Lesson
37
Transmission Of Air In Air Conditioning Ducts

The specific objectives of this chapter are to:

1. Describe an Air Handling Unit (AHU) and its functions (*Section 37.1*)

2. Discuss the need for studying transmission aspects of air in air conditioning (*Section 37.2*)

3. Discuss airflow through air conditioning ducts, Bernoulli and modified Bernoulli equations, Static, dynamic, datum and total head, Fan Total Pressure (FTP) and power input to fan (*Section 37.3*)

4. Discuss estimation of pressure loss through air conditioning ducts (*Section 37.4*)

5. Estimation of frictional pressure drop of circular and rectangular ducts using friction charts and equations (*Section 37.4*)

6. Estimation of dynamic pressure drop in various types of fittings (*Section 37.5*)

7. Static regain (*Section 37.6*)

At the end of the lecture, the student should be able to:

1. Apply Bernoulli equation and modified Bernoulli equation to air conditioning ducts and estimate various pressure heads, pressure loss, FTP and fan power input

2. Estimate frictional pressure drops through circular and non-circular ducts using friction charts and equations

3. Estimate dynamic pressure drop through various fittings used in air conditioning ducts using tables, charts and equations

4. Define static regain and calculate static regain factor for various types of duct enlargements

37.1. Introduction:

In air conditioning systems that use air as the fluid in the thermal distribution system, it is essential to design the Air Handling Unit (AHU) properly. The primary function of an AHU is to transmit processed air from the air conditioning plant to the conditioned space and distribute it properly within the conditioned space. A typical AHU consists of:

1. A duct system that includes a supply air duct, return air duct, cooling and/or heating coils, humidifiers/dehumidifiers, air filters and dampers

2. An air distribution system comprising various types of outlets for supply air and inlets for return air

3. Supply and return air fans which provide the necessary energy to move the air throughout the system

37.2. Transmission of air:

In an AHU, air is transmitted through various ducts and other components with the help of fans. Since the fan motor consumes a large amount of power, and the duct system occupies considerable building space, the design of air transmission system is an important step in the complete design of air conditioning systems. In the end the success of any air conditioning system depends on the design of individual components as well as a good matching between them under all conditions. In order to design the system for transmission of air, it is important to understand the fundamentals of fluid (air) flow through ducts. These aspects have been dealt with to some extent in Chapter 6 on Fundamentals of Fluid Flow.

37.3. Flow of air through ducts:

As mentioned in Chapter 6, the fundamental equation to be used in the analysis of air conditioning ducts is the Bernoulli's equation. Bernoulli's equation is **valid between any two points in the flow field** when the **flow is *steady, irrotational, inviscid and incompressible***. The equation is **valid along a streamline** *for **rotational, steady and incompressible flows***. Between any two points 1 and 2 in the flow field for irrotational flows, the Bernoulli's equation is written as:

$$\frac{p_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\rho g}+\frac{V_2^2}{2g}+z_2=\frac{p_T}{\rho g}=\text{total head} \qquad (37.1)$$

where $\frac{p}{\rho g}$ is the pressure head, $\frac{V^2}{2g}$ is the velocity head and **Z** is the static head, respectively. Each of the heads has units of length as explained before. The above equation can be written in terms of static, velocity, datum and total pressures as:

$$p_1+\frac{\rho V_1^2}{2}+\rho g z_1=p_2+\frac{\rho V_2^2}{2}+\rho g z_2=p_T=\text{total pressure} \qquad (37.2)$$

The above equation implies that for frictionless flow through a duct, the total pressure remains constant along the duct. Since **all real fluids have finite viscosity**, i.e. in all actual fluid flows, some energy will be lost in overcoming friction. This is referred to as **head loss**, i.e. if the fluid were to rise in a vertical pipe it will rise to a lower height than predicted by Bernoulli's equation. The head loss will cause the total pressure to decrease in the flow direction. If the head loss is denoted by H_l, then Bernoulli's equation can be modified to:

$$\frac{p_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{p_2}{\rho g}+\frac{V_2^2}{2g}+z_2+H_l \qquad (37.3)$$

To overcome the fluid friction and the resulting head, a fan is required in air conditioning systems. When a fan is introduced into the duct through which air is flowing, then the static and total pressures at the section where the fan is located rise. This rise is called as **Fan Total Pressure (FTP)**. Then the required power input to the fan is given by:

$$W_{fan} = \frac{\dot{Q}_{air}.FTP}{\eta_{fan}} \qquad (37.4)$$

The FTP should be such that it overcomes the pressure drop of air as it flows through the duct and the air finally enters the conditioned space with sufficient momentum so that a good air distribution can be obtained in the conditioned space. Evaluation of FTP is important in the selection of a suitable fan for a given application. It can be easily shown that when applied between any two sections 1 and 2 of the duct, in which the fan is located, the FTP is given by:

$$FTP = (p_2 - p_1) + \frac{\rho(V_2^{\,2} - V_1^{\,2})}{2g} + \rho g(z_2 - z_1) + \rho g H_l \qquad (37.5)$$

Thus to evaluate FTP, one needs to know the static pressures at sections 1 and 2 (p_1, p_2), air velocities at 1 and 2 (V_1, V_2), datum at 1 and 2 (Z_1, Z_2) and the head loss H_l. Normally, compared to the other terms, the pressure change due to datum $\rho g(z_2 - z_1)$ is negligible. If the static pressures at the inlet and exit are equal, say, to atmospheric pressure ($p_1 = p_2 = p_{atm}$) and the duct has a uniform cross section ($v_1 = v_2$), then FTP is equal to the pressure loss due to friction. Thus to find FTP, one has to estimate the total pressure loss as air flows through the duct from one section to other.

37.4. Estimation of pressure loss in ducts:

As air flows through a duct its total pressure drops in the direction of flow. The pressure drop is due to:

1. Fluid friction
2. Momentum change due to change of direction and/or velocity

The pressure drop due to friction is known as **frictional pressure drop or friction loss, Δp_f**. The pressure drop due to momentum change is known as **momentum pressure drop or dynamic loss, Δp_d**. Thus the total pressure drop **Δp_t** is given by:

$$\Delta p_t = \Delta p_f + \Delta p_d \qquad (37.6)$$

37.4.1. Evaluation of frictional pressure drop in ducts

The Darcy-Weisbach equation is one of the most commonly used equations for estimating frictional pressure drops in internal flows. This equation is given by:

$$\Delta p_f = f\frac{L}{D}\left(\frac{\rho V^2}{2}\right) \qquad (37.7)$$

where f is the dimensionless friction factor, L is the length of the duct and D is the diameter in case of a circular duct and hydraulic diameter in case of a non-circular duct. The friction factor is a function of **Reynolds number**, $\mathbf{Re_D} = \left(\frac{\rho \mathbf{VD}}{\mu}\right)$ and the relative surface roughness of the pipe or duct surface in contact with the fluid.

For turbulent flow, the friction factor can be evaluated using the empirical correlation suggested by Colebrook and White is used, the correlation is given by:

$$\frac{1}{\sqrt{f}} = -2\log_{10}\left[\frac{k_s}{3.7D} + \frac{2.51}{(Re_D)\sqrt{f}}\right] \qquad (37.8)$$

where k_S is the average surface roughness of inner duct expressed in same units as the diameter D. Evaluation of f from the above equation requires iteration since f occurs on both the sides of it.

In general in air conditioning ducts, the fluid flow is turbulent. It is seen from the above equation that when the flow is turbulent, the friction factor is a function of Reynolds number, hydraulic diameter and inner surface roughness of the duct material. Table 37.1 shows absolute roughness values of some of the materials commonly used in air conditioning:

Material	Absolute roughness , ε (m)
Galvanized Iron (GI) sheet	0.00015
Concrete	0.0003 to 0.003
Riveted steel	0.0009 to 0.009
Cast Iron (CI)	0.00026
Commercial steel	0.00046

Table 37.1: *Average surface roughness of commonly used duct materials*

Of the different materials, the GI sheet material is very widely used for air conditioning ducts. Taking **GI as the reference material** and properties of air at 20^{o}C and 1 atm. pressure, the frictional pressure drop in a circular duct is given by:

$$\Delta p_f = \frac{0.022243\,\dot{Q}_{air}^{\;1.852}\,L}{D^{4.973}} \quad \text{in } N/m^2 \qquad (37.9)$$

where $\dot{Q}_{air}$ is the volumetric flow rate of air in m^3/s, L is the length and D is the inner diameter of the duct in meters, respectively.

Using the above equation, friction charts have been created for estimation of frictional pressure drop of standard air through circular ducts made of GI sheets. Figure 37.1 shows the standard chart for estimating frictional pressure drop in circular ducts made of GI sheets at standard air conditions.

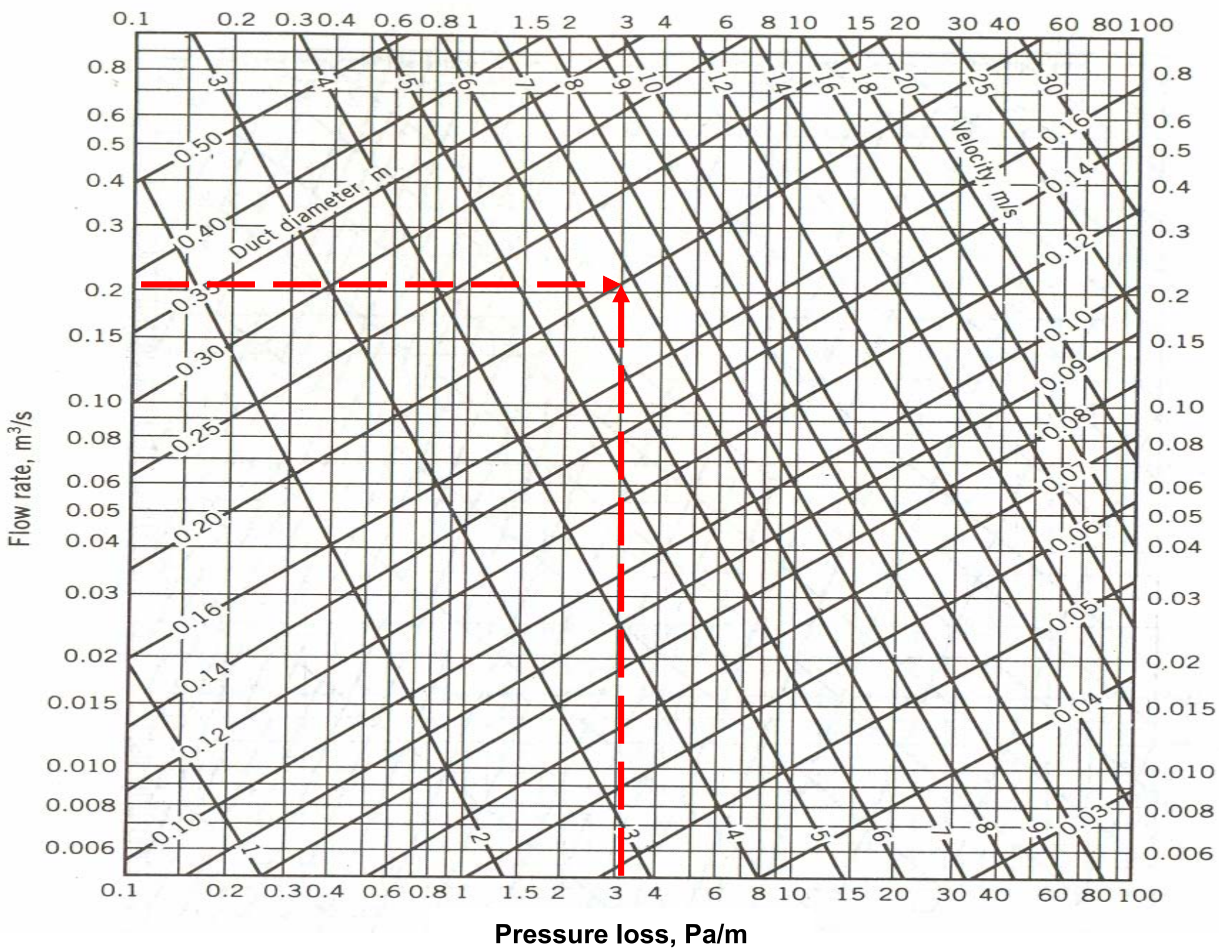

Fig.37.1. *Chart for estimating frictional pressure drop in circular GI ducts*

It can be seen from the chart that one can estimate frictional pressure drop per unit length if any two parameters out of the three parameters, i.e., flow rate $\dot{\mathbf{Q}}_{\mathbf{air}}$, diameter **D** and velocity **V** are known. Correction factors have to be applied to the pressure drop values for ducts made of other materials and/or for air at other conditions. For small changes in air density (ρ) and temperature (T in K), one can use the following relation to obtain frictional pressure drop from the standard chart.

$$\left(\frac{\Delta p_{f,1}}{\Delta p_{f,2}}\right)=\left(\frac{\rho_1}{\rho_2}\right) \quad \textbf{and} \quad \left(\frac{\Delta p_{f,1}}{\Delta p_{f,2}}\right)=\left(\frac{T_2}{T_1}\right)^{0.857} \qquad (37.10)$$

The chart shown above is valid only for circular ducts. For other shapes, an equivalent diameter has to be used to estimate the frictional pressure drop.

37.4.2. Rectangular ducts:

Even though circular ducts require the least material for a given flow rate and allowable pressure drop, **rectangular ducts are generally preferred** in practice as they fit easily into the building construction thus occupying less space, and they are also easy to fabricate. The ratio of the two sides 'a' and 'b' of the rectangle (a/b) is called as aspect ratio of the duct. Since square ducts with aspect ratio 1.0 come close in performance to a circular duct, it is preferable to use an aspect ratio as close to unity as possible for best performance.

One can use equation (37.9) and friction chart for circular ducts for estimating pressure drop through a rectangular duct by using an equivalent diameter. **A rectangular duct is said to be equivalent to a circular duct, if the volumetric flow rate $\dot{Q}_{air}$ and frictional pressure drop per unit length ($\Delta P_f/L$) are same for both.** Equating these two parameters for a rectangular duct and an equivalent circular duct, it can be shown that the equivalent diameter is given by:

$$D_{eq} = 1.3\frac{(ab)^{0.625}}{(a+b)^{0.25}} \qquad (37.11)$$

The above equation is found to be **valid for aspect ratio less than or equal to 1:8.** Thus from the known values of the two sides of the duct 'a' and 'b', one can find the equivalent diameter D_{eq}. From the equivalent diameter and the air flow rate, one can estimate the frictional pressure drop per unit length by using either Eq.(37.9) or the friction chart Fig. 37.1. However, when using equivalent diameter and flow rate to find the frictional pressure drop from the chart, the velocity values shown on the chart are not the actual velocities. The **actual velocities have to be obtained from the flow rate and the actual cross-sectional area of the rectangular duct**. If a rectangular duct has to be designed for a given flow rate and a given frictional pressure drop, then one can first find the equivalent diameter from the friction chart or from Eq.(37.9) and then find the required dimensions of the duct either by fixing the aspect ratio or one of the sides.

37.5. Dynamic losses in ducts:

Dynamic pressure loss takes place whenever there is a change in either the velocity or direction of airflow due to the use of a variety of bends and fittings in air conditioning ducts. Some of the commonly used fittings are: **enlargements, contractions, elbows, branches, dampers etc**. Since in general these fittings and bends are rather short in length (< 1 m), the major pressure drop as air flows through these fittings is not because of viscous drag (friction) but due to momentum change. Pressure drop in bends and fittings could be considerable, and hence should be evaluated properly. However, exact analytical evaluation of dynamic pressure drop through actual bends and fittings is quite complex. Hence for almost all the cases, the dynamic losses are determined from experimental data. In turbulent flows, the dynamic loss is proportional to square of velocity. Hence these are expressed as:

$$\Delta p_d = K\frac{\rho V^2}{2} \qquad (37.12)$$

where K is the dynamic loss coefficient, which is normally obtained from experiments.

Sometimes, an equivalent length L_{eq} is defined to estimate the dynamic pressure loss through bends and fittings. The dynamic pressure loss is obtained from the equivalent length and the frictional pressure drop equation or chart, i.e.,

$$\Delta p_d = K\left(\frac{\rho V^2}{2}\right) = \left(\frac{f.L_{eq}}{D_{eq}}\right)\left(\frac{\rho V^2}{2}\right) \qquad (37.13)$$

where f is the friction factor and L_{eq} is the equivalent length.

37.5.1. Evaluation of dynamic pressure loss through various fittings:

a) Turns, bends or elbows: The most common type of bends used in air conditioning ducts are 90^o turns shown in Fig. 37.2(a).

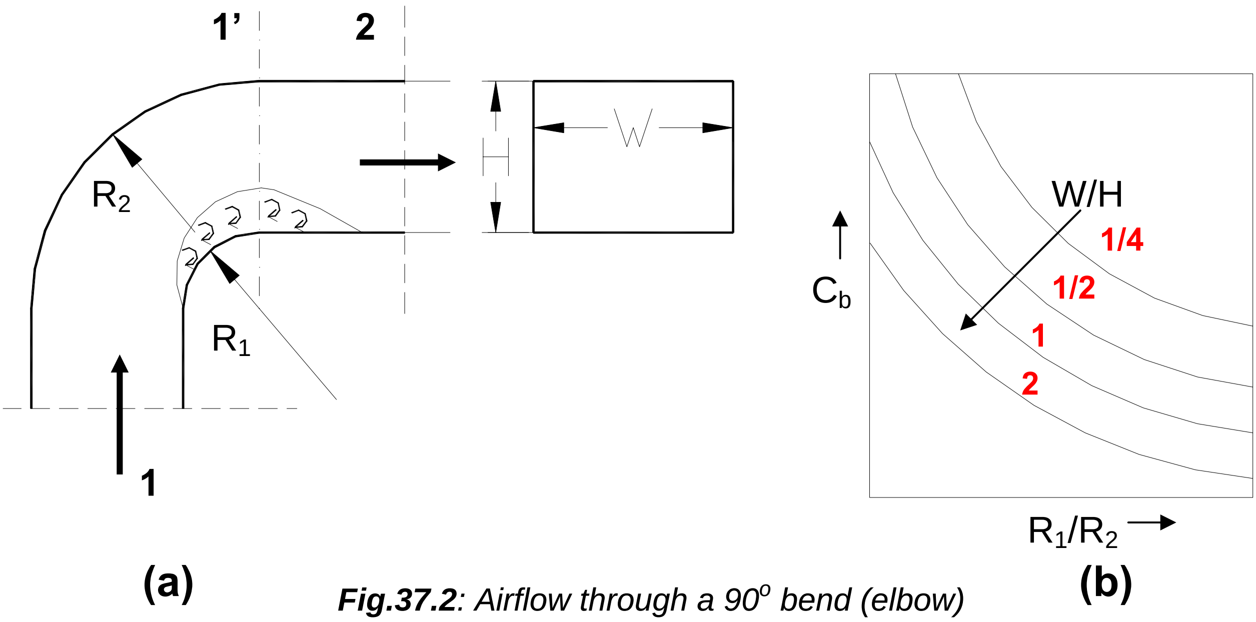

***Fig.37.2**: Airflow through a 90^o bend (elbow)*

The cross-section of the elbow could be circular or rectangular. Weisbach proposed that the dynamic pressure loss in an elbow is due to the sudden expansion from the *vena contracta* region (1') to full cross-section 2 as shown in Fig.37.2(a). The dynamic pressure drop due to the elbow or 90^o turn is found to be a function of the aspect ratio (W/H), inner and outer radii of the turn (R_1 and R_2) and the velocity pressure $\rho V^2/2$, i.e.,

$$\Delta p_{d,b} = C_b\left(\frac{\rho V^2}{2}\right) = f((W/H), R_1, R_2)\left(\frac{\rho V^2}{2}\right) \qquad (37.14)$$

The value of dynamic loss coefficient C_b as a function of aspect ratio (W/H), inner and outer radii of the turn (R_1 and R_2) is available in the form of tables and graphs (Fig.37.2(b)). It can be seen from Fig. 37.2(b) that the pressure loss increases as (R_1/R_2) decreases and/or the aspect ratio W/H decreases. As a result, installing turning vanes in the bends reduces the dynamic pressure drop as it is equivalent to increasing W/H, as shown in Fig. 37.2(c).

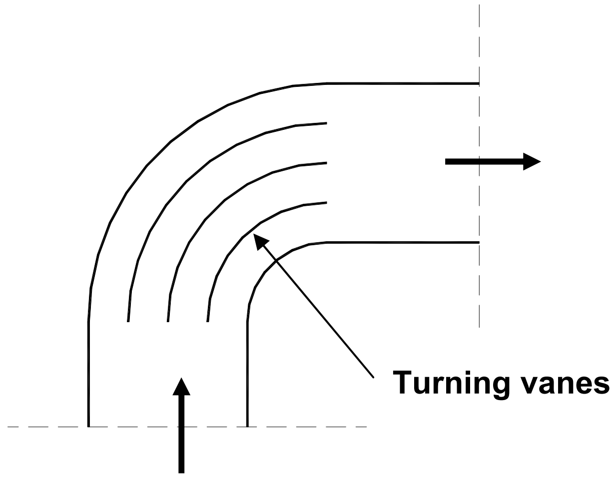

Fig.37.2(c): Use of turning vanes in a 90° bend (elbow)

The equivalent lengths are available as function of geometry for other types of turns and bends.

b) Branch take-offs: Branch take-offs (Fig. 37.3) are commonly used in air conditioning ducts for splitting the airflow into a branch and a downstream duct. The dynamic pressure drop from the upstream (u) to downstream (d), Δp_{u-d} is given by:

$$\Delta p_{u-d} = 0.4\left(\frac{\rho V_d^{\,2}}{2}\right)\left(1-\frac{V_d}{V_u}\right)^2 \qquad (37.15)$$

where V_d and V_u are the air velocities in the downstream and upstream ducts, respectively.

The dynamic pressure drop from the upstream (u) to branch (b), Δp_{u-b} is given by:

$$\Delta p_{u-b} = C_{u-b}\left(\frac{\rho V_d^{\,2}}{2}\right) \qquad (37.16)$$

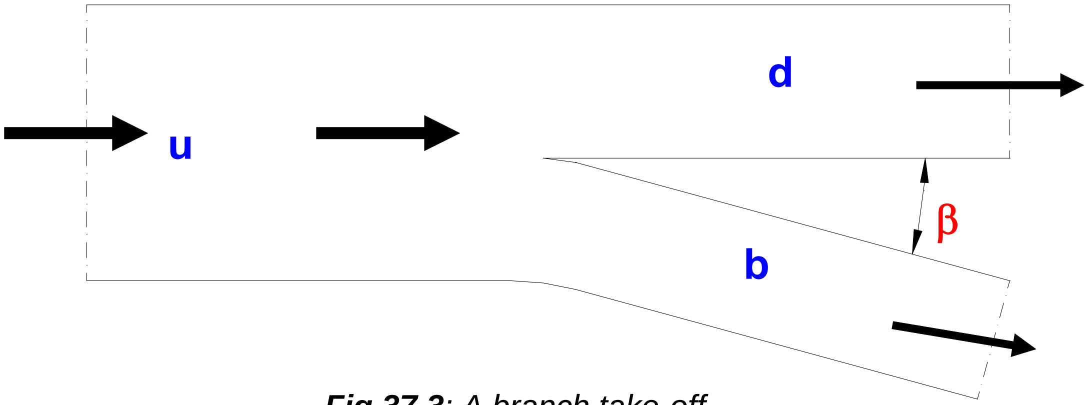

***Fig.37.3**: A branch take-off*

The value of dynamic loss coefficient C_{u-b} is available in the form of tables and graphs as a function of the angle β and the ratio of branch-to-upstream velocity, V_b/V_u. **C_{u-b} is found to increase as β and V_b/V_u increase.**

c) Branch entries: Branch entries (Fig. 37.4) are commonly used in return air ducts. Similar to branch take-offs, the values of dynamic pressure loss coefficients from upstream-to-downstream (**C_{u-d}**) and from branch-to-downstream (**C_{u-d}**) are available in the form of tables and graphs as functions of upstream, branch and downstream velocities and the angle β.

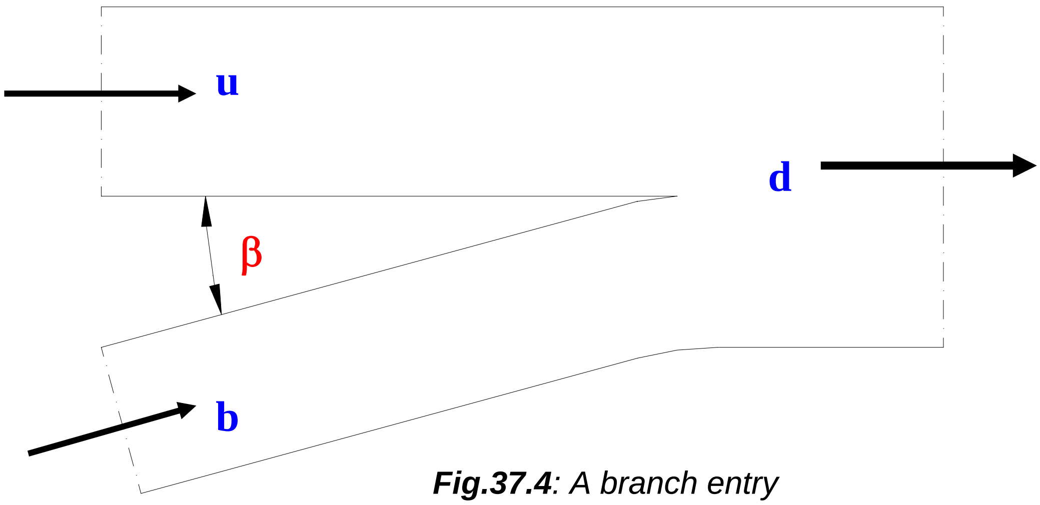

***Fig.37.4**: A branch entry*

d) Sudden enlargement: The pressure loss due to sudden enlargement, shown in Fig. 37.5(a), $\Delta P_{d,enl}$ is given by **Borda-Carnot equation** as:

$$\Delta p_{d,enl} = \left(\frac{\rho V_1^2}{2}\right)\left(1 - \frac{A_1}{A_2}\right)^2 \qquad (37.17)$$

where V_1 is the velocity before enlargement, and A_1 and A_2 are the areas before and after enlargement, respectively. The above expression, which is obtained analytically using modified Bernouille's equation and momentum balance equation is found to over-predict the pressure loss when the air flow rates are high and under-predict when the flow rate is low. Correction factors are available in the form of tables for different enlargements.

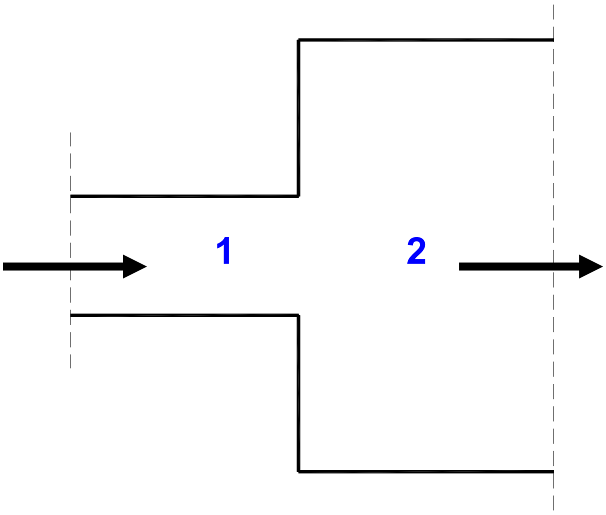

Fig.37.5(a): Sudden enlargement

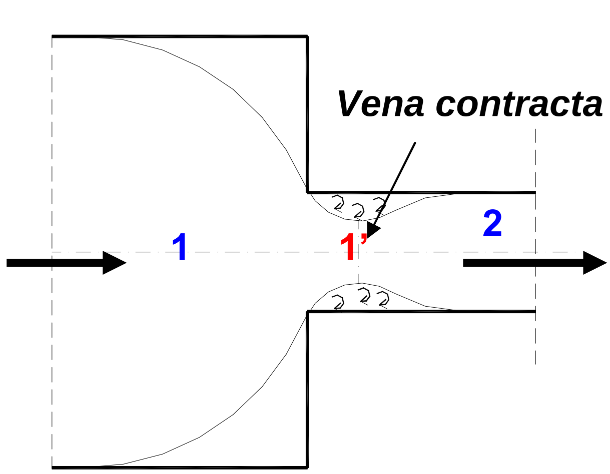

***Fig.37.5(b)**: Sudden contraction*

e) Sudden contraction: A sudden contraction is shown in Fig. 37.5(b). Similar to sudden enlargement, the dynamic pressure loss due to sudden contraction $\Delta P_{d,con}$ can be obtained analytically. This expression is also known as Borda-Carnot equation. It is given by:

$$\Delta p_{d,con} = \left(\frac{\rho V_2^2}{2}\right)\left(\frac{A_2}{A_{1'}} - 1\right)^2 = \left(\frac{\rho V_2^2}{2}\right)\left(\frac{1}{C_c} - 1\right) \qquad (37.18)$$

where V_2 is the velocity in the downstream, and $A_{1'}$ and A_2 are the areas at *vena contracta* and after contraction, respectively. The coefficient C_c is known as contraction coefficient and is seen to be equal to area ratio $A_{1'}/A_2$. The contraction coefficient C_c is found to be a function of the area ratio A_2/A_1, and the values of C_c as obtained by Weisbach are shown in Table 37.3.

A_2/A_1	C_c
0.1	0.624
0.5	0.681
0.8	0.813
1.0	1.000

Table 37.3: Values of contraction coefficient C_c for different area ratios

Comparing the expressions of pressure loss for sudden enlargement and sudden contraction, it can be seen that for the same flow rates and area ratios, the pressure drop due to sudden enlargement is higher than that due to sudden contraction.

f) Miscellaneous fittings, openings etc.: The dynamic pressure loss coefficients for other types of fittings, such as suction and discharge openings are also available in the form of tables. These values depend on the design of the fitting/opening. For abrupt suction opening the dynamic loss coefficient (K) is found to be about 0.85, while it is about 0.03 for a formed entrance. For discharge openings where the downstream pressure is atmospheric, all the kinetic energy of the air stream is dissipated at the exit, hence, the dynamic loss coefficient is equal to 1.0 in this case.

Filters, cooling and heating coils, dampers etc.: The pressure drop across air handling unit equipment, such as, air filters, dampers, cooling and heating coils depend on several factors. Hence, normally these values have to be obtained from the manufacturer's data.

37.6. Static regain:

Whenever there is an enlargement in the cross-sectional area of the duct, the velocity of air decreases, and the velocity pressure is converted into static pressure. The increase in static pressure due to a decrease in velocity pressure is known as static regain. In an ideal case, when there are no pressure losses, the increase in static pressure **(Δp_s)** is exactly equal to the decrease in velocity pressure **(Δp_v)** and the total pressure **(p_t)** remains constant as shown in Fig.37.6(a). Thus for the ideal case:

$$\begin{aligned} \Delta p_v &= p_{v,1} - p_{v,2} = \Delta p_s = p_{s,2} - p_{s,1} \\ p_{t,1} &= p_{t,2} \end{aligned} \qquad (37.19)$$

However, for sudden enlargements or for other non-ideal enlargements, the decrease in velocity pressure will be greater than the increase in static pressure, and the total pressure decreases in the direction flow due to pressure losses as shown in Fig. 37.6(b). The pressure loss is due to separation of the boundary layer and the formation of eddies as shown in Fig.37.6(b). Thus, for sudden or non-ideal enlargement:

$$\begin{aligned} \Delta p_v &= p_{v,1} - p_{v,2} > \Delta p_s = p_{s,2} - p_{s,1} \\ p_{t,1} &= p_{t,2} + \Delta p_{loss} \end{aligned} \qquad (37.20)$$

The pressure loss due to enlargement Δp_{loss} is expressed in terms of a Static Regain Factor, R as:

$$\Delta p_{loss} = (1-R)\Delta p_v = (1-R)(p_{v,1} - p_{v,2}) \qquad (37.21)$$

where the static regain factor R is given by:

$$R = \frac{\Delta p_s}{\Delta p_v} = \frac{(p_{s,2} - p_{s,1})}{(p_{v,1} - p_{v,2})} \qquad (37.21)$$

Thus for ideal enlargement the Static Regain Factor R is equal to 1.0, whereas it is less than 1.0 for non-ideal enlargement.

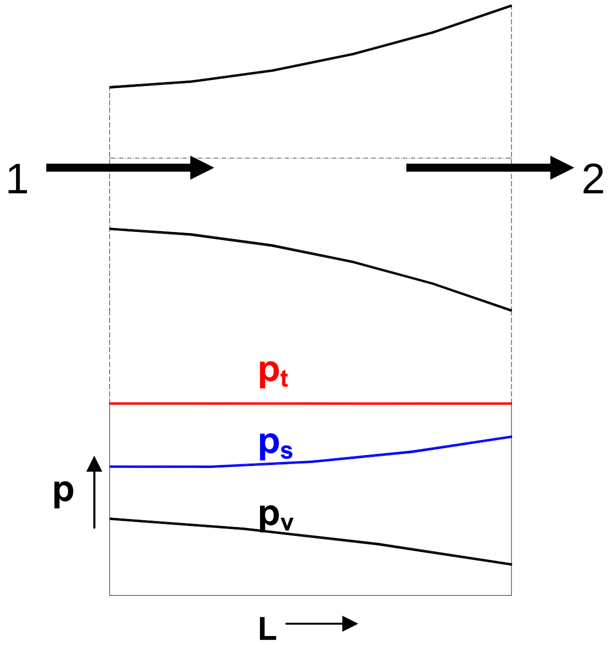

Fig.37.6(a): *Ideal enlargement*

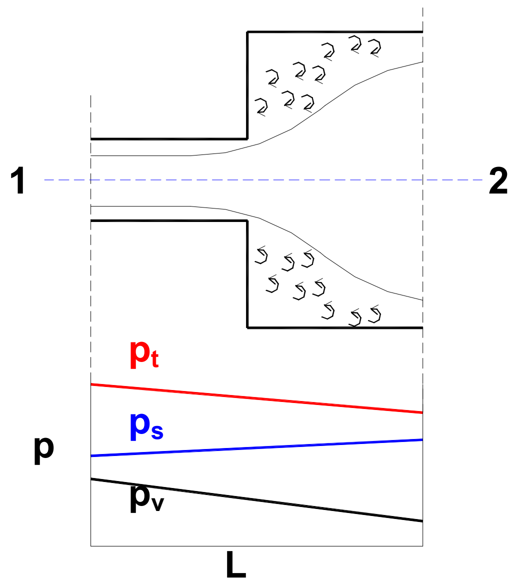

Fig.37.6(b): *Sudden enlargement*

Questions and answers:

1. State which of the following statements are TRUE?

a) An air handling unit conveys air between the conditioned space and the plant
b) An air handling unit consists of supply and return air fans
c) The fan used in an air conditioning system consumes large amount of power
d) All of the above

Ans.: d)

2. State which of the following statements are TRUE?

a) Under ideal conditions, the static pressure through an air conditioning duct remains constant
b) Under ideal conditions, the total pressure through an air conditioning duct remains constant
c) A fan is required in an air conditioning duct to overcome static pressure loss
d) A fan is required in an air conditioning duct to overcome total pressure loss

Ans.: b) and d)

3. State which of the following statements are TRUE?

a) In a duct of uniform cross section, the static pressure remains constant
b) In a duct of uniform cross section, the static pressure decreases along length
c) In a duct of uniform cross section, the total pressure decreases along length
d) In a duct of uniform cross section, the dynamic pressure remains constant

Ans.: b), c) and d)

4. State which of the following statements are TRUE?

a) The pressure drop in an air conditioning duct is due to frictional effects
b) The pressure drop in an air conditioning duct is due to friction as well as momentum change
c) Frictional pressure drop increases with duct length
d) Momentum pressure drop takes place over relatively short lengths

Ans.: b), c) and d)

5. Rectangular ducts are generally preferred over circular ducts in buildings as:

a) For a given flow rate, the pressure drop is less compared to a circular duct
b) For a given pressure drop, it requires less material compared to a circular duct
c) Rectangular ducts are easier to fabricate
d) Rectangular ducts match better with building profile

Ans.: c) and d)

6. State which of the following statements are TRUE?

a) Dynamic pressure drop in an elbow of rectangular cross-section reduces as the aspect ratio increases
b) Use of turning vanes increase the aspect ratio
c) Compared to sudden enlargement, the dynamic pressure drop in sudden contraction is less
d) Compared to sudden enlargement, the dynamic pressure drop in sudden contraction is more

Ans.: a), b) and c)

7. State which of the following statements are TRUE?

a) The static regain factor always lies between 0 and 1
b) The static regain factor is 0 for an ideal enlargement
c) The static regain factor is 1 for an ideal enlargement
d) In an actual enlargement, reduction in dynamic pressure is always greater than increase in static pressure

Ans.: a), c) and d)

8. 1 m^3/s of air is conveyed through a straight, horizontal duct of uniform cross-section and a length of 40 m. If the velocity of air through the duct is 5 m/s, find the required fan power input when a) A circular duct is used, and b) A rectangular duct of aspect ratio 1:4 is used. Take the efficiency of the fan to be 0.7. If a GI sheet of 0.5 mm thick with a density of 8000 kg/m3 is used to construct the duct, how many kilograms of sheet metal is required for circular and rectangular cross sections? Assume standard conditions and the static pressure at the inlet and exit of the duct to be same.

Ans.: From continuity equation; the required cross-sectional area of the duct is given by:

$$\mathbf{A_{cs} = (Flow\ rate,\ Q/Velocity,\ V) = (1.0/5.0) = 0.2\ m^2}$$

<u>a) Circular duct:</u>

The required diameter of the duct, D = $(4A_{cs}/\pi)$ = 0.50463 m

Then using the equation for frictional pressure drop;

$$\Delta p_f = \frac{0.022243\,\dot{Q}_{air}^{\,1.852}\,L}{D^{4.973}} = \frac{0.022243\,(1.0)^{1.852}\,40}{(0.50463)^{4.973}} = 26.7\ N/m^2$$

Since the duct is straight, the dynamic pressure drop is zero in the absence of any fittings, hence:

Fan Total Pressure, FTP = ΔPf = 26.7 N/m²

Hence the required fan power input, W_{fan} is:

$$W_{fan} = \frac{\dot{Q}_{air}.FTP}{\eta_{fan}} = \frac{1.0 \times 26.7}{0.7} = 38.14 \text{ W} \qquad \textbf{(Ans.)}$$

The required mass of the duct is given by:

$$m_{circular} = (\pi D)\text{x(thickness)x(density)} = 6.34 \text{ kg} \qquad \textbf{(Ans.)}$$

b) Rectangular duct: of aspect ratio (a:b) = (1:4)

As before, cross sectional area, $A_{cs} = 0.2 \text{ m}^2 = a \times b = a \times 4a$

$$\Rightarrow a = 0.22361 \text{ m and } b = 4a = 0.89440 \text{ m}$$

The equivalent diameter, Deq is given by:

$$D_{eq} = 1.3 \frac{(ab)^{0.625}}{(a+b)^{0.25}} = 1.3 \frac{(0.2)^{0.625}}{(0.22361 + 0.8944)^{0.25}} = 0.46236 \text{ m}$$

Using the friction equation, the frictional pressure drop is:

$$\Delta p_f = \frac{0.022243 \dot{Q}_{air}^{1.852} L}{D^{4.973}} = \frac{0.022243 (1.0)^{1.852} 40}{(0.46236)^{4.973}} = 41.24 \text{ N/m}^2$$

Hence, the required fan power is:

$$W_{fan} = \frac{\dot{Q}_{air}.FTP}{\eta_{fan}} = \frac{1.0 \times 41.24}{0.7} = 58.91 \text{ W} \qquad \textbf{(Ans.)}$$

The required mass of the rectangular duct is given by:

$$m_{rectangular} = 2(a+b)\text{x(thickness)x(density)} = 8.94 \text{ kg} \qquad \textbf{(Ans.)}$$

Thus for the same flow rate and velocity, a **rectangular duct consumes 54.5% higher fan power** and **weighs 41%** compared to a circular duct.

9. Air at a flow rate of 1.2 kg/s flows through a fitting with sudden enlargement. The area before and after the enlargements are 0.1 m^2 and 1 m^2, respectively. Find the pressure drop due to sudden enlargement using Borda-Carnot Equation. What is the pressure drop if the same amount of air flows through a sudden contraction with area changing from 0.1 m^2 and 1 m^2.

Ans.: Assuming standard air conditions, the density of air is approximately equal to 1.2 kg/m^3. Hence the volumetric flow rate of air, Q is given by:

Q = (mass flow rate/density) = (1.2/1.2) = 1.0 m^3/s

Sudden enlargement:

From Borda-Carnot Equation; pressure drop due to sudden enlargement is given by:

$$\Delta p_{d,enl} = \left(\frac{\rho V_1^2}{2}\right)\left(1 - \frac{A_1}{A_2}\right)^2$$

Velocity $V_1 = Q/A_1 = 1.0/0.1 = 10$ m/s and $A_1/A_2 = 0.1/1.0 = 0.1$

Substituting the above values in the equation, we get:

$$\Delta p_{d,enl} = \left(\frac{\rho V_1^2}{2}\right)\left(1 - \frac{A_1}{A_2}\right)^2 = \left(\frac{1.2 \times 10^2}{2}\right)(1 - 0.1)^2 = 48.6 \text{ N/m}^2 \quad \textbf{(Ans.)}$$

Sudden contraction:

Pressure drop due to sudden contraction is given by Borda-Carnot equation:

$$\Delta p_{d,con} = \left(\frac{\rho V_2^2}{2}\right)\left(\frac{A_2}{A_{1'}} - 1\right)^2 = \left(\frac{\rho V_2^2}{2}\right)\left(\frac{1}{C_c} - 1\right)$$

From Table 37.3, for an area ratio (A_2/A_1) of 0.1, the contraction coefficient **C_c is 0.624.** The velocity after contraction (V_2) is 10 m/s. Hence substituting these values in the above equation:

$$\Delta p_{d,con} = \left(\frac{\rho V_2^2}{2}\right)\left(\frac{1}{C_c} - 1\right) = \left(\frac{1.2 \times 10^2}{2}\right)\left(\frac{1.0}{0.624} - 1\right) = 36.15 \text{ N/m}^2 \quad \textbf{(Ans.)}$$

It can be seen from the example that for the same area ratio and flow rate, the pressure drop due to sudden enlargement is larger than that due to sudden contraction by about **34.4%.**

10. Air at a flow rate of 1 m^3/s flows through a fitting whose cross-sectional area increases gradually from 0.08 m^2 to 0.12 m^2. If the static regain factor **(R)** of the fitting is 0.8, what is the rise in static pressure (static regain) and total pressure loss as air flows through the fitting?

Ans.: The velocity of air at the inlet and exit of the fitting are:

$$\mathbf{V_{in} = 1/A_{in} = 1/0.08 = 12.5\ m/s \text{ and } V_{out} = 1/A_{out} = 1/0.12 = 8.33\ m/s}$$

Taking a value of 1.2 kg/m^3 for the density of air, the velocity pressure at the inlet and exit are given by:

$$\mathbf{P_{v,in} = (\rho V_{in}^2)/2 = (1.2 \times 12.5^2)/2 = 93.75\ N/m^2}$$

$$\mathbf{P_{v,out} = (\rho V_{out}^2)/2 = (1.2 \times 8.33^2)/2 = 41.63\ N/m^2}$$

Static pressure rise through the fitting (static regain) is given by:

$(P_{s,out} – P_{s,in}) = R(P_{v,in} – P_{v,out}) = 0.7 \times (93.75 – 41.63) = 36.484$ **N/m^2** **(Ans.)**

The loss in total pressure is given by:

$\Delta P_{t,loss} = (1–R)\ (P_{v,in} – P_{v,out}) = 0.3 \times (93.75 – 41.63) = 15.636$ **N/m^2** **(Ans.)**