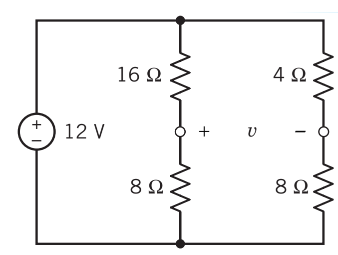


FIGURE P 3.3-4

P 3.3-5 The model of a cable and load resistor connected to a source is shown in Figure P 3.3-5. Determine the appropriate cable resistance, R, so that the output voltage, v_o, remains between 9 V and 13 V when the source voltage, v_s, varies between 20 V and 28 V. The cable resistance can only assume integer values in the range $20 < R < 100\ \Omega$.

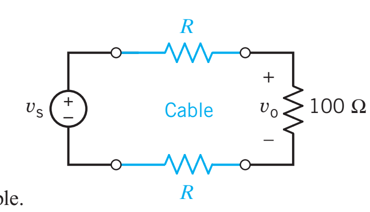


FIGURE P 3.3-5
Circuit with a cable.

P 3.3-6 The input to the circuit shown in Figure P 3.3-6 is the voltage of the voltage source, v_a. The output of this circuit is the voltage measured by the voltmeter, v_b. This circuit produces an output that is proportional to the input, that is

$$v_b = k\, v_a$$

where k is the constant of proportionality.

(a) Determine the value of the output, v_b, when $R = 240\ \Omega$ and $v_a = 18$ V.

(b) Determine the value of the power supplied by the voltage source when $R = 240\ \Omega$ and $v_a = 18$ V.

(c) Determine the value of the resistance, R, required to cause the output to be $v_b = 2$ V when the input is $v_a = 18$ V.

(d) Determine the value of the resistance, R, required to cause $v_b = 0.2 v_a$ (that is, the value of the constant of proportionality is $k = \frac{2}{10}$).

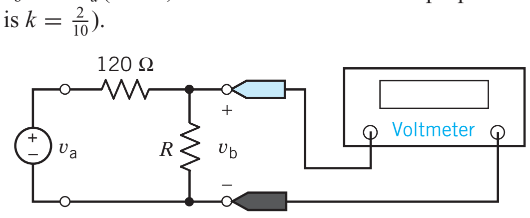


FIGURE P 3.3-6

P 3.3-7 Determine the value of voltage v in the circuit shown in Figure P 3.3-7.

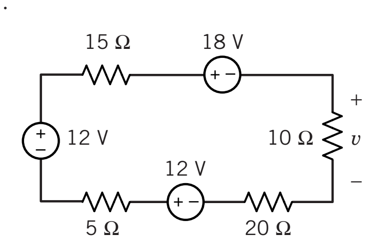


FIGURE P 3.3-7

P3.3-8 Determine the power supplied by the dependent source in the circuit shown in Figure P 3.3-8.

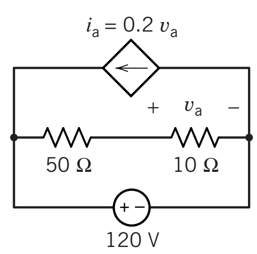


FIGURE P 3.3-8

P3.3-9 A potentiometer can be used as a transducer to convert the rotational position of a dial to an electrical quantity. Figure P 3.3-9 illustrates this situation. Figure P 3.3-9*a* shows a potentiometer having resistance R_p connected to a voltage source. The potentiometer has three terminals, one at each end and one connected to a sliding contact called a wiper. A voltmeter measures the voltage between the wiper and one end of the potentiometer.

Figure P 3.3-9*b* shows the circuit after the potentiometer is replaced by a model of the potentiometer that consists of two resistors. The parameter a depends on the angle, θ, of the dial. Here $a = \frac{\theta}{360°}$, and θ is given in degrees. Also, in Figure P 3.3-9*b*, the voltmeter has been replaced by an open circuit and the voltage measured by the voltmeter, v_m, has been labeled. The input to the circuit is the angle θ, and the output is the voltage measured by the meter, v_m.

(a) Show that the output is proportional to the input.

(b) Let $R_p = 1\ \text{k}\Omega$ and $v_s = 24$ V. Express the output as a function of the input. What is the value of the output when $\theta = 45°$? What is the angle when $v_m = 10$ V?

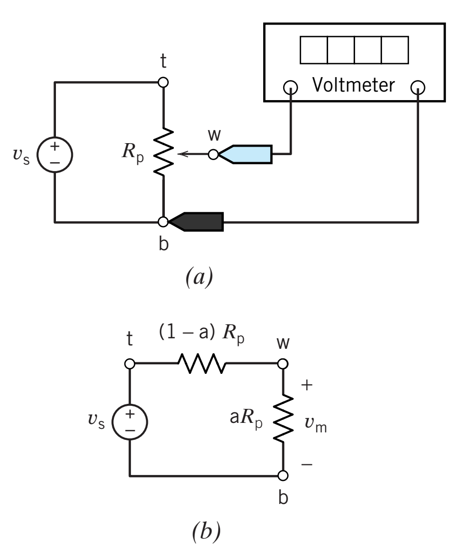


FIGURE P 3.3-9

P3.3-10 Determine the value of the voltage measured by the meter in Figure P 3.3-10.

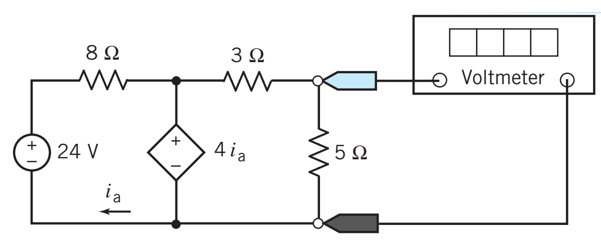


FIGURE P 3.3-10

P3.3-11 For the circuit of Figure P 3.3-11, find the voltage v_3 and the current i and show that the power delivered to the three resistors is equal to that supplied by the source.
Answer: $v_3 = 3$ V, $i = 1$ A

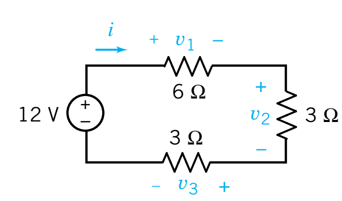


FIGURE P 3.3-11

P 3.3-12 Consider the voltage divider shown in Figure P 3.3-12 when $R_1 = 6\ \Omega$. It is desired that the output power absorbed by $R_1 = 6\ \Omega$ be 6 W. Find the voltage v_o and the required source v_s.
Answer: $v_s = 14$ V, $v_o = 6$ V

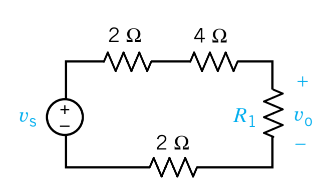


FIGURE P 3.3-12

Section 3.4 Parallel Resistors and Current Division

P 3.4-1 Use current division to determine the currents i_1, i_2, i_3, and i_4 in the circuit shown in Figure P 3.4-1.

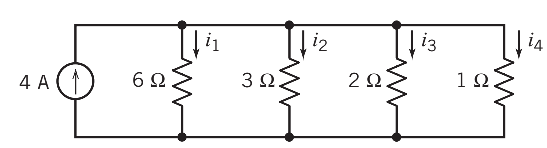


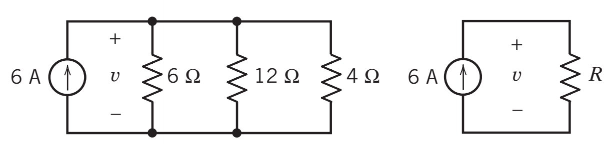
FIGURE P 3.4-1

P 3.4-2 Consider the circuits shown in Figure P 3.4-2.

(a) Determine the value of the resistance R in Figure P 3.4-2*b* that makes the circuit in Figure P 3.4-2*b* equivalent to the circuit in Figure P 3.4-2*a*.

(b) Determine the voltage v in Figure P 3.4-2b. Because the circuits are equivalent, the voltage v in Figure P 3.4-2a is equal to the voltage v in Figure P 3.4-2b.

(c) Determine the power supplied by the current source.



(a) (b)

FIGURE P 3.4-2

P 3.4-3 The ideal voltmeter in the circuit shown in Figure P 3.4-3 measures the voltage v.

(a) Suppose $R_2 = 12\,\Omega$. Determine the value of R_1 and of the current i.

(b) Suppose, instead, $R_1 = 12\,\Omega$. Determine the value of R_2 and of the current i.

(c) Instead, choose R_1 and R_2 to minimize the power absorbed by any one resistor.

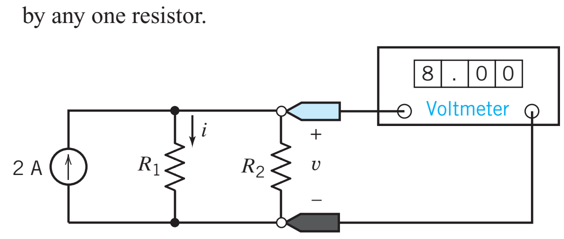


FIGURE P 3.4-3

P 3.4-4 Determine the current i in the circuit shown in Figure P 3.4-4.

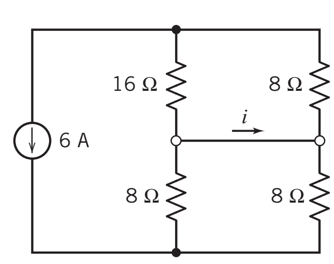


FIGURE P 3.4-4

P 3.4-5 Consider the circuit shown in Figure P 3.4-5 when $4\,\Omega \le R_1 \le 6\,\Omega$ and $R_2 = 10\,\Omega$. Select the source i_s so that v_o remains between 9 V and 13 V.

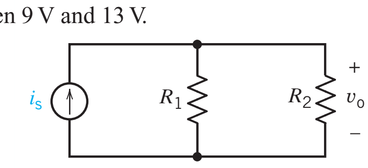


FIGURE P 3.4-5

P 3.4-6 The input to the circuit shown in Figure P 3.4-6 is the current of the current source, i_a. The output of this circuit is the current measured by the ammeter, i_b. This circuit produces an output that is proportional to the input, that is

$$i_b = k\, i_a$$

where k is the constant of proportionality.

(a) Determine the value of the output, i_b, when $R = 24\,\Omega$ and $i_a = 1.8$ A.

(b) Determine the value of the resistance, R, required to cause the output to be $i_b = 1.6$ A when the input is $i_a = 2$ A.

(c) Determine the value of the resistance, R, required to cause $i_b = 0.4\, i_a$ (that is, the value of the constant of proportionality is $k = \frac{4}{10}$).

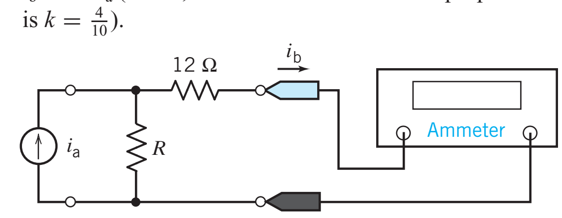


FIGURE P 3.4-6

***P 3.4-7** Figure P 3.4-7 shows a transistor amplifier. The values of R_1 and R_2 are to be selected. Resistances R_1 and R_2 are used to bias the transistor, that is, to create useful operating conditions. In this problem, we want to select R_1 and R_2 so that $v_b =$ 5 V. We expect the value of i_b to be approximately 10 μA. When $i_1 \le 10 i_b$, it is customary to treat i_b as negligible, that is, to assume $i_b = 0$. In that case R_1 and R_2 comprise a voltage divider.

(a) Select values for R_1 and R_2 so that v_b = 5 V and the total power absorbed by R_1 and R_2 is no more than 5 mW.

(b) An inferior transistor could cause i_b to be larger than expected. Using the values of R_1 and R_2 from part (a), determine the value of v_b that would result from i_b = 15 μA.

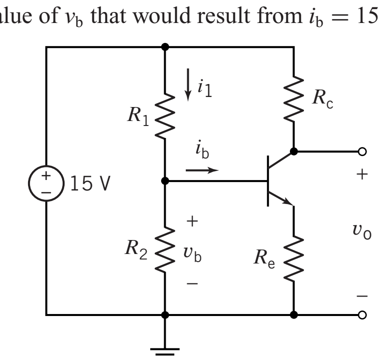


FIGURE P 3.4-7

P 3.4-8 Determine the value of the current i in the circuit shown in Figure P 3.4-8.

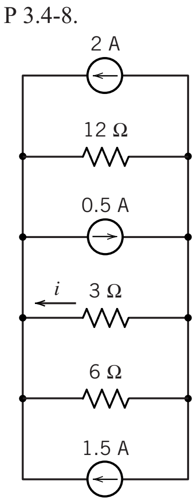


FIGURE P 3.4-8

P 3.4-9 Determine the value of the voltage v in Figure P 3.4-9.

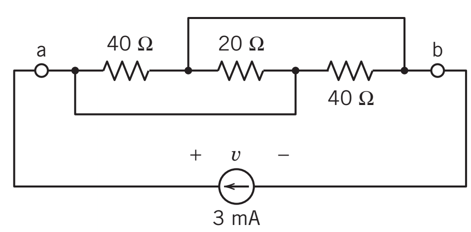


FIGURE P 3.4-9

P 3.4-10 A solar photovoltaic panel may be represented by the circuit model shown in Figure P 3.4-10, where R_L is the load resistor. Determine the values of the resistances R_1 and R_L.

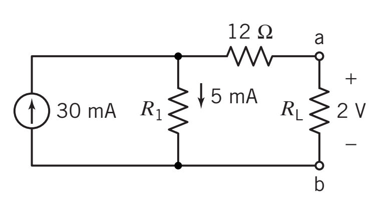


FIGURE P 3.4-10

P 3.4-11 Determine the power supplied by the dependent source in Figure P 3.4-11.

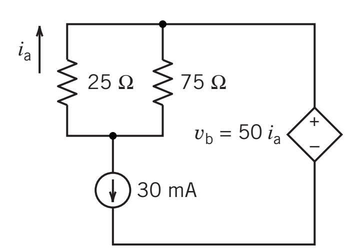


FIGURE P 3.4-11

P 3.4-12 The voltmeter in Figure P 3.4-12 measures the value of the voltage v_m.

(a) Determine the value of the resistance R.

(b) Determine the value of the power supplied by the current source.

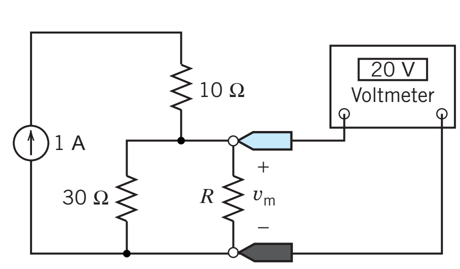


FIGURE P 3.4-12

***P 3.4-13** Determine the values of the resistances R_1 and R_2 for the circuit shown in Figure P 3.4-13.

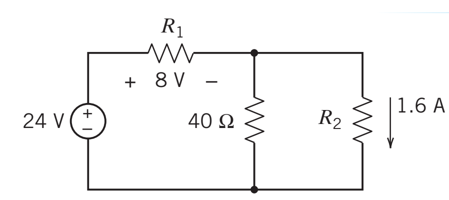


FIGURE P 3.4-13

***P 3.4-14** Determine the values of the resistances R_1 and R_2 for the circuit shown in Figure P 3.4-14.

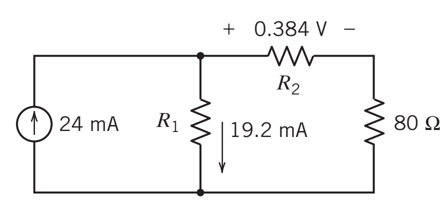


FIGURE P 3.4-14

P 3.4-15 Determine the value of the current measured by the meter in Figure P 3.4-15.

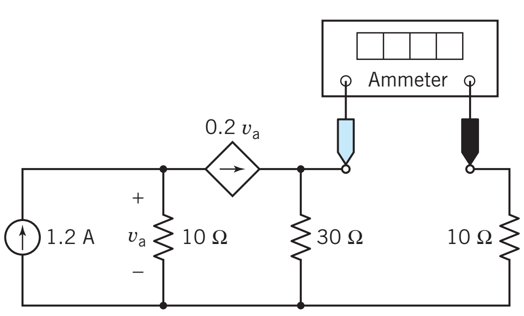


FIGURE P 3.4-15

Section 3.5 Series Voltage Sources and Parallel Current Sources

P 3.5-1 Determine the power supplied by each source in the circuit shown in Figure P 3.5-1.

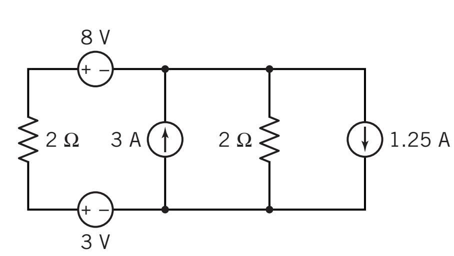


FIGURE P 3.5-1

P 3.5-2 Determine the power supplied by each source in the circuit shown in Figure P 3.5-2.

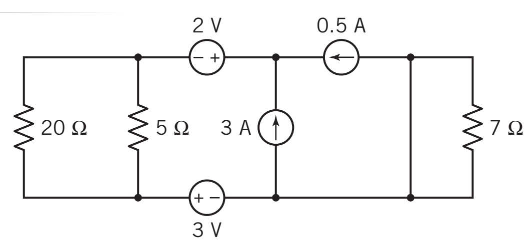


FIGURE P 3.5-2

P 3.5-3 Determine the power received by each resistor in the circuit shown in Figure P 3.5-3.

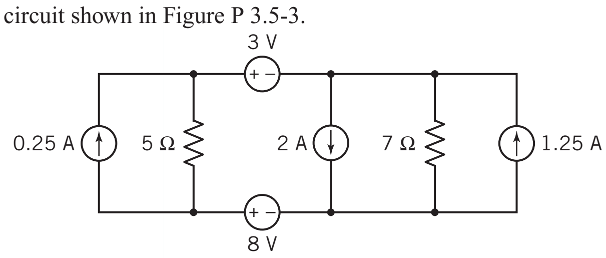


FIGURE P 3.5-3

Section 3.6 Circuit Analysis

P 3.6-1 The circuit shown in Figure P 3.6-1*a* has been divided into two parts. In Figure P 3.6-1*b*, the right-hand part has been replaced with an equivalent circuit. The left-hand part of the circuit has not been changed.

(a) Determine the value of the resistance R in Figure P 3.6-1*b* that makes the circuit in Figure P 3.6-1*b* equivalent to the circuit in Figure P 3.6-1*a*.

(b) Find the current i and the voltage v shown in Figure P 3.6-1*b*. Because of the equivalence, the current i and the voltage v shown in Figure P 3.6-1*a* are equal to the current i and the voltage v shown in Figure P 3.6-1*b*.

(c) Find the current i_2 shown in Figure P 3.6-1*a* using current division.

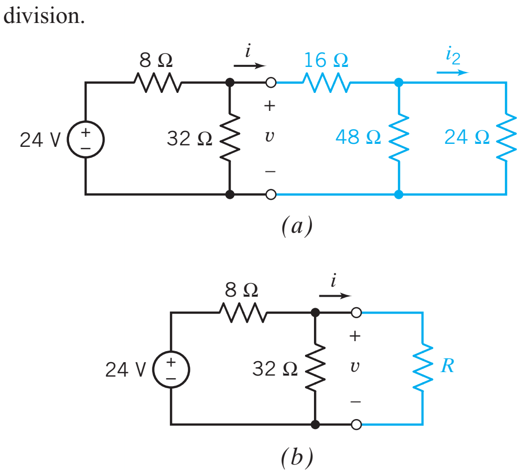


FIGURE P 3.6-1

P 3.6-2 The circuit shown in Figure P 3.6-2*a* has been divided into three parts. In Figure P 3.6-2*b*, the rightmost part has been replaced with an equivalent circuit. The rest of the circuit has not been changed. The circuit is simplified further in Figure 3.6-2*c*. Now the middle and rightmost parts have been replaced by a single equivalent resistance. The leftmost part of the circuit is still unchanged.

(a) Determine the value of the resistance R_1 in Figure P 3.6-2*b* that makes the circuit in Figure P 3.6-2*b* equivalent to the circuit in Figure P 3.6-2*a*.

(b) Determine the value of the resistance R_2 in Figure P 3.6-2*c* that makes the circuit in Figure P 3.6-2*c* equivalent to the circuit in Figure P 3.6-2*b*.

(c) Find the current i_1 and the voltage v_1 shown in Figure P 3.6-2*c*. Because of the equivalence, the current i_1 and the voltage v_1 shown in Figure P 3.6-2*b* are equal to the current i_1 and the voltage v_1 shown in Figure P 3.6-2*c*.
Hint: $24 = 6(i_1 - 2) + i_1 R_2$

(d) Find the current i_2 and the voltage v_2 shown in Figure P 3.6-2*b*. Because of the equivalence, the current i_2 and the voltage v_2 shown in Figure P 3.6-2*a* are equal to the current i_2 and the voltage v_2 shown in Figure P 3.6-2*b*.
Hint: Use current division to calculate i_2 from i_1.

(e) Determine the power absorbed by the 3-Ω resistance shown at the right of Figure P 3.6-2*a*.

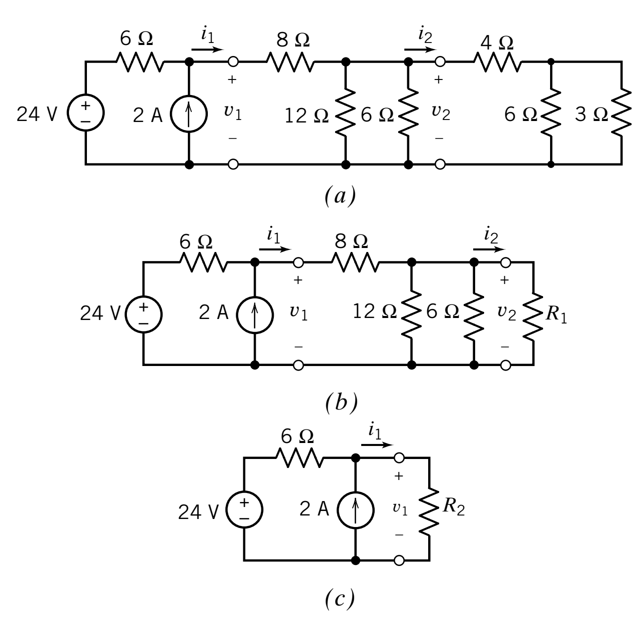


FIGURE P 3.6-2

P 3.6-3 Find i using appropriate circuit reductions and the current divider principle for the circuit of Figure P 3.6-3.

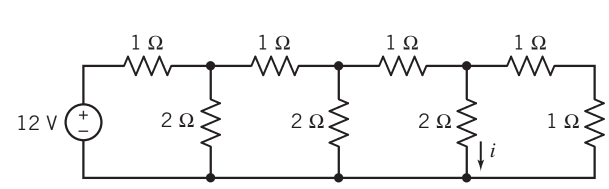


FIGURE P 3.6-3

P 3.6-4

(a) Determine values of R_1 and R_2 in Figure P 3.6-4*b* that make the circuit in Figure P 3.6-4*b* equivalent to the circuit in Figure P 3.6-4*a*.

(b) Analyze the circuit in Figure P 3.6-4*b* to determine the values of the currents i_a and i_b

(c) Because the circuits are equivalent, the currents i_a and i_b shown in Figure P 3.6-4*b* are equal to the currents i_a and i_b shown in Figure P 3.6-4*a*. Use this fact to determine values of the voltage v_1 and current i_2 shown in Figure P 3.6-4*a*.

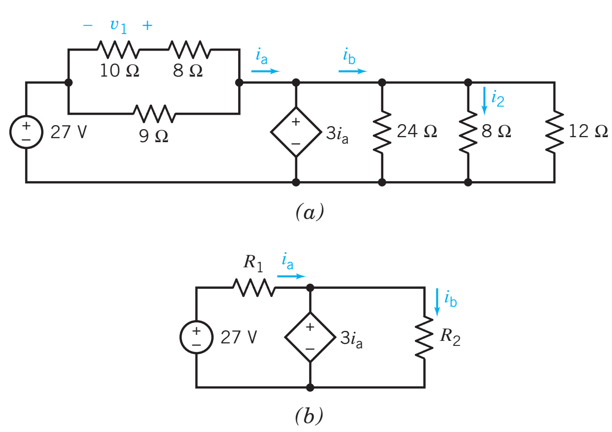


FIGURE P 3.6-4

P 3.6-5 The voltmeter in the circuit shown in Figure P 3.6-5 shows that the voltage across the 30-Ω resistor is 6 volts. Determine the value of the resistance R_1.
Hint: Use the voltage division twice.
Answer: $R_1 = 40\ \Omega$

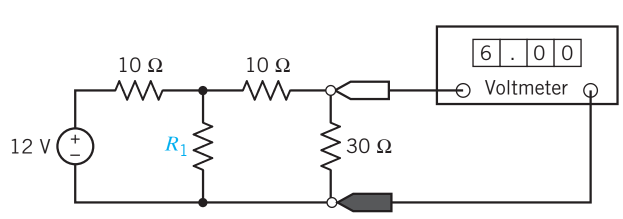


FIGURE P 3.6-5

P 3.6-6 Determine the voltages v_a and v_c and the currents i_b and i_d for the circuit shown in Figure P 3.6-6.
Answer: $v_a = -2$ V, $v_c = 6$ V, $i_b = -16$ mA, and $i_d = 2$ mA

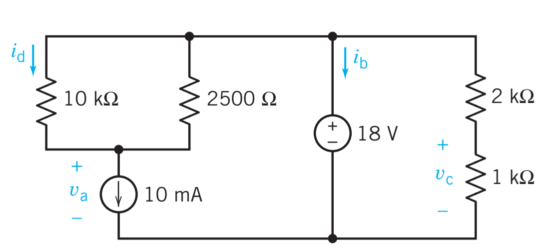


FIGURE P 3.6-6

P 3.6-7 Determine the value of the resistance R in Figure P 3.6-7.
Answer: $R = 28\ \text{k}\Omega$

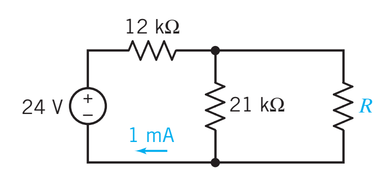


FIGURE P 3.6-7

P 3.6-8 Most of us are familiar with the effects of a mild electric shock. The effects of a severe shock can be devastating and often fatal. Shock results when current is passed through the body. A person can be modeled as a network of resistances. Consider the model circuit shown in Figure P 3.6-8. Determine the voltage developed across the heart and the current flowing through the heart of the person when he or she firmly grasps one end of a voltage source whose other end is connected to the floor. The heart is represented by R_h. The floor has resistance to current flow equal to R_f, and the person is standing barefoot on the floor. This type of accident might occur at a swimming pool or boat dock. The upper-body resistance R_u and lower-body resistance R_L vary from person to person.

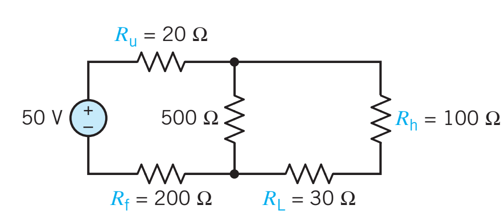


FIGURE P 3.6-8

P 3.6-9 Determine the value of the current i in Figure 3.6-9.
Answer: $i = 0.5$ mA

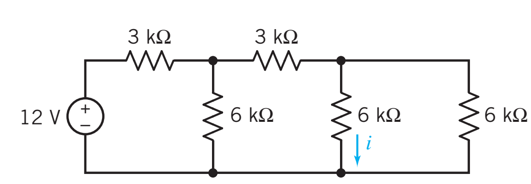


FIGURE P 3.6-9

P 3.6-10 Determine the values of i_a, i_b, and v_c in Figure P 3.6-10.

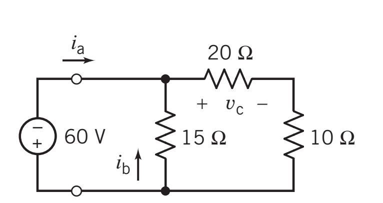


FIGURE P 3.6-10

P 3.6-11 Find i and $R_{\text{eq a-b}}$ if $v_{ab} = 40$ V in the circuit of Figure P 3.6-11.
Answer: $R_{\text{eq a-b}} = 8\ \Omega$, $i = 5/6$ A

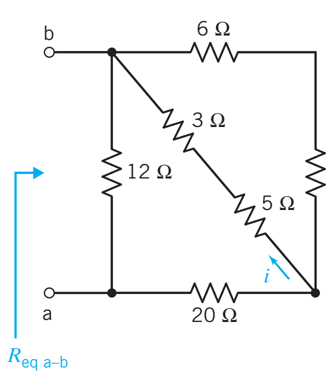


FIGURE P 3.6-11

P 3.6-12 The ohmmeter in Figure P 3.6-12 measures the equivalent resistance, R_{eq}, of the resistor circuit. The value of the equivalent resistance, R_{eq}, depends on the value of the resistance R.

(a) Determine the value of the equivalent resistance, R_{eq}, when $R = 18\ \Omega$.

(b) Determine the value of the resistance R required to cause the equivalent resistance to be $R_{eq} = 18\ \Omega$.

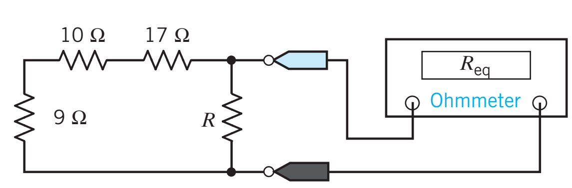


FIGURE P 3.6-12

P 3.6-13 The source $v_s = 240$ volts is connected to three equal resistors as shown in Figure P 3.6-13. Determine R when the voltage source delivers 1920 W to the resistors.
Answer: $R = 45\ \Omega$

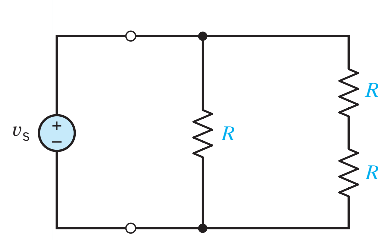


FIGURE P 3.6-13

P 3.6-14 Find the R_{eq} at terminals a–b in Figure P 3.6-14. Also determine i, i_1, and i_2.
Answer: $R_{eq} = 8\ \Omega$, $i = 5$ A, $i_1 = 5/3$ A, $i_2 = 5/2$ A

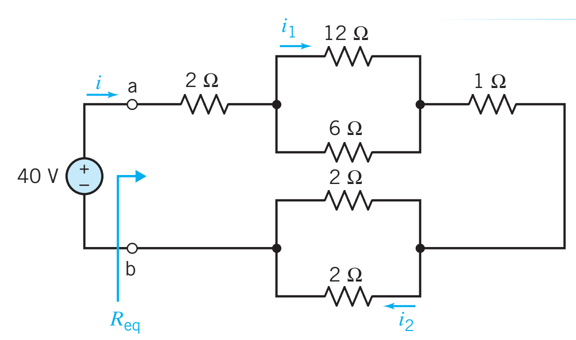


FIGURE P 3.6-14

P 3.6-15 All of the resistances in the circuit shown in Figure P 3.6-15 are multiples of R. Determine the value of R.

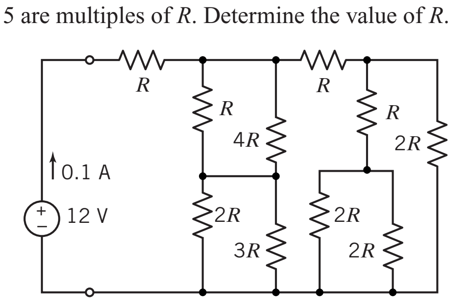


FIGURE P 3.6-15

***P 3.6-16** The circuit shown in Figure P 3.6-16 contains seven resistors, each having resistance R. The input to this circuit is the voltage source voltage, v_s. The circuit has two outputs, v_a and v_b. Express each output as a function of the input.

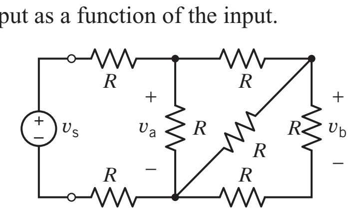


FIGURE P 3.6-16

P 3.6-17 The circuit shown in Figure P 3.6-17 contains three 10-Ω, 1/4-W resistors. (Quarter-watt resistors can dissipate 1/4 W safely.) Determine the range of voltage source voltages, v_s, such that none of the resistors absorbs more than 1/4 W of power.

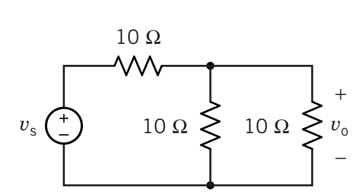


FIGURE P 3.6-17

P 3.6-18 The four resistors shown in Figure P 3.6-18 represent strain gauges. Strain gauges are transducers that measure the strain that results when a resistor is stretched or compressed. Strain gauges are used to measure force, displacement, or pressure. The four strain gauges in Figure P 3.6-18 each have a nominal (unstrained) resistance of 120 Ω and can each absorb 0.2 mW safely. Determine the range of voltage source voltages, v_s, such that no strain gauge absorbs more than 0.2 mW of power.

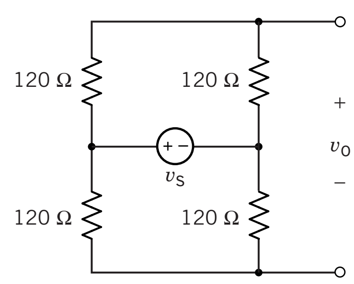


FIGURE P 3.6-18

P 3.6-19 The circuit shown in Figure P 3.6-19*b* has been obtained from the circuit shown in Figure P 3.6-19*a* by replacing series and parallel combinations of resistances by equivalent resistances.

(a) Determine the values of the resistances R_1, R_2, and R_3 in Figure P 3.6-19*b* so that the circuit shown in Figure P 3.6-19*b* is equivalent to the circuit shown in Figure P 3.6-19*a*.

(b) Determine the values of v_1, v_2, and i in Figure P 3.6-19*b*.

(c) Because the circuits are equivalent, the values of v_1, v_2, and i in Figure P 3.6-19*a* are equal to the values of v_1, v_2, and i in Figure P 3.6-19*b*. Determine the values of v_4, i_5, i_6, and v_7 in Figure P 3.6-19*a*.

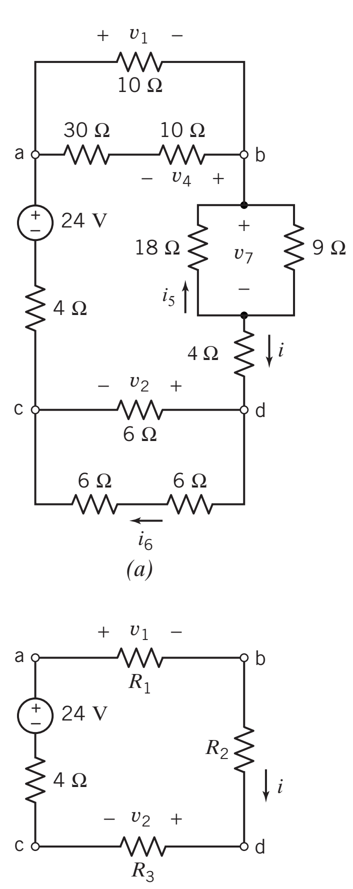


FIGURE P 3.6-19

P 3.6-20 Determine the values of v_1, v_2, i_3, v_4, v_5, and i_6 in Figure P 3.6-20.

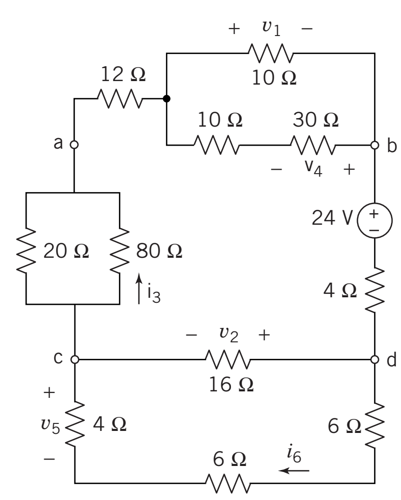


FIGURE P 3.6-20

P 3.6-21 Determine the values of i, v, and R_{eq} by the circuit model shown in Figure P 3.6-21, given that $v_{ab} = 18$ V.

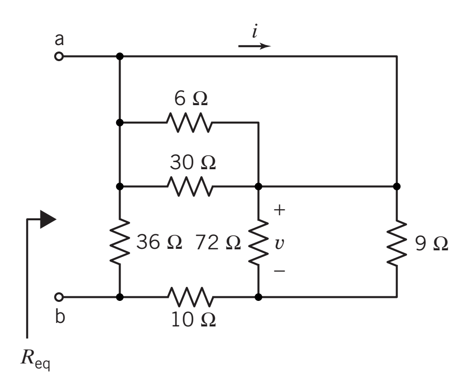


FIGURE P 3.6-21

P 3.6-22 Determine the value of the resistance R in the circuit shown in Figure P 3.6-22, given that $R_{eq} = 9$ Ω.
Answer: $R = 15$ Ω

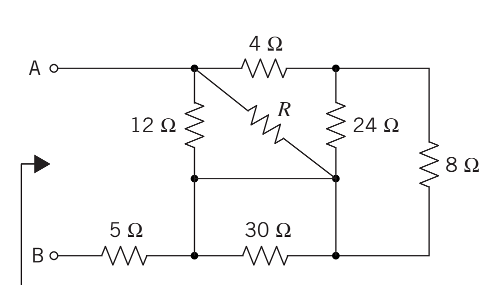


FIGURE P 3.6-22

P 3.6-23 Determine the value of the resistance R in the circuit shown in Figure P 3.6-23, given that $R_{eq} = 50\ \Omega$.

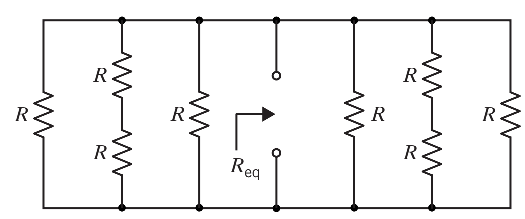


FIGURE P 3.6-23

P 3.6-24 Determine the values of r, the gain of the CCVS, and g, the gain of the VCCS, for the circuit shown in Figure P 3.6-24.

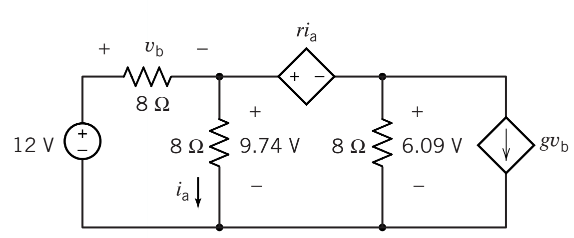


FIGURE P 3.6-24

P 3.6-25 The input to the circuit in Figure P 3.6-25 is the voltage of the voltage source, v_s. The output is the voltage measured by the meter, v_o. Show that the output of this circuit is proportional to the input. Determine the value of the constant of proportionality.

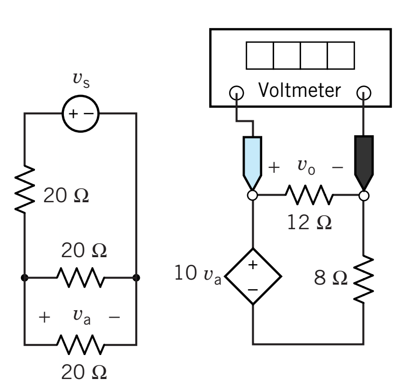


FIGURE P 3.6-25

P 3.6-26 The input to the circuit in Figure P 3.6-26 is the voltage of the voltage source, v_s. The output is the current measured by the meter, i_o. Show that the output of this circuit is proportional to the input. Determine the value of the constant of proportionality.

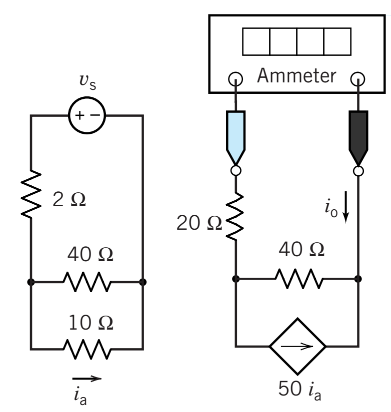


FIGURE P 3.6-26

P 3.6-27 Determine the voltage measured by the voltmeter in the circuit shown in Figure P 3.6-27.

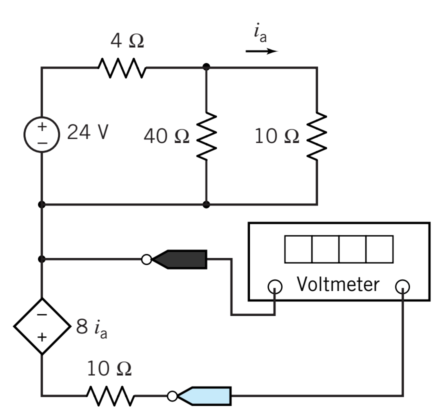


FIGURE P 3.6-27

P 3.6-28 Determine the current measured by the ammeter in the circuit shown in Figure P 3.6-28.

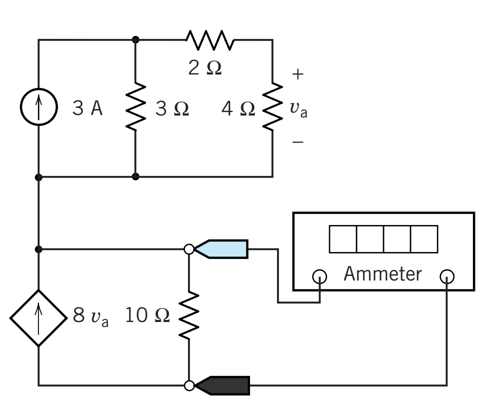


FIGURE P 3.6-28

P 3.6-29 Determine the value of the resistance R that causes the voltage measured by the voltmeter in the circuit shown in Figure P 3.6-29 to be 4 V.

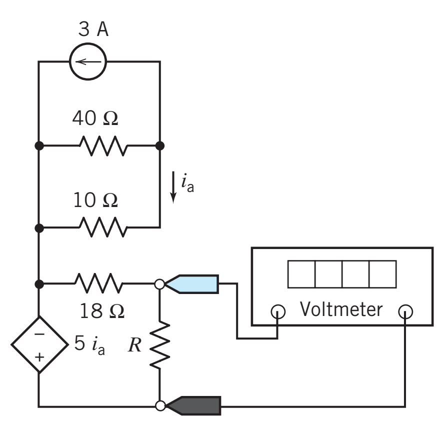


FIGURE P 3.6-29

P 3.6-30 The input to the circuit shown in Figure P 3.6-30 is the voltage of the voltage source, v_s. The output is the current measured by the meter, i_m.

(a) Suppose $v_s = 15$ V. Determine the value of the resistance R that causes the value of the current measured by the meter to be $i_m = 5$ A.

(b) Suppose $v_s = 15$ V and $R = 24\ \Omega$. Determine the current measured by the ammeter.

(c) Suppose $R = 24\ \Omega$. Determine the value of the input voltage, v_s, that causes the value of the current measured by the meter to be $i_m = 3$ A.

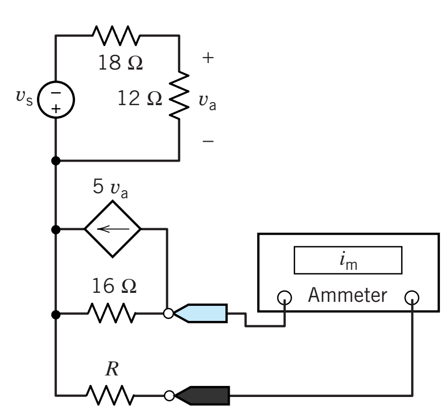


FIGURE P 3.6-30

P3.6-31 The ohmmeter in Figure P 3.6-31 measures the equivalent resistance of the resistor circuit connected to the meter probes.

(a) Determine the value of the resistance R required to cause the equivalent resistance to be $R_{eq} = 12\ \Omega$.

(b) Determine the value of the equivalent resistance when $R = 14\ \Omega$.

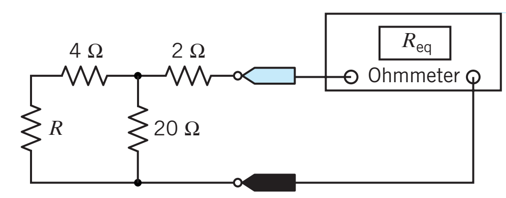


FIGURE P 3.6-31

P 3.6-32 The voltmeter in Figure P 3.6-32 measures the voltage across the current source.

(a) Determine the value of the voltage measured by the meter.

(b) Determine the power supplied by each circuit element.

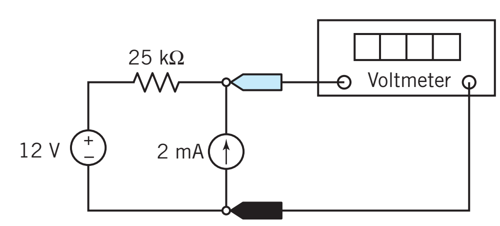


FIGURE P 3.6-32

P3.6-33 Determine the resistance measured by the ohmmeter in Figure P 3.6-33.

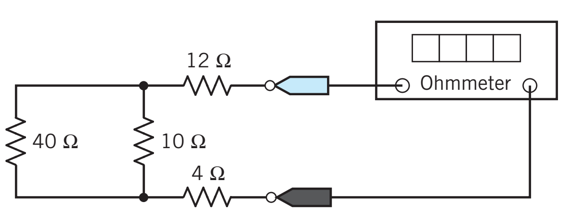


FIGURE P 3.6-33

P3.6-34 Determine the resistance measured by the ohmmeter in Figure P 3.6-34.

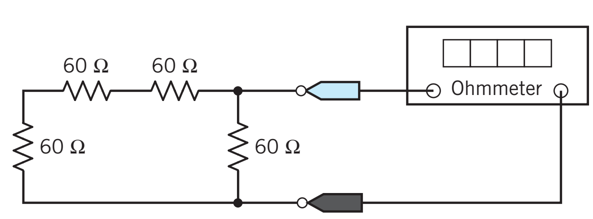


FIGURE P 3.6-34

Section 3.8 How Can We Check . . . ?

P 3.8-1 A computer analysis program, used for the circuit of Figure P 3.8-1, provides the following branch currents and voltages: i_1 A = −0.833, i_2 A = −0.333, i_3 A = −1.167, and v = −2.0 V. Are these answers correct?

Hint: Verify that KCL is satisfied at the center node and that KVL is satisfied around the outside loop consisting of the two 6-Ω resistors and the voltage source.

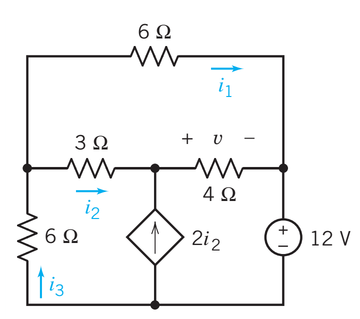


FIGURE P 3.8-1

P 3.8-2 The circuit of Figure P 3.8-2 was assigned as a homework problem. The answer in the back of the textbook says the current, i, is 1.25 A. Verify this answer using current division.

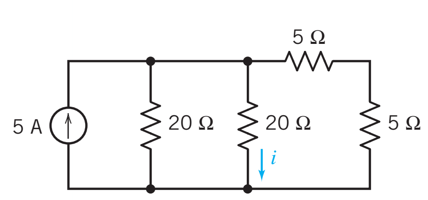


FIGURE P 3.8-2

P 3.8-3 The circuit of Figure P 3.8-3 was built in the lab and v_o was measured to be 6.25 V. Verify this measurement using the voltage divider principle.

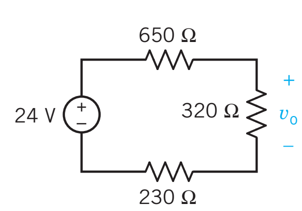


FIGURE P 3.8-3

P 3.8-4 The circuit of Figure P 3.8-4 represents an auto's electrical system. A report states that $i_H = 9$ A, $i_B = -9$ A, and $i_A = 19.1$ A. Verify that this result is correct.

Hint: Verify that KCL is satisfied at each node and that KVL is satisfied around each loop.

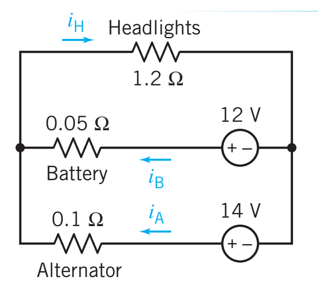


FIGURE P 3.8-4
Electric circuit model of an automobile's electrical system.

P 3.8-5 Computer analysis of the circuit in Figure P 3.8-5 shows that $i_a = -0.5$ mA and $i_b = -2$ mA. Was the computer analysis done correctly?

Hint: Verify that the KVL equations for all three meshes are satisfied when $i_a = -0.5$ mA and $i_b = -2$ mA.

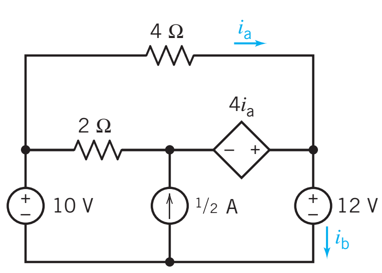


FIGURE P 3.8-5

P 3.8-6 Computer analysis of the circuit in Figure P 3.8-6 shows that $i_a = 0.5$ mA and $i_b = 4.5$ mA. Was the computer analysis done correctly?

Hint: First, verify that the KCL equations for all five nodes are satisfied when $i_a = 0.5$ mA and $i_b = 4.5$ mA. Next, verify that the KVL equation for the lower left mesh (a-e-d-a) is satisfied. (The KVL equations for the other meshes aren't useful because each involves an unknown voltage.)

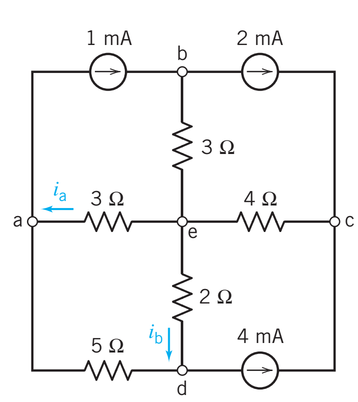


FIGURE P 3.8-6

P 3.8-7 Verify that the element currents and voltages shown in Figure P 3.8-7 satisfy Kirchhoff's laws:

(a) Verify that the given currents satisfy the KCL equations corresponding to nodes a, b, and c.

(b) Verify that the given voltages satisfy the KVL equations corresponding to loops a-b-d-c-a and a-b-c-d-a.

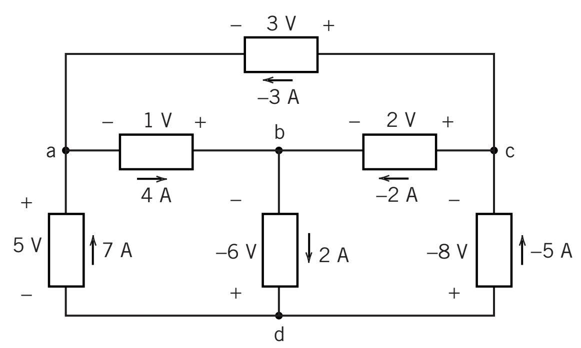


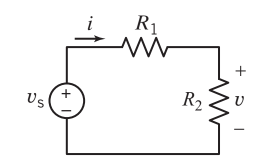
FIGURE P 3.8-7

***P 3.8-8** Figure P 3.8-8 shows a circuit and some corresponding data. The tabulated data provides values of the current, i, and voltage, v, corresponding to several values of the resistance R_2.

(a) Use the data in rows 1 and 2 of the table to find the values of v_s and R_1.

(b) Use the results of part (a) to verify that the tabulated data are consistent.

(c) Fill in the missing entries in the table.



(a)

R_2, Ω	i, A	v, V
0	2.4	0
10	1.2	12
20	0.8	16
30	?	18
40	0.48	?

(b)

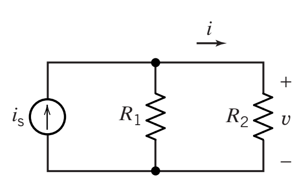
FIGURE P 3.8-8

***P 3.8-9** Figure P 3.8-9 shows a circuit and some corresponding data. The tabulated data provide values of the current, i, and voltage, v, corresponding to several values of the resistance R_2.

(a) Use the data in rows 1 and 2 of the table to find the values of i_s and R_1.

(b) Use the results of part (a) to verify that the tabulated data are consistent.

(c) Fill in the missing entries in the table.



(a)

R_2, Ω	i, A	v, V
10	4/3	40/3
20	6/7	120/7
40	1/2	20
80	?	?

(b)

FIGURE P 3.8-9

Design Problems

DP 3-1 The circuit shown in Figure DP 3-1 uses a potentiometer to produce a variable voltage. The voltage v_m varies as a knob connected to the wiper of the potentiometer is turned. Specify the resistances R_1 and R_2 so that the following three requirements are satisfied:

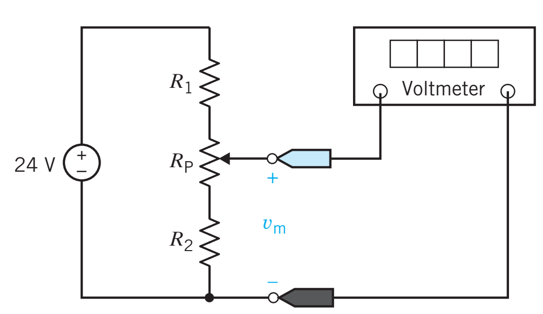


FIGURE DP 3-1

1. The voltage v_m varies from 8 V to 12 V as the wiper moves from one end of the potentiometer to the other end of the potentiometer.
2. The voltage source supplies less than 0.5 W of power.
3. Each of R_1, R_2, and R_P dissipates less than 0.25 W.

DP 3-2 The resistance R_L in Figure DP 3-2 is the equivalent resistance of a pressure transducer. This resistance is specified to be 200 Ω ± 5 percent. That is, $190\ \Omega \leq R_L \leq 210\ \Omega$. The voltage source is a 12 V ± 1 percent source capable of supplying 5 W. Design this circuit, using 5 percent, 1/8-watt resistors for R_1 and R_2, so that the voltage across R_L is

$$v_o = 4\ \text{V} \pm 10\%$$

(A 5 percent, 1/8-watt 100-Ω resistor has a resistance between 95 and 105 Ω and can safely dissipate 1/8-W continuously.)

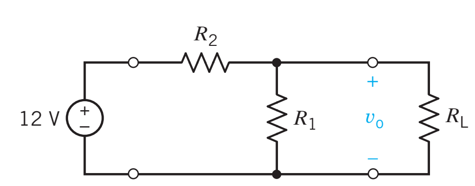


FIGURE DP 3-2

DP 3-3 A phonograph pickup, stereo amplifier, and speaker are shown in Figure DP 3-3*a* and redrawn as a circuit model as shown in Figure DP 3-3*b*. Determine the resistance R so that the voltage v across the speaker is 16 V. Determine the power delivered to the speaker.

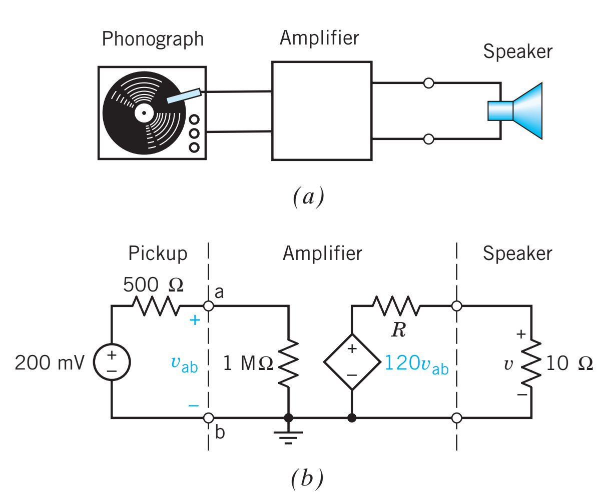


FIGURE DP 3-3 A photograph stereo system.

DP 3-4 A Christmas tree light set is required that will operate from a 6-V battery on a tree in a city park. The heavy-duty battery can provide 9A for the four-hour period of operation each night. Design a parallel set of lights (select the maximum number of lights) when the resistance of each bulb is 12 Ω.

DP 3-5 The input to the circuit shown in Figure DP 3-5 is the voltage source voltage, v_s. The output is the voltage v_o. The output is related to the input by

$$v_o = \frac{R_2}{R_1 + R_2} v_s = g v_s$$

The output of the voltage divider is proportional to the input. The constant of proportionality, g, is called the gain of the voltage divider and is given by

$$g = \frac{R_2}{R_1 + R_2}$$

The power supplied by the voltage source is

$$p = v_s i_s = v_s \left(\frac{v_s}{R_1 + R_2} \right) = \frac{{v_s}^2}{R_1 + R_2} = \frac{{v_s}^2}{R_{in}}$$

where

$$R_{in} = R_1 + R_2$$

is called the input resistance of the voltage divider.

(a) Design a voltage divider to have a gain, $g = 0.65$.
(b) Design a voltage divider to have a gain, $g = 0.65$, and an input resistance, $R_{in} = 2500\ \Omega$.

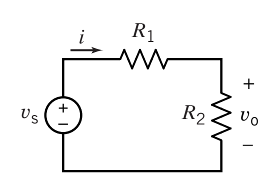


FIGURE DP 3-5

DP 3-6 The input to the circuit shown in Figure DP 3-6 is the current source current, i_s. The output is the current i_o. The output is related to the input by

$$i_o = \frac{R_1}{R_1 + R_2} i_s = g i_s$$

The output of the current divider is proportional to the input. The constant of proportionality, g, is called the gain of the current divider and is given by

$$g = \frac{R_1}{R_1 + R_2}$$

The power supplied by the current source is

$$p = v_s i_s = \left[i_s \left(\frac{R_1 R_2}{R_1 + R_2} \right) \right] i_s = \frac{R_1 R_2}{R_1 + R_2} {i_s}^2 = R_{in} {i_s}^2$$

where

$$R_{in} = \frac{R_1 R_2}{R_1 + R_2}$$

is called the input resistance of the current divider.

(a) Design a current divider to have a gain, $g = 0.65$.
(b) Design a current divider to have a gain, $g = 0.65$, and an input resistance, $R_{in} = 10000\ \Omega$.

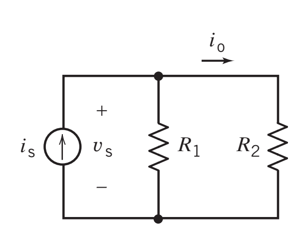


FIGURE DP 3-6

P1: IML/OVY P2: IML/OVY QC: IML/OVY T1: IML
GTBL002-dorfc03 GTBL002-Dorf-v32 October 14, 2005 21:2

DP 3-7 Design the circuit shown in Figure DP 3-7 to have an output $v_o = 8.5$ V when the input is $v_s = 12$ V. The circuit should require no more than 1 mW from the voltage source.

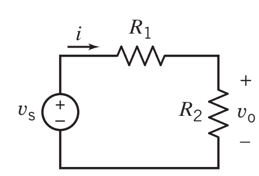


FIGURE DP 3-7

DP 3-8 Design the circuit shown in Figure DP 3-8 to have an output $i_o = 1.8$ mA when the input is $i_s = 5$ mA. The circuit should require no more than 1 mW from the current source.

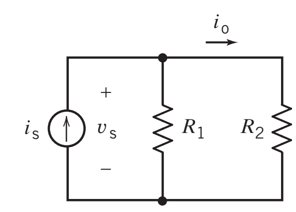


FIGURE DP 3-8