Lesson
13
Multi-Evaporator And Cascade Systems

The objectives of this lesson are to:

1. Discuss the advantages and applications of multi-evaporator systems compared to single stage systems (*Section 13.1*)
2. Describe multi-evaporator systems using single compressor and a pressure reducing valve with:
    a) Individual expansion valves (*Section 13.2.1*)
    b) Multiple expansion valves (*Section 13.2.2*)
3. Describe multi-evaporator systems with multi-compression, intercooling and flash gas removal (*Section 13.3*)
4. Describe multi-evaporator systems with individual compressors and multiple expansion valves (*Section 13.4*)
5. Discuss limitations of multi-stage systems (*Section 13.5*)
6. Describe briefly cascade systems (Section 13.6)
7. Describe briefly the working principle of auto-cascade cycle (*Section 13.7*)

At the end of the lecture, the student should be able to:

1. Explain the need for multi-evaporator systems
2. Evaluate the performance of:
    a) Multi-evaporator systems with single compressor and individual expansion valves
    b) Multi-evaporator systems with single compressor and multiple expansion valves
3. Evaluate the performance of multi-evaporator systems with multi-compression, intercooling and flash gas removal
4. Evaluate the performance of multi-evaporator systems with individual compressors and multiple or individual expansion valves
5. Evaluate the performance of cascade systems
6. Describe the working principle of auto-cascade systems

13.1. Introduction

As mentioned in Chapter 12, there are many applications where refrigeration is required at different temperatures. For example, in a typical food processing plant, cold air may be required at –30^{o}C for freezing and at +7^{o}C for cooling of food products or space cooling. One simple alternative is to use different refrigeration systems to cater to these different loads. However, this may not be economically viable due to the high total initial cost. Another alternative is to use a single refrigeration system with one compressor and two evaporators both operating at –30^{o}C. The schematic of such a system and corresponding operating cycle on P-h diagram are shown in Figs. 13.1(a) and (b). As shown in the figure the system consists of a single compressor and a single condenser but two evaporators. Both evaporators-I and II operate at same evaporator temperature (-30^{o}C) one evaporator (say Evaporator-I) caters to freezing while the other (Evaporator-II) caters to product cooling/space conditioning at 7^{o}C. It can be seen that operating the evaporator at –30^{o}C when refrigeration is required at +7^{o}C is thermodynamically inefficient as the system irreversibilities increase with increasing temperature difference for heat transfer.

The COP of this simple system is given by:

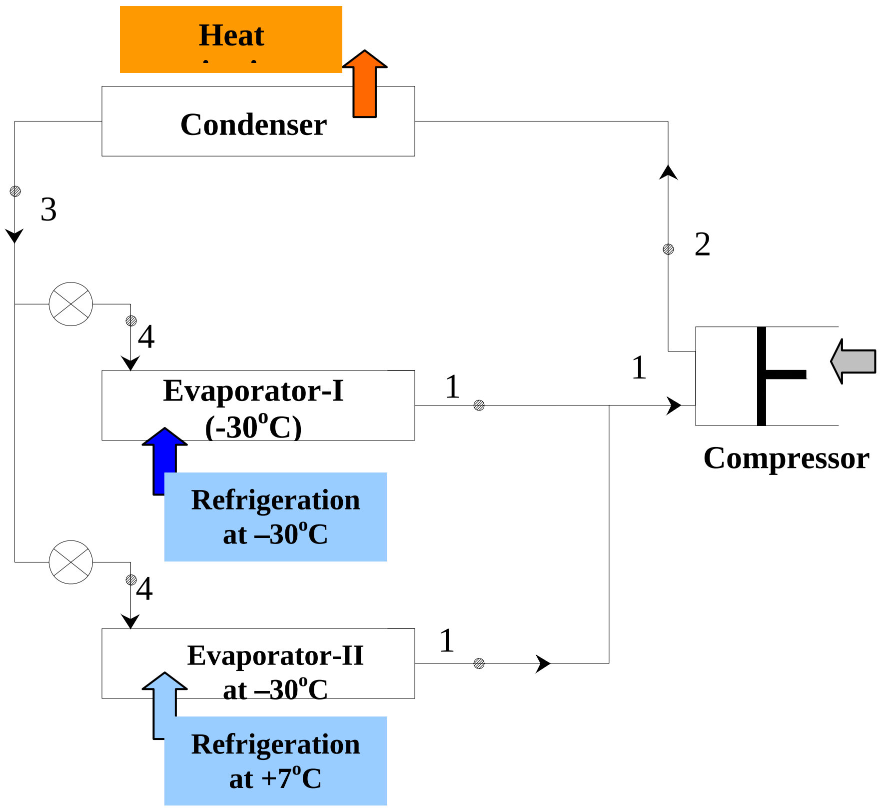
$$COP = \frac{Q_{e,I} + Q_{e,II}}{W_c} = \frac{(h_1 - h_4)}{(h_2 - h_1)} \qquad (13.1)$$

In addition to this there will also be other difficulties such as: evaporator catering to space cooling (7^{o}C) may collect frost leading to blockage of air-flow passages, if a liquid is to chilled then it may freeze on the evaporator and the moisture content of air may become too low leading to water losses in the food products. In such cases multi-stage systems with multiple evaporators can be used. Several multi-evaporator combinations are possible in practice. Some of the most common ones are discussed below.

13.2. Individual evaporators and a single compressor with a pressure-reducing valve

13.2.1. Individual expansion valves:

Figures 13.2 (a) and (b) show system schematic and P-h diagram of a multi-evaporator system that uses two evaporators at two different temperatures and a single compressor. This system also uses individual expansion valves and a pressure regulating valve (PRV) for reducing the pressure from that corresponding to the high temperature evaporator to the compressor suction pressure. The PRV also maintains the required pressure in high temperature evaporator (Evaporator-II). Compared to the earlier system, this system offers the advantage of higher refrigeration effect at the high temperature evaporator [(h_6-h_4) against (h_7-h_5)]. However, this advantage is counterbalanced by higher specific work input due to the operation of compressor in



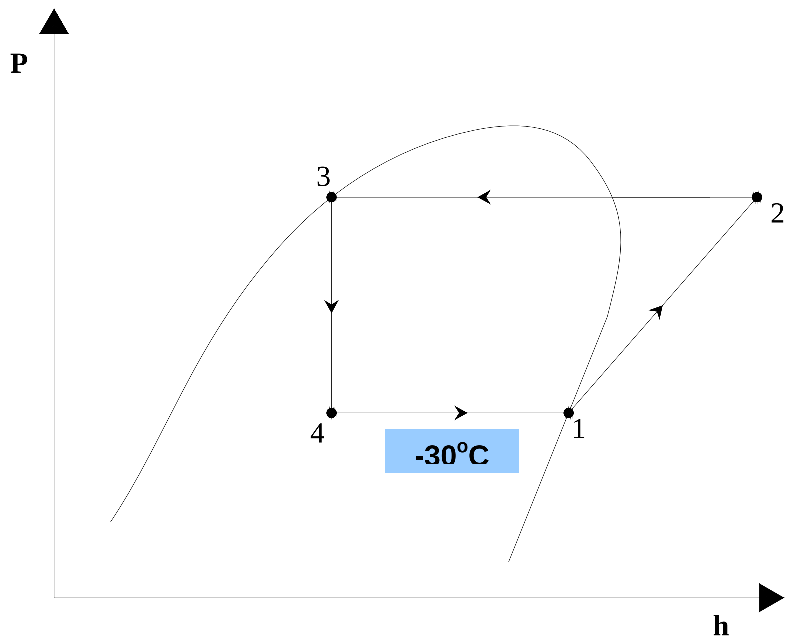


Fig.13.1(a) & (b)*: A single stage system with two evaporators*

superheated region. Thus ultimately there may not be any improvement in system COP due to this arrangement. It is easy to see that this modification does not result in significant improvement in performance due to the fact that the refrigerant vapour at the intermediate pressure is reduced first using the PRV and again increased using compressor. Obviously this is inefficient. However, this system is still preferred to the earlier system due to proper operation of high temperature evaporator.

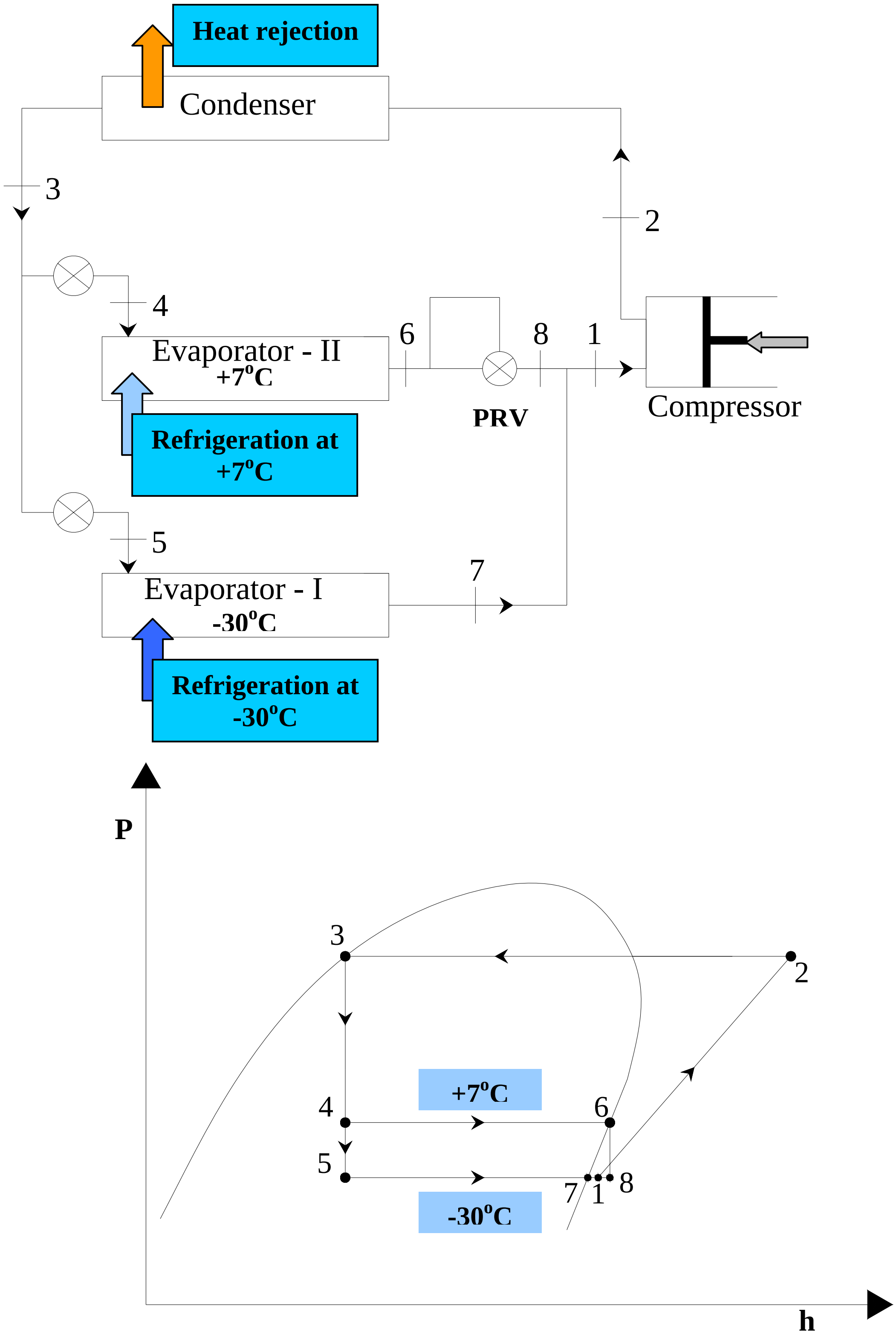


***Fig.13.2(a) & (b)**: Multi-evaporator system with single compressor and individual expansion valves*

The COP of the above system is given by:

$$COP = \frac{Q_{e,I} + Q_{e,II}}{W_c} = \frac{\dot{m}_I (h_7 - h_5) + \dot{m}_{II} (h_6 - h_4)}{(\dot{m}_I + \dot{m}_{II})(h_2 - h_1)} \quad (13.2)$$

where $\dot{m}_I$ and $\dot{m}_{II}$ are the refrigerant mass flow rates through evaporator I and II respectively. They are given by:

$$\dot{m}_I = \frac{Q_{e,I}}{(h_7 - h_5)} \quad (13.3)$$

$$\dot{m}_{II} = \frac{Q_{e,II}}{(h_6 - h_4)} \quad (13.4)$$

Enthalpy at point 2 (inlet to compressor) is obtained by applying mass and energy balance to the mixing of two refrigerant streams, i.e.,

$$h_2 = \frac{\dot{m}_I h_7 + \dot{m}_{II} h_8}{\dot{m}_I + \dot{m}_{II}} \quad (13.5)$$

If the expansion across PRV is isenthalpic, then specific enthalpy h_8 will be equal to h_6.

13.2.2. Multiple expansion valves:

Figures 13.3 (a) and (b) show system schematic and P-h diagram of a multi-evaporator with a single compressor and multiple expansion valves. It can be seen from the P-h diagram that the advantage of this system compared to the system with individual expansion valves is that the refrigeration effect of the low temperature evaporator increases as saturated liquid enters the low stage expansion valve. Since the flash gas is removed at state 4, the low temperature evaporator operates more efficiently.

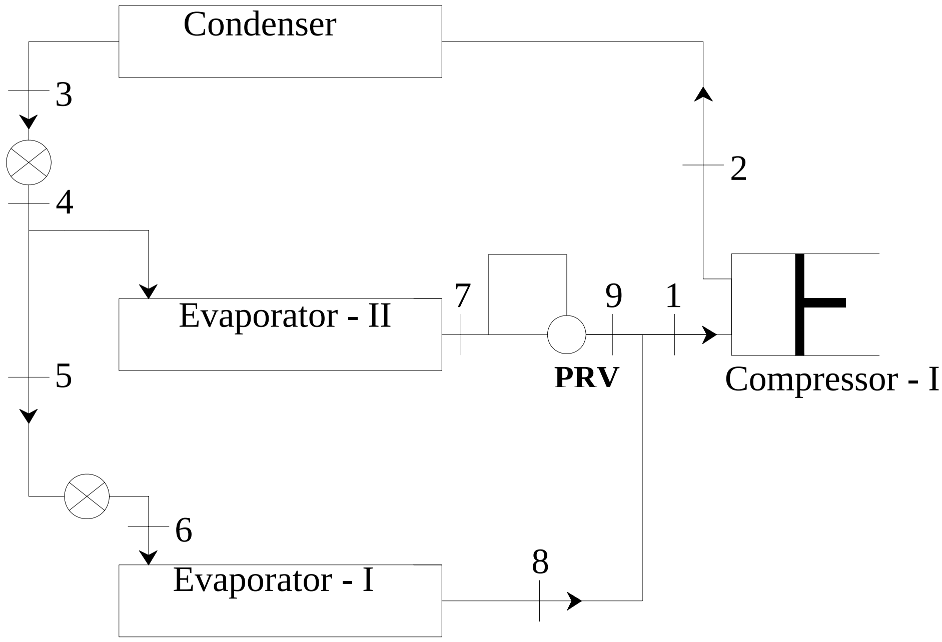
The COP of this system is given by:

$$COP = \frac{Q_{e,I} + Q_{e,II}}{W_c} = \frac{\dot{m}_I (h_8 - h_6) + \dot{m}_{II} (h_7 - h_4)}{(\dot{m}_I + \dot{m}_{II})(h_2 - h_1)} \quad (13.6)$$

where $\dot{m}_I$ and $\dot{m}_{II}$ are the refrigerant mass flow rates through evaporator I and II respectively. They are given by:

$$\dot{m}_I = \frac{Q_{e,I}}{(h_8 - h_6)} \quad (13.7)$$

$$\dot{m}_{II} = \frac{Q_{e,II}}{(h_7 - h_4)} \quad (13.8)$$



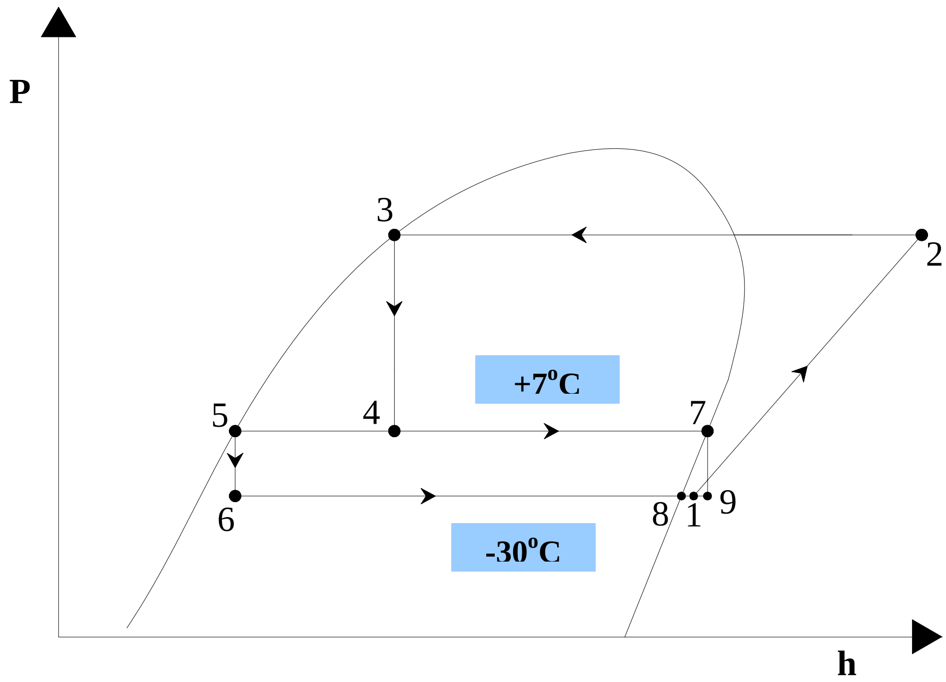


***Fig.13.3(a) & (b)**: Multi-evaporator system with single compressor and multiple expansion valves*

Enthalpy at point 2 (inlet to compressor) is obtained by applying mass and energy balance to the mixing of two refrigerant streams, i.e.,

$$h_2 = \frac{\dot{m}_I h_8 + \dot{m}_{II} h_9}{\dot{m}_I + \dot{m}_{II}} \qquad (13.9)$$

If the expansion across PRV is isenthalpic, then specific enthalpy h_7 will be equal to h_9.

COP obtained using the above multi-evaporator systems is not much higher compared to single stage system as refrigerant vapour at intermediate pressure is first throttled then compressed, and compressor inlet is in superheated region. Performance can be improved significantly if multiple compressors are used in place of a single compressor.

13.3. Multi-evaporator system with multi-compression, intercooling and flash gas removal

Figures 13.4(a) and (b) show the schematic and P-h diagram of a multi-evaporator system which employs multiple compressors, a flash tank for flash gas removal and intercooling. This system is good for low temperature lift applications with different refrigeration loads. For example one evaporator operating at say –40^{o}C for quick freezing of food products and other evaporator operating at –25^{o}C for storage of frozen food. As shown in the system schematic, the pressure in the high temperature evaporator (Evaporator-II) is same as that of flash tank. Superheated vapour from the low-stage compressor is cooled to the saturation temperature in the flash tank. The low temperature evaporator operates efficiently as flash gas is removed in the flash tank. In addition the high-stage compressor (Compressor-II) operates efficiently as the suction vapour is saturated. Even though the high stage compressor has to handle higher mass flow rate due to de-superheating of refrigerant in the flash tank, still the total power input to the system can be reduced substantially, especially with refrigerants such as ammonia.

The COP of this system is given by:

$$COP = \frac{Q_{e,I} + Q_{e,II}}{W_{c,I} + W_{c,II}} = \frac{\dot{m}_I (h_1 - h_8) + \dot{m}_{e,II} (h_3 - h_6)}{\dot{m}_I (h_2 - h_1) + \dot{m}_{II} (h_4 - h_3)} \qquad (13.10)$$

where $\dot{m}_I$ and $\dot{m}_{e,II}$ are the refrigerant mass flow rates through evaporator I and II respectively. They are given by:

$$\dot{m}_I = \frac{Q_{e,I}}{(h_8 - h_6)} \qquad (13.11)$$

$$\dot{m}_{e,II} = \frac{Q_{e,II}}{(h_3 - h_6)} \qquad (13.12)$$

$\dot{m}_{II}$ is the mass flow rate of refrigerant through the high-stage compressor which can be obtained by taking a control volume which includes the flash tank and high temperature evaporator (as shown by dashed line in the schematic) and applying mass and energy balance:

mass balance:

$$\dot{m}_5 + \dot{m}_2 = \dot{m}_7 + \dot{m}_3;\; \dot{m}_5 = \dot{m}_{II} = \dot{m}_3 \;\; \& \;\; \dot{m}_2 = \dot{m}_I = \dot{m}_7 \qquad (13.13)$$

energy balance:

$$\dot{m}_5 h_5 + \dot{m}_2 h_2 + Q_{e,II} = m_7 h_7 + m_3 h_3 \qquad (13.14)$$

from known operating temperatures and evaporator loads ($Q_{e,I}$ and $Q_{e,II}$) one can get the mass flow rate through the high stage compressor and system COP from the above equations.

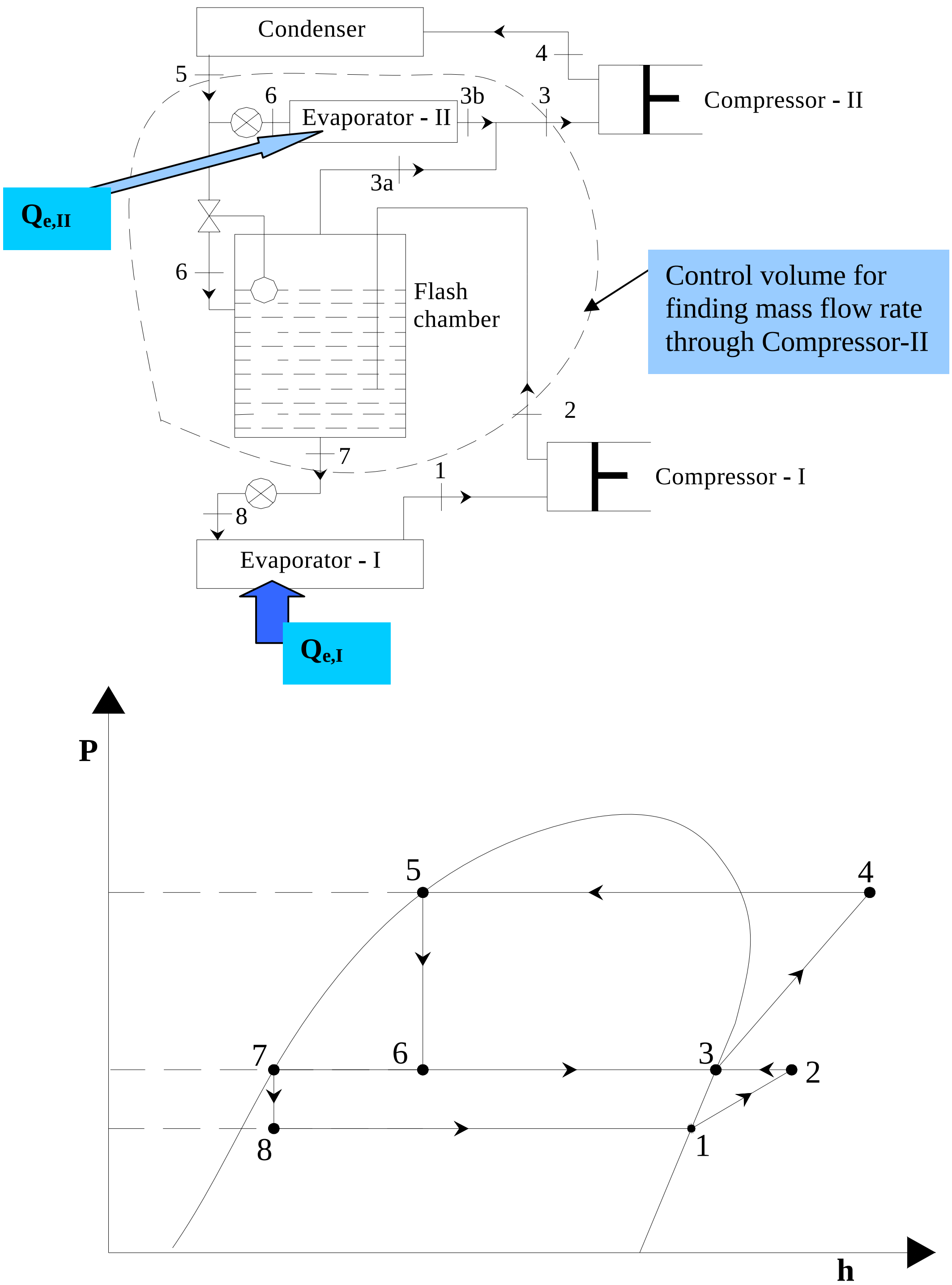


***Fig.13.4(a) & (b)**: Multi-evaporator system with multiple compressors and a flash tank for flash gas removal and intercooling*

13.4. Multi-evaporator system with individual compressors and multiple expansion valves

Figures 13.5(a) and (b) show the schematic and P-h diagram of a multi-evaporator system which employs individual compressors and multiple expansion valves.

The COP of this combined system is given by:

$$COP = \frac{Q_{e,I} + Q_{e,II}}{W_{c,I} + W_{c,II}} = \frac{\dot{m}_I(h_3 - h_9) + \dot{m}_{II}(h_1 - h_7)}{\dot{m}_I(h_4 - h_3) + \dot{m}_{II}(h_2 - h_1)} \qquad (13.15)$$

where $\dot{m}_I$ and $\dot{m}_{II}$ are the refrigerant mass flow rates through evaporator I and II respectively. They are given by:

$$\dot{m}_I = \frac{Q_{e,I}}{(h_3 - h_9)} \qquad (13.16)$$

$$\dot{m}_{II} = \frac{Q_{e,II}}{(h_1 - h_7)} \qquad (13.17)$$

The inlet to the condenser (state 5) is obtained by applying mass and energy balance to the process of mixing of refrigerant vapours from Compressors I and II.

13.5. Limitations of multi-stage systems

Though multi-stage systems have been very successful, they have certain limitations. These are:

a) Since only one refrigerant is used throughout the system, the refrigerant used should have high critical temperature and low freezing point.

b) The operating pressures with a single refrigerant may become too high or too low. Generally only R12, R22 and NH_3 systems have been used in multi-stage systems as other conventional working fluids may operate in vacuum at very low evaporator temperatures. Operation in vacuum leads to leakages into the system and large compressor displacement due to high specific volume.

c) Possibility of migration of lubricating oil from one compressor to other leading to compressor break-down.

The above limitations can be overcome by using cascade systems.

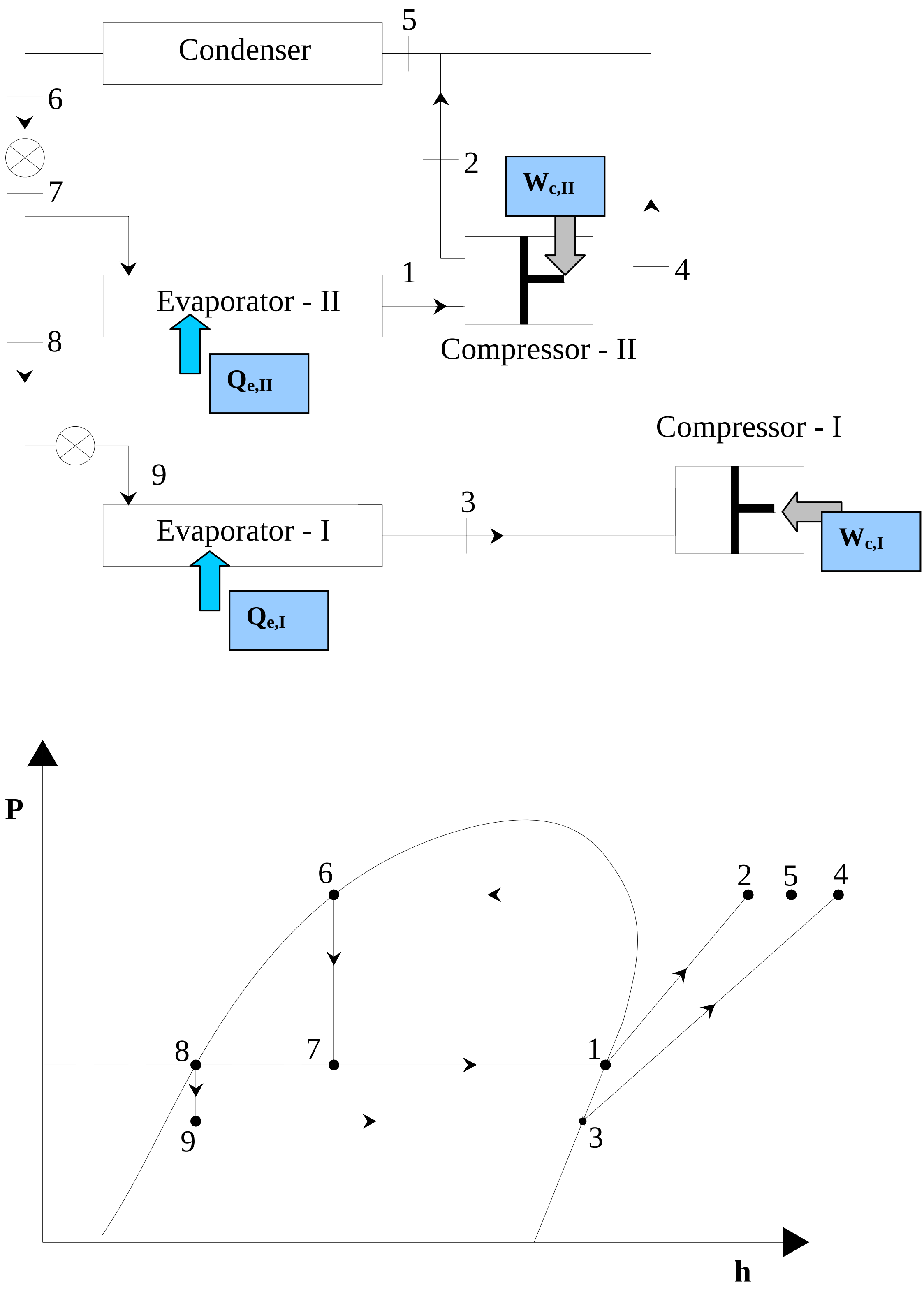


***Fig.13.5(a) & (b)**: Multi-evaporator system with individual compressors and multiple expansion valves*

13.6. Cascade Systems

In a cascade system a series of refrigerants with progressively lower boiling points are used in a series of single stage units. The condenser of lower stage system is coupled to the evaporator of the next higher stage system and so on. The component where heat of condensation of lower stage refrigerant is supplied for vaporization of next level refrigerant is called as *cascade condenser*. Figures 13.6(a) and (b) show the schematic and P-h diagrams of a two-stage cascade refrigeration system. As shown, this system employs two different refrigerants operating in two individual cycles. They are thermally coupled in the cascade condenser. The refrigerants selected should have suitable pressure-temperature characteristics. An example of refrigerant combination is the use of carbon dioxide (NBP = -78.4^{o}C, T_{cr} = 31.06^{o}C) in low temperature cascade and ammonia (NBP = -33.33^{o}C, T_{cr} = 132.25^{o}C) in high temperature cascade. It is possible to use more than two cascade stages, and it is also possible to combine multi-stage systems with cascade systems.

Applications of cascade systems:

i. Liquefaction of petroleum vapours
ii. Liquefaction of industrial gases
iii. Manufacturing of dry ice
iv. Deep freezing etc.

Advantages of cascade systems:

i. Since each cascade uses a different refrigerant, it is possible to select a refrigerant that is best suited for that particular temperature range. Very high or very low pressures can be avoided
ii. Migration of lubricating oil from one compressor to the other is prevented

In practice, matching of loads in the cascade condenser is difficult, especially during the system pull-down. Hence the cascade condensers are normally oversized. In addition, in actual systems a temperature difference between the condensing and evaporating refrigerants has to be provided in the cascade condenser, which leads to loss of efficiency. In addition, it is found that at low temperatures, superheating (useful or useless) is detrimental from volumetric refrigeration effect point-of-view, hence in cascade systems, the superheat should be just enough to prevent the entry of liquid into compressor, and no more for all refrigerants.

Optimum cascade temperature:

For a two-stage cascade system working on Carnot cycle, the optimum cascade temperature at which the COP will be maximum, $T_{cc,opt}$ is given by:

$$T_{cc,opt} = \sqrt{T_e.T_c} \tag{13.18}$$

where T_e and T_c are the evaporator temperature of low temperature cascade and condenser temperature of high temperature cascade, respectively.

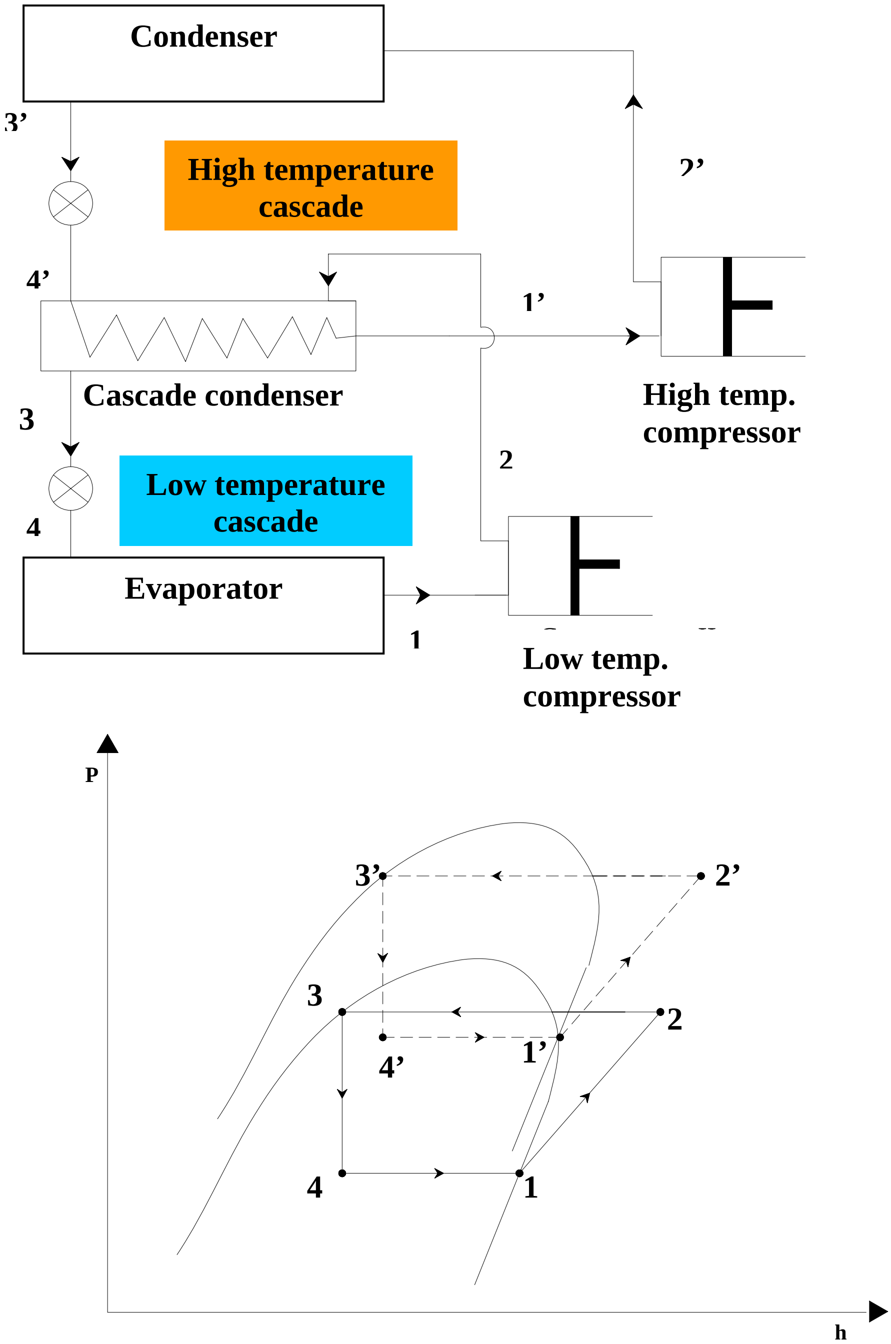


***Fig.13.6(a) & (b)**: A two-stage cascade refrigeration system*

For cascade systems employing vapour compression refrigeration cycle, the optimum cascade temperature assuming equal pressure ratios between the stages is given by:

$$T_{cc,opt} = \left(\frac{b_1 + b_2}{\frac{b_2}{T_c} + \frac{b_1}{T_e}} \right) \qquad (13.19)$$

where b_1 and b_2 are the constants in Clausius-Clayperon equation: $\ln P = a - \frac{b}{T}$ for low and high temperature refrigerants, respectively.

13.7. Auto-cascade systems

An auto-cascade system may be considered as a variation of cascade system, in which a single compressor is used. The concept of auto-cascade system was first proposed by Ruhemann in 1946. Figure 13.7(a) shows the schematic of a two-stage auto-cascade cycle and Fig.137(b) shows the vapour pressure curves of the two

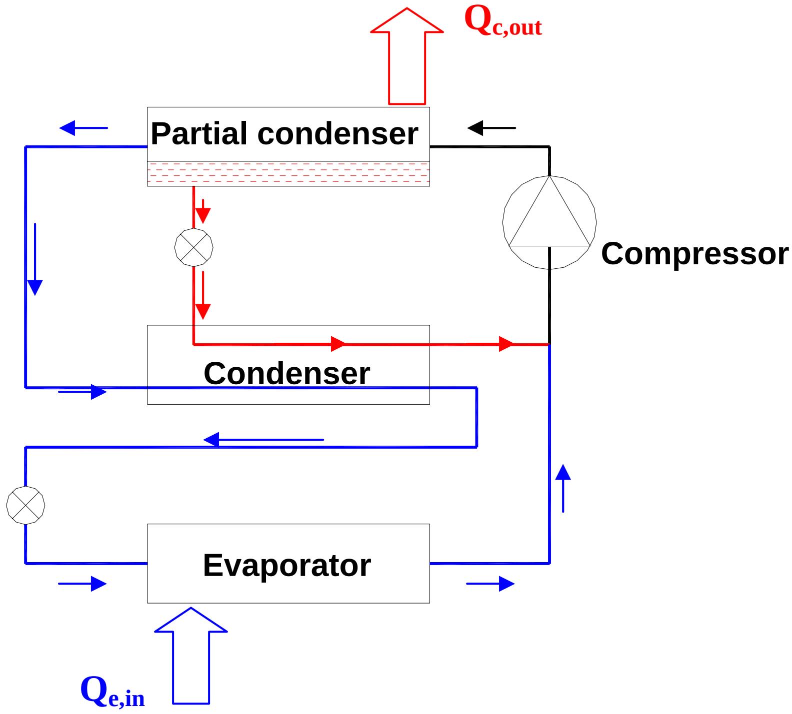


***Fig.13.7(a)**: Schematic of a two-stage auto-cascade system*

refrigerants used in the cycle on D˘hring plot.

In a two-stage auto-cascade system two different working fluids; a low boiling point (low temperature) refrigerant and a high boiling point (high temperature) refrigerant are used. The vapour mixture consisting of both these refrigerants is compressed in the compressor to a discharge pressure ($P_{discharge}$). When this high pressure mixture flows through the partial condenser, the high temperature refrigerant

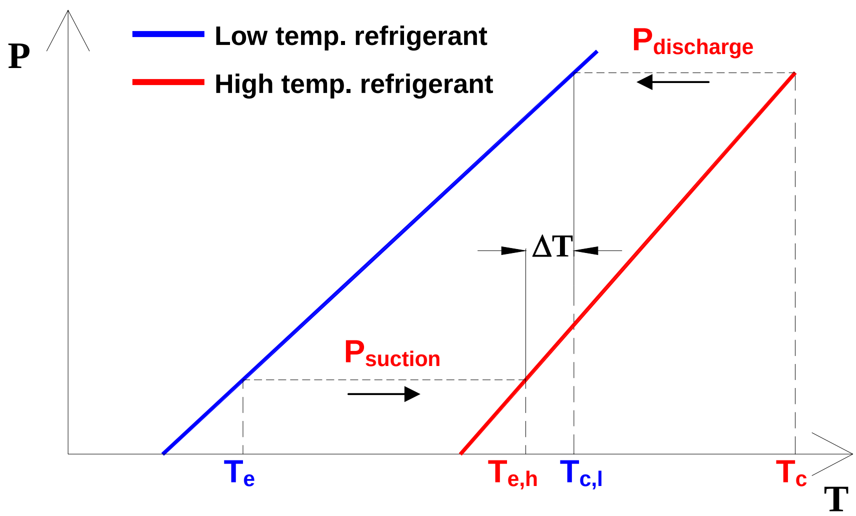


Fig.13.7(b): Schematic illustrating principle of two-stage auto-cascade system on D˘hring plot

can condense by rejecting heat ($Q_{c,out}$) to the external heat sink, if its partial pressure in the mixture is such that the saturation temperature corresponding to the partial pressure is higher than the external heat sink temperature. Since the saturation temperature of the low temperature refrigerant is much lower than the external heat sink temperature at its partial pressure, it cannot condense in the partial condenser, hence, remains as vapour. Thus it is possible theoretically to separate the high temperature refrigerant in liquid form from the partial condenser. Next this high temperature, high pressure liquid is expanded through the expansion valve into the condenser operating at a pressure $P_{suction}$. Due to the expansion of the high temperature refrigerant liquid from $P_{discharge}$ to $P_{suction}$, its temperature drops to a sufficiently low value ($T_{e,h}$) so that when the low temperature, high pressure refrigerant vapour comes in contact with the high temperature, low pressure refrigerant in the condenser it can condense at a temperature $T_{c,l}$. This condensed, high pressure, low temperature refrigerant is then throttled to the suction pressure and is then made to flow through the evaporator, where it can provide the required refrigeration effect at a very low temperature T_e. Both the high temperature refrigerant from condenser and low temperature refrigerant vapour from evaporator can be mixed as they are at the same pressure. This mixture is then compressed in the compressor to complete the cycle. Thus using a single compressor, it is possible to obtain refrigeration at very low temperatures using the auto-cascade system. In practice, more than two stages with more than two refrigerants can be used to achieve very high temperature lifts. However, in actual systems, it is not possible to separate pure refrigerants in the partial condenser as some amount of low temperature refrigerant condenses in the partial condenser and some amount of high temperature refrigerant leaves the partial condenser in vapour form. Thus everywhere in the system, one encounters refrigerant mixtures of varying composition. These systems are widely used in the liquefaction of natural gas.

Questions:

1. Multi-evaporator systems are:

a) Widely used when refrigeration is required at different temperatures
b) When humidity control in the refrigerated space is required
c) When the required temperature lift is small
d) All of the above

Ans.: a) and b)

2. Multi-evaporator systems with a single compressor and a pressure reducing valve:

a) Yield very high COPs compared to multi-evaporator, single stage systems
b) Yield lower compressor discharge temperature compared to single stage systems
c) Yield slightly higher refrigeration effect in the low temperature evaporator compared to single stage systems
d) Yield slightly higher refrigeration effect in the high temperature evaporator compared to single stage systems

Ans.: d)

3. Compared to individual expansion valves, multiple expansion valves:

a) Yield higher refrigeration effect in the low temperature evaporator
b) Yield higher refrigeration effect in the high temperature evaporator
c) Yield lower compressor discharge temperature
d) Decrease the quality of refrigerant at the inlet to low temperature evaporator

Ans.: a) and d)

4. Compared to multi-evaporator and single compressor systems, multi-evaporator systems with multiple compressors:

a) Yield higher COP
b) Decrease maximum cycle temperature
c) Yield higher refrigeration effect
d) All of the above

Ans.: a) and b)

5. In multi-stage systems:

a) The refrigerant used should have high critical temperature and high freezing point
b) The refrigerant used should have high critical temperature and low freezing point
c) There is a possibility of migration of lubricating oil from one compressor to other
d) Operating pressures can be too high or too low

Ans.: b), c) and d)

6. In cascade systems:

a) Different refrigerants are used in individual cascade cycles
b) There is no mixing of refrigerants and no migration of lubricating oil
c) Higher COPs compared to multi-stage systems can be obtained
d) Operating pressures need not be too high or too low

Ans.: a), b) and d)

7. Cascade systems are widely used for:

a) Large refrigeration capacity systems
b) Applications requiring large temperature lifts
c) Applications requiring very high efficiencies
d) All of the above

Ans.: b)

8. For a two-stage cascade system working on Carnot cycle and between low and high temperatures of -90^oC and 50^oC, the optimum cascade temperature at which the COP will be maximum is given by:

a) -20^oC
b) -30^oC

c) –67^{o}C
d) 0^{o}C

Ans.: b)

9. In a two stage, auto-cascade system:

a) Two compressors and two refrigerants are used
b) A single compressor and a single refrigerant are used
c) A single compressor and two refrigerants are used
d) Two compressors and a single refrigerant are used

Ans.: c)

10. In a two stage, auto-cascade system:

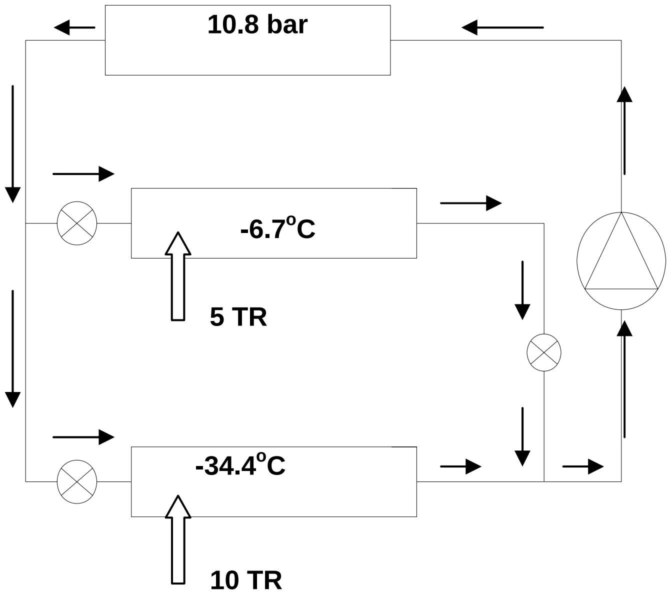
a) Compressor compresses refrigerant mixture
b) Refrigerants are separated in partial condenser
c) Condensing temperature of low temperature refrigerant at discharge pressure is higher than the boiling temperature of high temperature refrigerant at suction pressure
d) Condensing temperature of low temperature refrigerant at discharge pressure is lower than the boiling temperature of high temperature refrigerant at suction pressure

Ans.: a), b) and c)

11. The figure given below shows a multi-evaporator, vapour compression refrigeration system working with ammonia. The refrigeration capacity of the high temperature evaporator operating at –6.7^{o}C is 5 TR, while it is 10 TR for the low temperature evaporator operating at –34.4^{o}C. The condenser pressure is 10.8 bar. Assuming saturated conditions at the exit of evaporators and condenser, ammonia vapour to behave as an ideal gas with a gas constant of 0.4882 kJ/kg.K and isentropic index (c_p/c_v) of 1.29, and isentropic compression:

a) Find the required power input to compressor in kW
b) Find the required power input if instead of using a single compressor, individual compressors are used for low and high temperature evaporators.

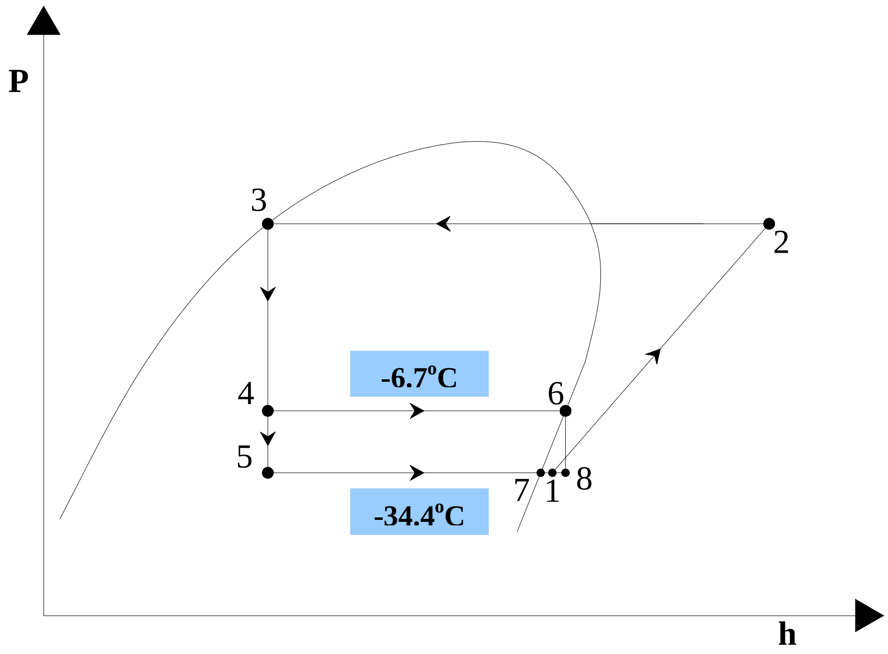
Use the data given in the table:



T,°C	P_{sat} (kPa)	h_f (kJ/kg) (sat.liquid)	h_g(kJ/kg) sat. vapour
-34.4	95.98	44.0	1417
-6.7	331.8	169.1	1455
27.7	1080.0	330.4	1485

Data for Problem 11

Ans.:
a) Single compressor: The P-h diagram for the above system is shown below:



The required mass flow rate through the low temperature evaporator ($m_{r,l}$) is given by:

$$m_{r,l} = Q_{e,l}/(h_7 - h_5) = (10 \text{ X } 3.517)/(1417 - 330.4) = \mathbf{0.03237\ kg/s}$$

The required mass flow rate through the high temperature evaporator ($m_{r,h}$) is given by:

$$m_{r,h} = Q_{e,h}/(h_6 - h_4) = (5 \text{ X } 3.517)/(1455 - 330.4) = \mathbf{0.01564\ kg/s}$$

Assuming the refrigerant vapour to behave as an ideal gas, and assuming the variation in specific heat of the vapour to be negligible, the temperature of the refrigerant after mixing, i.e., at point 1 is given by:

$$T_1 = (m_{r,l}.T_7 + m_{r,h}.T_6)/(m_{r,l} + m_{r,h}) = \mathbf{247.6\ K}$$

Assuming isentropic compression and ideal gas behaviour, the power input to the compressor,W_c is given by:

$$W_c = m_r.R.T_1\left(\frac{k}{k-1}\right)\left[\left(\frac{P_c}{P_e}\right)^{\frac{k-1}{k}} - 1\right]$$

where m_r is the refrigerant flow rate through the compressor ($m_r = m_{r,l} + m_{r,h}$), R is the gas constant (0.4882 kJ/kg.K), P_c and P_e are the discharge and suction pressures and k is the isentropic index of compression (= 1.29).

Substituting these values, the power input to the compressor is found to be:

$$W_c = 18.67 \text{ kW} \qquad \text{(Ans.)}$$

Since the refrigerant vapour is assumed to behave as an ideal gas with constant specific heat, and the compression process is assumed to be isentropic, the discharge temperature T_2 can be obtained using the equation:

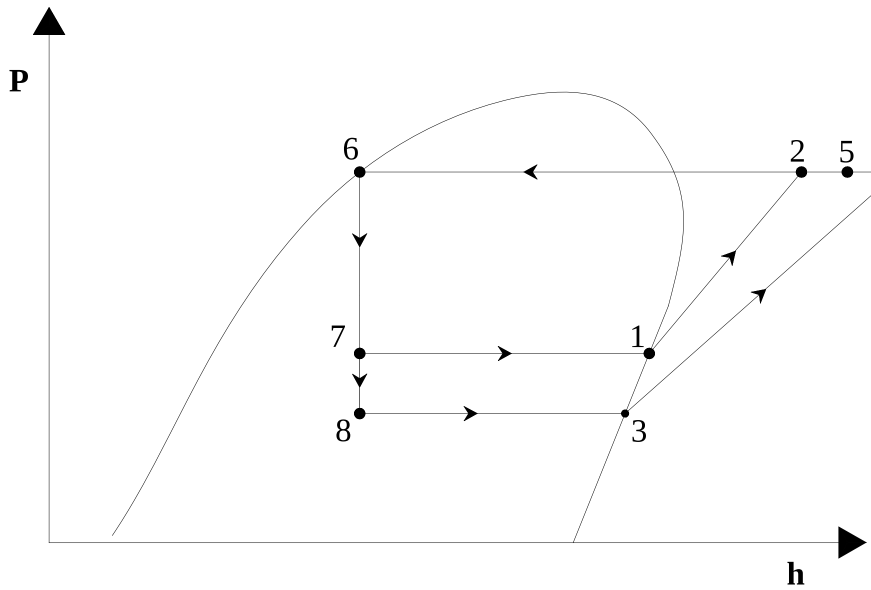
$$W_c = m_r.C_p(T_2 - T_1) = 18.67 \text{ kW}$$

Substituting the values of m_r, C_p (=2.1716 kJ/kg.K) and T_1, the discharge temperature is found to be:

$$T_2 = 427.67 \text{ K} = 153.5^oC$$

b) Individual compressors:

The P-h diagram with individual compressors is shown below:



The mass flow rates through evaporators will be same as before.

The power input to low temperature compressor (process 3 to 4), $W_{c,l}$ is given by:

$$W_{c,l} = m_{r,l}.R.T_3\left(\frac{k}{k-1}\right)\left[\left(\frac{P_c}{P_e}\right)^{\frac{k-1}{k}} - 1\right]$$

substituting the values, we obtain:

$$W_{c,l} = 12.13 \text{ kW}$$

Similarly, for the high temperature compressor (process 1-2), the power input $W_{c,h}$ is given by:

$$W_{c,h} = m_{r,h}.R.T_1\left(\frac{k}{k-1}\right)\left[\left(\frac{P_c}{P_{e,h}}\right)^{\frac{k-1}{k}} - 1\right] = 2.75 \text{ kW}$$

Therefore total power input is given by:

$$\mathbf{W_c = W_{c,l} + W_{c,h} = 12.13 + 2.75 = 14.88 \text{ kW} \qquad (Ans.)}$$

The compressor discharge temperatures for the low temperature and high temperature compressor are found to be:

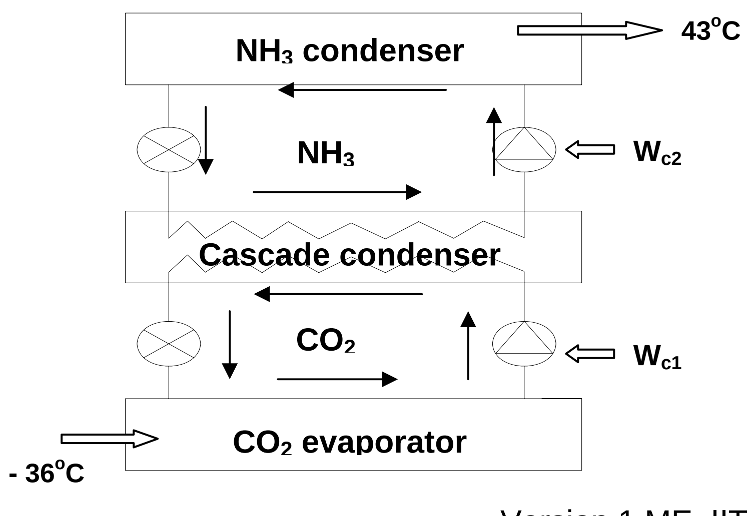
$$\mathbf{T_4 = 411.16 \text{ K} = 138.0^oC}$$
$$\mathbf{T_2 = 347.27 \text{ K} = 74.10^oC}$$

Comments:

1. Using individual compressors in place of a single compressor, the power input to the system could be reduced considerably (≈ 20.3%).
2. In addition, the maximum compressor discharge temperature also could be reduced by about 15^{o}C.
3. In addition to this, the high temperature compressor operates at much lower compression ratio, leading to low discharge temperatures and high volumetric efficiency.

These are the advantages one could get by using individual compressors, instead of a pressure regulating valve and a single compressor. However, in actual systems these benefits will be somewhat reduced since smaller individual compressors generally have lower isentropic and volumetric efficiencies.

4. A cascade refrigeration system shown in the figure given below uses CO_2 as refrigerant for the low-stage and NH_3 as the refrigerant for the high-stage. The system has to provide a refrigeration capacity of **10 TR** and maintain the refrigerated space at **–36^{o}C**, when the ambient temperature (heat sink) is at **43^{o}C**. A temperature difference of **7 K** is required for heat transfer in the evaporator, condenser and the cascade condenser. Assume the temperature lift (T_{cond}-T_{evap}) to be same for both CO_2 and NH_3 cycles and find a) Total power input to the system; b) Power input if the cascade system is replaced with a single stage NH_3 system operating between same refrigerated space and heat sink.
The actual COP of the vapour compression system (COP_{act}) can be estimated using



the equation:

$$COP_{act} = 0.85\,COP_{Carnot}\left[1 - \frac{T_c - T_e}{265}\right]$$

where
COP_{Carnot} = Carnot COP
T_c =Condensing Temp.,
T_e= Evaporator Temp.

Ans.: Since a temperature difference of & K is required for heat transfer, the CO_2 evaporator and NH_3 condenser temperatures are given by:

$$\mathbf{T_{e,CO2} = -36 - 7 = -43^oC = 230\ K}$$
$$\mathbf{T_{c,NH3} = 43 + 7 = 50^oC = 323\ K}$$

In the cascade condenser,

$$\mathbf{T_{c,CO2} = T_{e,NH3} + 7}$$

Since the temperature lifts of CO2 and NH3 cycles are same,
$(T_{c,CO2} - T_{e,CO2}) = (T_{c,NH3} - T_{e,NH3})$

From the above 4 equations, we obtain:

$$\mathbf{T_{c,CO2} = 280\ K}$$
$$\mathbf{T_{e,NH3} = 273\ K}$$

Substituting the values of temperatures in the expression for actual COP, we obtain:

$$\mathbf{COP_{CO2} = 3.17,}\ \text{and}$$
$$\mathbf{COP_{NH3} = 3.77}$$

The power input to CO_2 compressor is given by,

$$W_{c,CO2} = Q_{e,CO2}/COP_{CO2} = 10\ X\ 3.517\ /3.17 = \mathbf{11.1\ kW}$$

Since the heat rejected by the condenser of CO2 system is the refrigeration load for the evaporator of NH3 system, the required refrigeration capacity of NH3 system is given by:

$$Q_{e,NH3} = Q_{c,CO2} = Q_{e,CO2} + W_{c,CO2} = \mathbf{46.27\ kW}$$

Hence power input to NH3 compressor is given by:

$$W_{c,NH3} = Q_{e,NH3}/COP_{NH3} = 46.27\ /3.77 = \mathbf{12.27\ kW}$$

Therefore, the total power input to the system is given by:

$$W_{c.total} = W_{c,CO2} + W_{c,NH3} = \mathbf{23.37\ kW} \qquad \textbf{(Ans.)}$$

b) If instead of a cascade system, a single stage NH3 is used then, the actual COP of the system is:

$$\mathbf{COP_{NH3,1st} = 1.363}$$

Power input to single stage ammonia system is given by:

$$W_{c,NH3,1st} = Q_e / COP_{NH3,1st} = 35.17/1.363 = \mathbf{25.8\ kW} \qquad \textbf{(Ans.)}$$

Comments:

1) Using a cascade system the power consumption could be reduced by about 9.5 %.
2) More importantly, in actual systems, the compared to the single stage system, the compressors of cascade systems will be operating at much smaller pressure ratios, yielding high volumetric and isentropic efficiencies and lower discharge temperatures. Thus cascade systems are obviously beneficial compared to single stage systems for large temperature lift applications.
3. The performance of the cascade system can be improved by reducing the temperature difference for heat transfer in the evaporator, condenser and cascade condenser, compared to larger compressors.