Lesson
7
Review of fundamentals: Heat and Mass transfer

The objective of this lesson is to review fundamentals of heat and mass transfer and discuss:

1. Conduction heat transfer with governing equations for heat conduction, concept of thermal conductivity with typical values, introduce the concept of heat transfer resistance to conduction
2. Radiation heat transfer and present Planck's law, Stefan-Boltzmann equation, expression for radiative exchange between surfaces and the concept of radiative heat transfer resistance
3. Convection heat transfer, concept of hydrodynamic and thermal boundary layers, Newton's law of cooling, convective heat transfer coefficient with typical values, correlations for heat transfer in forced convection, free convection and phase change, introduce various non-dimensional numbers
4. Basics of mass transfer – Fick's law and convective mass transfer
5. Analogy between heat, momentum and mass transfer
6. Multi-mode heat transfer, multi-layered walls, heat transfer networks, overall heat transfer coefficients
7. Fundamentals of heat exchangers

At the end of the lesson the student should be able to:

1. Write basic equations for heat conduction and derive equations for simpler cases
2. Write basic equations for radiation heat transfer, estimate radiative exchange between surfaces
3. Write convection heat transfer equations, indicate typical convective heat transfer coefficients. Use correlations for estimating heat transfer in forced convection, free convection and phase change
4. Express conductive, convective and radiative heat transfer rates in terms of potential and resistance.
5. Write Fick's law and convective mass transfer equation
6. State analogy between heat, momentum and mass transfer
7. Evaluate heat transfer during multi-mode heat transfer, through multi-layered walls etc. using heat transfer networks and the concept of overall heat transfer coefficient
8. Perform basic calculation on heat exchangers

7.1. Introduction

Heat transfer is defined as energy-in-transit due to temperature difference. Heat transfer takes place whenever there is a temperature gradient within a system or whenever two systems at different temperatures are brought into thermal contact. Heat, which is energy-in-transit cannot be measured or observed directly, but the effects produced by it can be observed and measured. Since heat transfer involves transfer and/or conversion of energy, all heat transfer processes must obey the first and second laws of thermodynamics. However unlike thermodynamics, heat transfer deals with systems not in thermal equilibrium and using the heat transfer laws it is possible to find the rate at which energy is transferred due to heat transfer. From the engineer's point of view, estimating the rate of heat transfer is a key requirement. Refrigeration and air conditioning involves heat transfer, hence a good understanding of the fundamentals of heat transfer is a must for a student of refrigeration and air conditioning. This section deals with a brief review of heat transfer relevant to refrigeration and air conditioning.

Generally heat transfer takes place in three different modes: conduction, convection and radiation. In most of the engineering problems heat transfer takes place by more than one mode simultaneously, i.e., these heat transfer problems are of multi-mode type.

7.2. Heat transfer

7.2.1. Conduction heat transfer:

Conduction heat transfer takes place whenever a temperature gradient exists in a stationary medium. Conduction is one of the basic modes of heat transfer. On a microscopic level, conduction heat transfer is due to the elastic impact of molecules in fluids, due to molecular vibration and rotation about their lattice positions and due to free electron migration in solids.

The fundamental law that governs conduction heat transfer is called Fourier's law of heat conduction, it is an empirical statement based on experimental observations and is given by:

$$Q_x = -k.A.\frac{dT}{dx} \tag{7.1}$$

In the above equation, Q_x is the rate of heat transfer by conduction in x-direction, (dT/dx) is the temperature gradient in x-direction, A is the cross-sectional area normal to the x-direction and k is a proportionality constant and is a property of the conduction medium, called thermal conductivity. The '-' sign in the above equation is a consequence of 2^{nd} law of thermodynamics, which states that in spontaneous process heat must always flow from a high temperature to a low temperature (i.e., dT/dx must be negative).

The thermal conductivity is an important property of the medium as it is equal to the conduction heat transfer per unit cross-sectional area per unit temperature gradient. Thermal conductivity of materials varies significantly. Generally it is very high for pure metals and low for non-metals. Thermal conductivity of solids is generally greater than that of fluids. Table 7.1 shows typical thermal conductivity values at 300 K. Thermal conductivity of solids and liquids vary mainly with temperature, while thermal conductivity of gases depend on both temperature and pressure. For isotropic materials the value of thermal conductivity is same in all directions, while for anisotropic materials such as wood and graphite the value of thermal conductivity is different in different directions. In refrigeration and air conditioning high thermal conductivity materials are used in the construction of heat exchangers, while low thermal conductivity materials are required for insulating refrigerant pipelines, refrigerated cabinets, building walls etc.

Table 7.1. Thermal conductivity values for various materials at 300 K

Material	**Thermal conductivity (W/m K)**
Copper (pure)	399
Gold (pure)	317
Aluminum (pure)	237
Iron (pure)	80.2
Carbon steel (1 %)	43
Stainless Steel (18/8)	15.1
Glass	0.81
Plastics	0.2 – 0.3
Wood (shredded/cemented)	0.087
Cork	0.039
Water (liquid)	0.6
Ethylene glycol (liquid)	0.26
Hydrogen (gas)	0.18
Benzene (liquid)	0.159
Air	0.026

General heat conduction equation:

Fourier's law of heat conduction shows that to estimate the heat transfer through a given medium of known thermal conductivity and cross-sectional area, one needs the spatial variation of temperature. In addition the temperature at any point in the medium may vary with time also. The spatial and temporal variations are obtained by solving the heat conduction equation. The heat conduction equation is obtained by applying first law of thermodynamics and Fourier's law to an elemental control volume of the conducting medium. In rectangular coordinates, the general heat conduction equation for a conducting media with constant thermo-physical properties is given by:

$$\frac{1}{\alpha}\frac{\partial T}{\partial \tau} = \left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right] + \frac{q_g}{k} \tag{7.2}$$

In the above equation, $\alpha = \frac{k}{\rho c_p}$ is a property of the media and is called as thermal diffusivity, q_g is the rate of heat generation per unit volume inside the control volume and τ is the time.

The general heat conduction equation given above can be written in a compact form using the Laplacian operator, ∇^2 as:

$$\frac{1}{\alpha}\frac{\partial T}{\partial \tau} = \nabla^2 T + \frac{q_g}{k} \quad (7.3)$$

If there is no heat generation inside the control volume, then the conduction equation becomes:

$$\frac{1}{\alpha}\frac{\partial T}{\partial \tau} = \nabla^2 T \quad (7.4)$$

If the heat transfer is steady and temperature does not vary with time, then the equation becomes:

$$\nabla^2 T = 0 \quad (7.5)$$

The above equation is known as Laplace equation.

The solution of heat conduction equation along with suitable initial and boundary conditions gives temperature as a function of space and time, from which the temperature gradient and heat transfer rate can be obtained. For example for a simple case of one-dimensional, steady heat conduction with no heat generation (Fig. 7.1), the governing equation is given by:

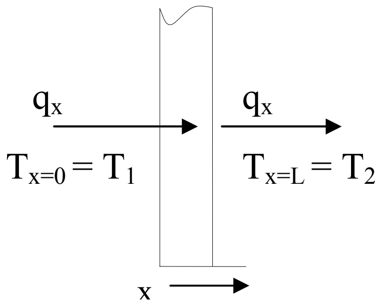

Fig. 7.1. *Steady 1-D heat conduction*

$$\frac{d^2 T}{dx^2} = 0 \quad (7.6)$$

The solution to the above equation with the specified boundary conditions is given by:

$$T = T_1 + (T_2 - T_1)\frac{x}{L} \quad (7.7)$$

and the heat transfer rate, Q_x is given by:

$$Q_x = -kA\frac{\mathbf{dT}}{\mathbf{dx}} = kA\left(\frac{T_1 - T_2}{L}\right) = \left(\frac{\Delta T}{R_{cond}}\right) \quad (7.8)$$

where $\Delta T = T_1 - T_2$ and resistance to conduction heat transfer, $R_{cond} = (L/kA)$

Similarly for one-dimensional, steady heat conduction heat transfer through a cylindrical wall the temperature profile and heat transfer rate are given by:

$$T = T_1 - (T_1 - T_2)\frac{\ln(r/r_1)}{\ln(r_2/r_1)} \quad (7.9)$$

$$Q_r = -kA\frac{dT}{dr} = 2\pi kL\frac{(T_1 - T_2)}{\ln(r_2/r_1)} = \left(\frac{\Delta T}{R_{cyl}}\right) \quad (7.10)$$

where r_1, r_2 and L are the inner and outer radii and length of the cylinder and $R_{cyl} = \frac{\ln(r_2/r_1)}{2\pi LK}$ is the heat transfer resistance for the cylindrical wall.

From the above discussion it is clear that the steady heat transfer rate by conduction can be expressed in terms of a potential for heat transfer (ΔT) and a resistance for heat transfer R, analogous to Ohm's law for an electrical circuit. This analogy with electrical circuits is useful in dealing with heat transfer problems involving multiplayer heat conduction and multimode heat transfer.

Temperature distribution and heat transfer rates by conduction for complicated, multi-dimensional and transient cases can be obtained by solving the relevant heat conduction equation either by analytical methods or numerical methods.

7.2.2. Radiation heat transfer:

Radiation is another fundamental mode of heat transfer. Unlike conduction and convection, radiation heat transfer does not require a medium for transmission as energy transfer occurs due to the propagation of electromagnetic waves. A body due to its temperature emits electromagnetic radiation, and it is emitted at all temperatures. It is propagated with the speed of light (3×10^8 m/s) in a straight line in vacuum. Its speed decreases in a medium but it travels in a straight line in homogenous medium. The speed of light, c is equal to the product of wavelength λ and frequency ν, that is,

$$\mathbf{c} = \lambda\nu \quad (7.11)$$

The wave length is expressed in Angstrom (1 $A^o = 10^{-10}$ m) or micron (1 μm = 10^{-6}m). Thermal radiation lies in the range of 0.1 to 100 μm, while visible light lies in the range of 0.35 to 0.75 μm. Propagation of thermal radiation takes place in the form of discrete quanta, each quantum having energy of

$$\mathbf{E} = \mathbf{h}\nu \quad (7.12)$$

Where, h is Plank's constant, h = 6.625×10^{-34} Js. The radiation energy is converted into heat when it strikes a body.

The radiation energy emitted by a surface is obtained by integrating Planck's equation over all the wavelengths. For a real surface the radiation energy given by Stefan-Boltzmann's law is:

$$Q_r = \varepsilon.\sigma.A.T_s^4 \quad (7.13)$$

where Q_r = Rate of thermal energy emission, W

ε	=	Emissivity of the surface
σ	=	Stefan-Boltzmann's constant, 5.669 X 10^{-8} W/m^2.K^4
A	=	Surface area, m^2
T_s	=	Surface Temperature, K

The emissivity is a property of the radiating surface and is defined as the emissive power (energy radiated by the body per unit area per unit time over all the wavelengths) of the surface to that of an ideal radiating surface. The ideal radiator is called as a "black body", whose emissivity is 1. A black body is a hypothetical body that absorbs all the incident (all wave lengths) radiation. The term 'black' has nothing to do with black colour. A white coloured body can also absorb infrared radiation as much as a black coloured surface. A hollow enclosure with a small hole is an approximation to black body. Any radiation that enters through the hole is absorbed by multiple reflections within the cavity. The hole being small very small quantity of it escapes through the hole.

The radiation heat exchange between any two surfaces 1 and 2 at different temperatures T_1 and T_2 is given by:

$$Q_{1\text{-}2}=\sigma.A.F_\varepsilon F_A(T_1^4-T_2^4) \quad (7.14)$$

where

$Q_{1\text{-}2}$	=	Radiation heat transfer between 1 and 2, W
F_ε	=	Surface optical property factor
F_A	=	Geometric shape factor
T_1,T_2	=	Surface temperatures of 1 and 2, K

Calculation of radiation heat transfer with known surface temperatures involves evaluation of factors F_ε and F_A.

Analogous to Ohm's law for conduction, one can introduce the concept of thermal resistance in radiation heat transfer problem by linearizing the above equation:

$$Q_{1-2}=\frac{(T_1-T_2)}{R_{rad}} \quad (7.15)$$

where the radiative heat transfer resistance R_{rad} is given by:

$$R_{rad} = \left(\frac{T_1\text{-}T_2}{\sigma A F_\varepsilon F_A (T_1^4\text{-}T_2^4)}\right) \quad (7.16)$$

7.2.3. Convection Heat Transfer:

Convection heat transfer takes place between a surface and a moving fluid, when they are at different temperatures. In a strict sense, convection is not a basic mode of heat transfer as the heat transfer from the surface to the fluid consists of two mechanisms operating simultaneously. The first one is energy transfer due to molecular motion (conduction) through a fluid layer adjacent to the surface, which remains stationary with respect to the solid surface due to no-slip condition. Superimposed upon this conductive mode is energy transfer by the macroscopic motion of fluid particles by virtue of an external force, which could be generated by a pump or fan (forced convection) or generated due to buoyancy, caused by density gradients.

When fluid flows over a surface, its velocity and temperature adjacent to the surface are same as that of the surface due to the no-slip condition. The velocity and temperature far away from the surface may remain unaffected. The region in which the velocity and temperature vary from that of the surface to that of the free stream are called as hydrodynamic and thermal boundary layers, respectively. Figure 7.2 show that fluid with free stream velocity U_∞ flows over a flat plate. In the vicinity of the surface as shown in Figure 7.2, the velocity tends to vary from zero (when the surface is stationary) to its free stream value U_∞. This happens in a narrow region whose thickness is of the order of $Re_L^{-0.5}$ ($Re_L = U_\infty L/\nu$) where there is a sharp velocity gradient. This narrow region is called hydrodynamic boundary layer. In the hydrodynamic boundary layer region the inertial terms are of same order magnitude as the viscous terms. Similarly to the velocity gradient, there is a sharp temperature gradient in this vicinity of the surface if the temperature of the surface of the plate is different from that of the flow stream. This region is called thermal boundary layer, δ_t whose thickness is of the order of $(Re_L Pr)^{-0.5}$, where Pr is the Prandtl number, given by:

$$Pr = \frac{c_{p,f}\mu_f}{k_f} = \frac{\nu_f}{\alpha_f} \tag{7.17}$$

In the expression for Prandtl number, all the properties refer to the flowing fluid.

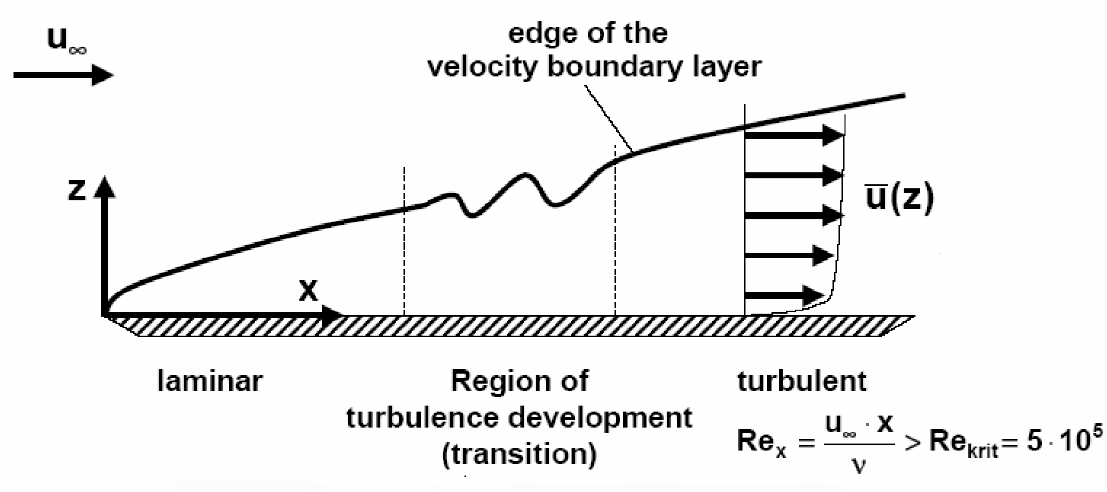

Fig. 7.2. *Velocity distribution of flow over a flat plate*

In the thermal boundary layer region, the conduction terms are of same order of magnitude as the convection terms.

The momentum transfer is related to kinematic viscosity ν while the diffusion of heat is related to thermal diffusivity α hence the ratio of thermal boundary layer to viscous boundary layer is related to the ratio ν/α, Prandtl number. From the expressions for boundary layer thickness it can be seen that the ratio of thermal boundary layer thickness to the viscous boundary layer thickness depends upon Prandtl number. For large Prandtl numbers $\delta_t < \delta$ and for small Prandtl numbers, $\delta_t > \delta$. It can also be seen that as the Reynolds number increases, the boundary layers become narrow, the temperature gradient becomes large and the heat transfer rate increases.

Since the heat transfer from the surface is by molecular conduction, it depends upon the temperature gradient in the fluid in the immediate vicinity of the surface, i.e.

$$Q = -kA\left(\frac{dT}{dy}\right)_{y=0} \tag{7.18}$$

Since temperature difference has been recognized as the potential for heat transfer it is convenient to express convective heat transfer rate as proportional to it, i.e.

$$Q = -k_f A\left(\frac{dT}{dy}\right)_{y=0} = h_c A(T_w - T_\infty) \tag{7.19}$$

The above equation defines the convective heat transfer coefficient h_c. This equation $Q = h_c A(T_w - T_\infty)$ is also referred to as Newton's law of cooling. From the above equation it can be seen that the convective heat transfer coefficient h_c is given by:

$$h_c = \frac{-k_f\left(\frac{dT}{dy}\right)_{y=0}}{(T_w - T_\infty)} \tag{7.20}$$

The above equation suggests that the convective heat transfer coefficient (hence heat transfer by convection) depends on the temperature gradient $\left(\frac{dT}{dy}\right)_{y=0}$ near the surface in addition to the thermal conductivity of the fluid and the temperature difference. The temperature gradient near the wall depends on the rate at which the fluid near the wall can transport energy into the mainstream. Thus the temperature gradient depends on the flow field, with higher velocities able to pressure sharper temperature gradients and hence higher heat transfer rates. Thus determination of convection heat transfer requires the application of laws of fluid mechanics in addition to the laws of heat transfer.

Table 7.2 Typical order-of magnitude values of convective heat transfer coefficients

Type of fluid and flow	**Convective heat transfer coefficient h_c, ($W/m^2\,K$)**
Air, free convection	6 – 30
Water, free convection	20 – 100
Air or superheated steam, forced convection	30 – 300
Oil, forced convection	60 – 1800
Water, forced convection	300 – 18000
Synthetic refrigerants, boiling	500 - 3000
Water, boiling	3000 – 60000
Synthetic refrigerants, condensing	1500 - 5000
Steam, condensing	6000 – 120000

Traditionally, from the manner in which the convection heat transfer rate is defined, evaluating the convective heat transfer coefficient has become the main objective of the problem. The convective heat transfer coefficient can vary widely depending upon the type of fluid and flow field and temperature difference. Table 7.2 shows typical order-of-magnitude values of convective heat transfer coefficients for different conditions.

Convective heat transfer resistance:

Similar to conduction and radiation, convective heat transfer rate can be written in terms of a potential and resistance, i.e.,

$$Q = h_c A(T_w - T_\infty) = \frac{(T_w - T_\infty)}{R_{conv}} \qquad (7.21)$$

where the convective heat transfer resistance, $R_{conv} = 1/(h_c A)$

Determination of convective heat transfer coefficient:

Evaluation of convective heat transfer coefficient is difficult as the physical phenomenon is quite complex. Analytically, it can be determined by solving the mass, momentum and energy equations. However, analytical solutions are available only for very simple situations, hence most of the convection heat transfer data is obtained through careful experiments, and the equations suggested for convective heat transfer coefficients are mostly empirical. Since the equations are of empirical nature, each equation is applicable to specific cases. Generalization has been made possible to some extent by using several non-dimensional numbers such as Reynolds number, Prandtl number, Nusselt number, Grashoff number, Rayleigh number etc. Some of the most important and commonly used correlations are given below:

Heat transfer coefficient inside tubes, ducts etc.:

When a fluid flows through a conduit such as a tube, the fluid flow and heat transfer characteristics at the entrance region will be different from the rest of the tube. Flow in the entrance region is called as developing flow as the boundary layers form and develop in this region. The length of the entrance region depends upon the type of flow, type of surface, type of fluid etc. The region beyond this entrance region is known as fully developed region as the boundary layers fill the entire conduit and the velocity and temperature profiles remains essentially unchanged. In general, the entrance effects are important only in short tubes and ducts. Correlations are available in literature for both entrance as well as fully developed regions. In most of the practical applications the flow will be generally fully developed as the lengths used are large. The following are some important correlations applicable to fully developed flows:

a) Fully developed laminar flow inside tubes (internal diameter D):

Constant wall temperature condition:

$$\text{Nusselt number, } Nu_D = \left(\frac{h_c D}{\mathbf{k_f}}\right) = 3.66 \qquad (7.22)$$

Constant wall heat flux condition:

$$\text{Nusselt number, } Nu_D = \left(\frac{h_c D}{\mathbf{k_f}}\right) = 4.364 \qquad (7.23)$$

b) Fully developed turbulent flow inside tubes (internal diameter D):

Dittus-Boelter Equation:

$$\text{Nusselt number, } Nu_D = \left(\frac{h_c D}{\mathbf{k_f}}\right) = 0.023\, Re_D^{\ 0.8}\, Pr^n \qquad (7.24)$$

where n = 0.4 for heating ($T_w > T_f$) and n = 0.3 for cooling ($T_w < T_f$).

The Dittus-Boelter equation is valid for smooth tubes of length L, with 0.7 < Pr < 160, Re_D > 10000 and (L/D) > 60.

Petukhov equation: This equation is more accurate than Dittus-Boelter and is applicable to rough tubes also. It is given by:

$$Nu_D = \frac{Re_D\, Pr}{X}\left(\frac{f}{8}\right)\left(\frac{\mu_b}{\mu_w}\right)^n$$
$$\text{where } X = 1.07 + 12.7(Pr^{2/3} - 1)\left(\frac{f}{8}\right)^{1/2} \qquad (7.25)$$

where n = 0.11 for heating with uniform wall temperature
n = 0.25 for cooling with uniform wall temperature, and
n = 0 for uniform wall heat flux or for gases

'f' in Petukhov equation is the friction factor, which needs to be obtained using suitable correlations for smooth or rough tubes. μ_b and μ_w are the dynamic viscosities of the fluid evaluated at bulk fluid temperature and wall temperatures respectively. Petukhov equation is valid for the following conditions:

$10^4 < Re_D < 5 \times 10^6$

0.5 < Pr < 200 with 5 percent error

0.5 < Pr < 2000 with 10 percent error

$0.08 < (\mu_b/\mu_w) < 40$

c) Laminar flow over a horizontal, flat plate ($Re_x < 5 \times 10^5$):

Constant wall temperature:

$$\text{Local Nusselt number, } Nu_x = \left(\frac{h_c x}{\mathbf{k_f}}\right) = 0.332\, Re_x^{\ 0.5}\, Pr^{1/3} \qquad (7.26)$$

Constant wall heat flux:

$$\text{Local Nusselt number, } Nu_x = \left(\frac{h_c x}{\mathbf{k_f}}\right) = 0.453\, Re_x^{0.5}\, Pr^{1/3} \qquad (7.27)$$

The average Nusselt number is obtained by integrating local Nusselt number from 0 to L and dividing by L

d) Turbulent flow over horizontal, flat plate ($Re_x > 5 \times 10^5$):

Constant wall temperature:

$$\text{Average Nusselt number, } \bar{Nu}_L = \left(\frac{\bar{h}_c L}{k_f}\right) = Pr^{1/3}\,(0.037\, Re_L^{0.8} - 850) \qquad (7.28)$$

e) Free convection over hot, vertical flat plates and cylinders:

Constant wall temperature:

$$\text{Average Nusselt number, } \bar{Nu}_L = \left(\frac{\bar{h}_c L}{\mathbf{k_f}}\right) = c\,(Gr_L\, Pr)^n = cRa_L^{\,n} \qquad (7.29)$$

where c and n are 0.59 and ¼ for laminar flow ($10^4 < Gr^L.Pr < 10^9$) and 0.10 and ⅓ for turbulent flow ($10^9 < Gr^L.Pr < 10^{13}$)

In the above equation, Gr_L is the average Grashoff number given by:

$$\text{Average Grashoff Number } Gr_L = \frac{g\beta\,(T_w - T_\infty)\,L^3}{\upsilon^2} \qquad (7.30)$$

where g is the acceleration due to gravity, β is volumetric coefficient of thermal expansion, T_w and T_∞ are the plate and the free stream fluid temperatures, respectively and ν is the kinematic viscosity.

Constant wall heat flux, q_W:

$$\text{Local Nusselt number, } Nu_x = \left(\frac{h_c x}{k_f}\right) = 0.60\,(Gr_x^{*} Pr)^{1/5} \qquad (7.31)$$

$$\text{where } Gr_x^{*} = \frac{g\beta\, q_w\, x^4}{k_f\, \upsilon^2}$$

The above equation is valid for $10^5 < Gr_x^{*}.Pr < 10^{11}$

f) Free convection over horizontal flat plates:

$$\text{Average Nusselt number, } \bar{Nu}_L = \left(\frac{\bar{h}_c L}{\mathbf{k_f}}\right) = c\,(Gr_L\, Pr)^n \qquad (7.32)$$

The values of c and n are given in Table 7.3 for different orientations and flow regimes.

Table 7.3 Values of c and n

Orientation of plate	**Range of Gr_LPr**	**c**	**n**	**Flow regime**
Hot surface facing up or cold surface facing down, **constant T_w**	10^5 to $2 X 10^7$	0.54	1/4	Laminar
	$2 X 10^7$ to $3 X 10^{10}$	0.14	1/3	Turbulent
Hot surface facing down or cold surface facing up, **constant T_w**	$3 X 10^5$ to $3 X 10^{10}$	0.27	1/4	Laminar
Hot surface facing up, **constant q_w**	$< 2 X 10^8$	0.13	1/3	
	$5 X 10^8$ to 10^{11}	0.16	1/3	
Hot surface facing down, **constant q_w**	10^6 to 10^{11}	0.58	1/5	

In the above free convection equations, the fluid properties have to be evaluated at a mean temperature defined as $T_m = T_w - 0.25(T_w - T_\infty)$.

g) Convection heat transfer with phase change:

Filmwise condensation over horizontal tubes of outer diameter D_o:

The heat transfer coefficient for film-wise condensation is given by Nusselt's theory that assumes the vapour to be still and at saturation temperature. The mean condensation heat transfer coefficient, h_m is given by:

$$h_m = 0.725\left[\frac{k_f^3\,\rho_f^2\,g\,h_{fg}}{ND_o\mu_f\Delta T}\right]^{1/4} \tag{7.33}$$

where, subscript f refers to saturated liquid state, N refers to number of tubes above each other in a column and $\Delta T = T_r - T_{wo}$, T_r and T_{wo} being refrigerant and outside wall temperatures respectively.

Filmwise condensation over a vertical plate of length L:

The mean condensation heat transfer coefficient, h_m is given by,

$$h_m = 0.943\left[\frac{k_f^3\,\rho_f^2\,g\,h_{fg}}{\mu_f L\Delta T}\right]^{1/4} \tag{7.34}$$

Nucleate pool boiling of refrigerants inside a shell:

$$h_r = C\,\Delta T^{2\,\text{to}\,3} \tag{7.35}$$

where ΔT is the temperature difference between surface and boiling fluid and C is a constant that depends on the nature of refrigerant etc.

The correlations for convective heat transfer coefficients given above are only few examples of some of the common situations. A large number of correlations are available for almost all commonly encountered convection problems. The reader should refer to standard text books on heat transfer for further details.

7.3. Fundamentals of Mass transfer

When a system contains two or more components whose concentration vary from point to point, there is a natural tendency for mass to be transferred, minimizing the concentration differences within the system. The transport of one constituent from a region of higher concentration to that of lower concentration is called *mass transfer*. A common example of mass transfer is drying of a wet surface exposed to unsaturated air. Refrigeration and air conditioning deal with processes that involve mass transfer. Some basic laws of mass transfer relevant to refrigeration and air conditioning are discussed below.

7.3.1. Fick's Law of Diffusion:

This law deals with transfer of mass within a medium due to difference in concentration between various parts of it. This is very similar to Fourier's law of heat conduction as the mass transport is also by molecular diffusion processes. According to this law, rate of diffusion of component A $\dot{m}_A$ (kg/s) is proportional to the concentration gradient and the area of mass transfer, i.e.

$$\dot{m}_A = -D_{AB} A \frac{dc_A}{dx} \tag{7.36}$$

where, D_{AB} is called diffusion coefficient for component A through component B, and it has the units of m^2/s just like those of thermal diffusivity α and the kinematic viscosity of fluid ν for momentum transfer.

7.3.2. Convective mass transfer:

Mass transfer due to convection involves transfer of mass between a moving fluid and a surface or between two relatively immiscible moving fluids. Similar to convective heat transfer, this mode of mass transfer depends on the transport properties as well as the dynamic characteristics of the flow field. Similar to Newton's law for convective heat transfer, he convective mass transfer equation can be written as:

$$\dot{m} = h_m A \, \Delta c_A \tag{7.37}$$

where h_m is the convective mass transfer coefficient and Δc_A is the difference between the boundary surface concentration and the average concentration of fluid stream of the diffusing species A.

Similar to convective heat transfer, convective mass transfer coefficient depends on the type of flow, i.e., laminar or turbulent and forced or free. In general the mass transfer coefficient is a function of the system geometry, fluid and flow properties and the concentration difference. Similar to momentum and heat transfers, concentration boundary layers develop whenever mass transfer takes place between a surface and a fluid. This suggests analogies between mass, momentum and energy transfers. In convective mass transfer the non-dimensional numbers corresponding to Prandtl and Nusselt numbers of convective heat transfer are called as Schmidt and Sherwood numbers. These are defined as:

$$\text{Sherwood number, } Sh_L = \frac{h_m L}{D} \tag{7.38}$$

$$\text{Schmidt number, } Sc = \frac{\nu}{D} \tag{7.39}$$

where h_m is the convective mass transfer coefficient, D is the diffusivity and ν is the kinematic viscosity.

The general convective mass transfer correlations relate the Sherwood number to Reynolds and Schmidt number.

7.4. Analogy between heat, mass and momentum transfer

7.4.1. Reynolds and Colburn Analogies

The boundary layer equations for momentum for a flat plate are exactly same as those for energy equation if Prandtl number, Pr = 1, pressure gradient is zero and viscous dissipation is negligible, there are no heat sources and for similar boundary conditions. Hence, the solution for non-dimensional velocity and temperature are also same. It can be shown that for such a case,

$$\text{Stanton number, } St = \left(\frac{Nu}{Re.Pr}\right) = \left(\frac{h_c}{\rho V c_p}\right) = \frac{f}{2} \tag{7.40}$$

where f is the friction factor and St is Stanton Number. The above equation, which relates heat and momentum transfers is known as Reynolds analogy.

To account for the variation in Prandtl number in the range of 0.6 to 50, the Reynolds analogy is modified resulting in Colburn analogy, which is stated as follows.

$$St.Pr^{2/3} = \frac{f}{2} \tag{7.41}$$

7.4.2. Analogy between heat, mass and momentum transfer

The role that thermal diffusivity plays in the energy equation is played by diffusivity D in the mass transfer equation. Therefore, the analogy between momentum and mass transfer for a flat plate will yield:

$$\frac{Sh}{Re.Sc} = \left(\frac{h_m L}{D}\right)\left(\frac{\nu}{VL}\right)\left(\frac{D}{\nu}\right) = \left(\frac{h_m}{V}\right) = \left(\frac{f}{2}\right) \tag{7.42}$$

To account for values of Schmidt number different from one, following correlation is introduced,

$$\frac{Sh}{\mathrm{Re}.Sc}Sc^{2/3}=\frac{f}{2} \tag{7.43}$$

Comparing the equations relating heat and momentum transfer with heat and mass transfer, it can be shown that,

$$\left(\frac{h_c}{\rho c_p h_m}\right)=\left(\frac{\alpha}{D}\right)^{2/3} \tag{7.44}$$

This analogy is followed in most of the chemical engineering literature and α/D is referred to as Lewis number. In air-conditioning calculations, for convenience Lewis number is defined as:

$$\text{Lewis number, Le}=\left(\frac{\alpha}{D}\right)^{2/3} \tag{7.45}$$

The above analogies are very useful as by applying them it is possible to find heat transfer coefficient if friction factor is known and mass transfer coefficient can be calculated from the knowledge of heat transfer coefficient.

7.5. Multimode heat transfer

In most of the practical heat transfer problems heat transfer occurs due to more than one mechanism. Using the concept of thermal resistance developed earlier, it is possible to analyze steady state, multimode heat transfer problems in a simple manner, similar to electrical networks. An example of this is transfer of heat from outside to the interiors of an air conditioned space. Normally, the walls of the air conditioned rooms are made up of different layers having different heat transfer properties. Once again the concept of thermal resistance is useful in analyzing the heat transfer through multilayered walls. The example given below illustrates these principles.

Multimode heat transfer through a building wall:
The schematic of a multimode heat transfer building wall is shown in Fig. 7.3. From the figure it can be seen that:

$$Q_{1\text{-}2}=\frac{(T_1\text{-}T_2)}{R_{total}} \tag{7.46a}$$

$$R_{total}=\left(\frac{R_{conv,2}R_{rad,2}}{R_{conv,2}+R_{rad,2}}\right)+\left(R_{w,3}+R_{w,2}+R_{w,1}\right)+\left(\frac{R_{conv,1}R_{rad,1}}{R_{conv,1}+R_{rad,1}}\right) \tag{7.46b}$$

$$R_{total}=\left(R_2\right)+\left(R_w\right)+\left(R_1\right) \tag{7.46c}$$

$$Q_{1\text{-}2}=UA(T_1\text{-}T_2) \tag{7.46d}$$

where, overall heat transfer coefficient, $U = \dfrac{1}{R_{total}A}$

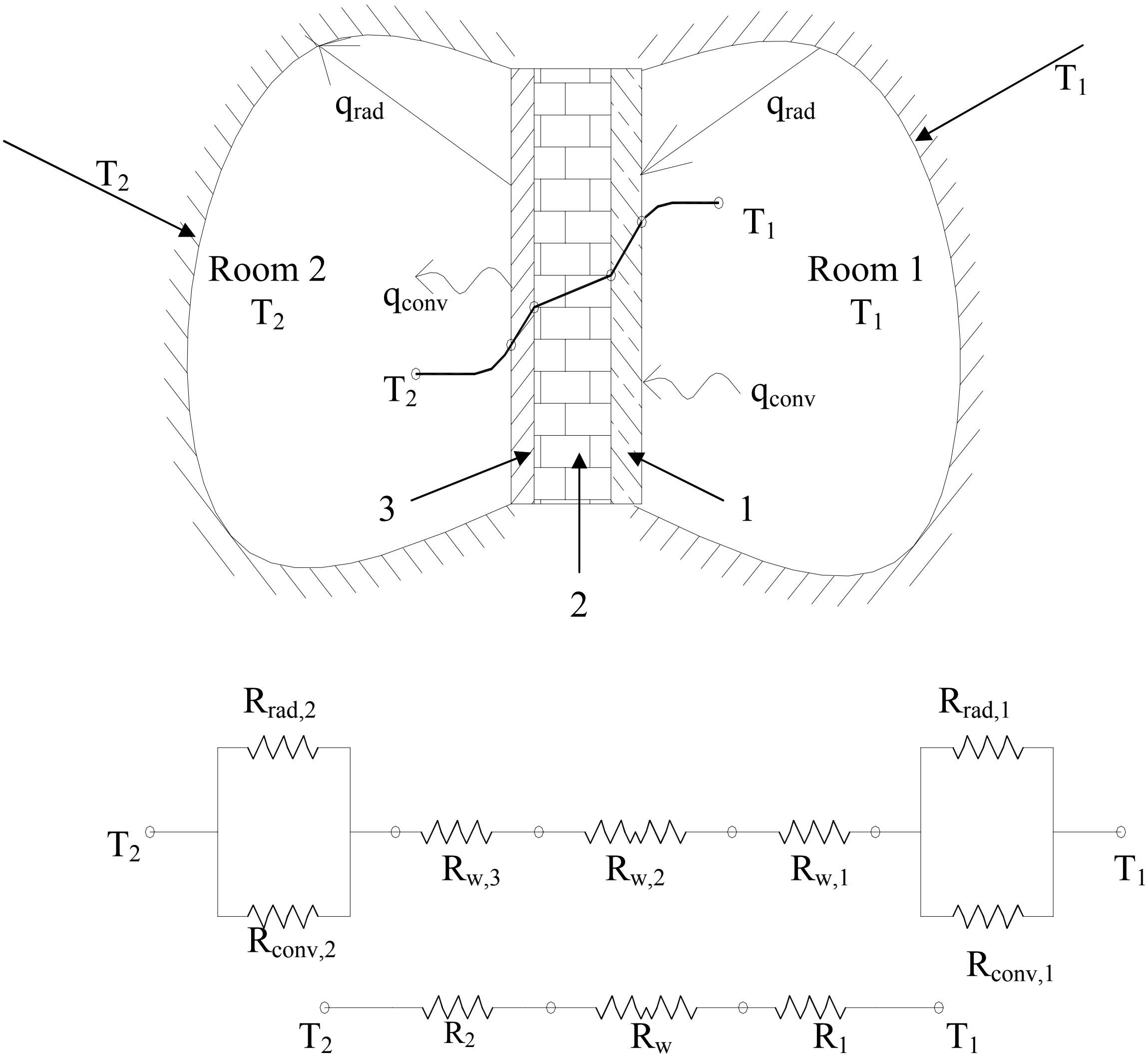

Fig. 7.3*. Schematic of a multimode heat transfer building wall*

Composite cylinders:

The concept of resistance networks is also useful in solving problems involving composite cylinders. A common example of this is steady state heat transfer through an insulated pipe with a fluid flowing inside. Since it is not possible to perfectly insulate the pipe, heat transfer takes place between the surroundings and the inner fluid when they are at different temperatures. For such cases the heat transfer rate is given by:

$$Q = U_o A_o (T_i - T_o) \tag{7.47}$$

where A_o is the outer surface area of the composite cylinder and U_o is the overall heat transfer coefficient with respect to the outer area given by:

$$\frac{1}{U_o A_o} = \frac{1}{h_i A_i} + \frac{\ln(r_2/r_1)}{2\pi L k_m} + \frac{\ln(r_3/r_2)}{2\pi L k_{in}} + \frac{1}{h_o A_o} \qquad (7.48)$$

In the above equation, h_i and h_o are the inner and outer convective heat transfer coefficients, A_i and A_o are the inner and outer surface areas of the composite cylinder, k_m and k_{in} are the thermal conductivity of tube wall and insulation, L is the length of the cylinder, r_1, r_2 and r_3 are the inner and outer radii of the tube and outer radius of the insulation respectively. Additional heat transfer resistance has to be added if there is any scale formation on the tube wall surface due to fouling.

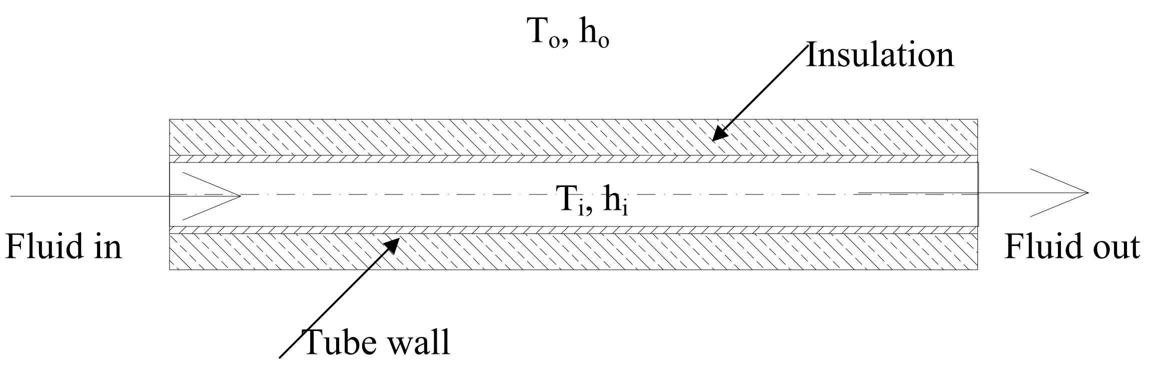

Fig. 7.4. *Composite cylindrical tube*

7.6. Heat exchangers:

A heat exchanger is a device in which heat is transferred from one fluid stream to another across a solid surface. Thus a typical heat exchanger involves both conduction and convection heat transfers. A wide variety of heat exchangers are extensively used in refrigeration and air conditioning. In most of the cases the heat exchangers operate in a steady state, hence the concept of thermal resistance and overall heat transfer coefficients can be used very conveniently. In general, the temperatures of the fluid streams may vary along the length of the heat exchanger. To take care of the temperature variation, the concept of Log Mean Temperature Difference (LMTD) is introduced in the design of heat exchangers. It is defined as:

$$LMTD = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)} \qquad (7.49)$$

where ΔT_1 and ΔT_2 are the temperature difference between the hot and cold fluid streams at two inlet and outlet of the heat exchangers.

If we assume that the overall heat transfer coefficient does not vary along the length, and specific heats of the fluids remain constant, then the heat transfer rate is given by:

$$Q = U_o A_o (LMTD) = U_o A_o \left(\frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)} \right)$$

also (7.50)

$$Q = U_i A_i (LMTD) = U_i A_i \left(\frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)} \right)$$

the above equation is valid for both parallel flow (both the fluids flow in the same direction) or counterflow (fluids flow in opposite directions) type heat exchangers. For other types such as cross-flow, the equation is modified by including a multiplying factor. The design aspects of heat exchangers used in refrigeration and air conditioning will be discussed in later chapters.

Questions:

1. Obtain an analytical expression for temperature distribution for a plane wall having uniform surface temperatures of T_1 and T_2 at x_1 and x_2 respectively. It may be mentioned that the thermal conductivity $k = k_0 (1+bT)$, where b is a constant. (Solution)

2. A cold storage room has walls made of 0.3 m of brick on outside followed by 0.1 m of plastic foam and a final layer of 5 cm of wood. The thermal conductivities of brick, foam and wood are 1, 0.02 and 0.2 W/mK respectively. The internal and external heat transfer coefficients are 40 and 20 W/m^{2}K. The outside and inside temperatures are 40^{0}C and -10^{0}C. Determine the rate of cooling required to maintain the temperature of the room at -10^{0}C and the temperature of the inside surface of the brick given that the total wall area is 100 m^2. (Solution)

3. A steel pipe of negligible thickness and having a diameter of 20 cm has hot air at 100^{0}C flowing through it. The pipe is covered with two layers of insulating materials each having a thickness of 10 cm and having thermal conductivities of 0.2 W/mK and 0.4 W/mK. The inside and outside heat transfer coefficients are 100 and 50 W/m^{2}K respectively. The atmosphere is at 35^{0}C. Calculate the rate of heat loss from a 100 m long pipe. (Solution)

4. Water flows inside a pipe having a diameter of 10 cm with a velocity of 1 m/s. the pipe is 5 m long. Calculate the heat transfer coefficient if the mean water temperature is at 40^{0}C and the wall is isothermal at 80^{0}C. (Solution)

5. A long rod having a diameter of 30 mm is to be heated from 400^{0}C to 600^{0}C. The material of the rod has a density of 8000 kg/m^3 and specific heat of 400 J/kgK. It is placed concentrically inside a long cylindrical furnace having an internal diameter of 150 mm. The inner side of the furnace is at a temperature of 1100^{0}C and has an

emissivity of 0.7. If the surface of the rod has an emissivity of 0.5, find the time required to heat the rod. (Solution)

6. Air flows over a flat plate of length 0.3 m at a constant temperature. The velocity of air at a distance far off from the surface of the plate is 50 m/s. Calculate the average heat transfer coefficient from the surface considering separate laminar and turbulent sections and compare it with the result obtained by assuming fully turbulent flow. (Solution)

Note: The local Nusselt number for laminar and turbulent flows is given by:

$$\text{laminar : } Nu_x = 0.331 Re_x^{1/2} Pr^{1/3}$$

$$\text{turbulent: } Nu_x = 0.0288 Re_x^{0.8} Pr^{1/3}$$

Transition occurs at $Re_{x.trans} = 2 \text{ X } 10^5$. The forced convection boundary layer flow begins as laminar and then becomes turbulent. Take the properties of air to be $\rho = 1.1 \text{ kg/m}^3$, $\mu = 1.7 \text{ X } 10^{-5}$ kg/m s, k = 0.03 W/mK and Pr = 0.7.

7. A vertical tube having a diameter of 80 mm and 1.5 m in length has a surface temperature of 80^{0}C. Water flows inside the tube while saturated steam at 2 bar condenses outside. Calculate the heat transfer coefficient. (Solution)

Note: Properties of saturated steam at 2 bar: $T_{sat} = 120.2^0C$, $h_{fg} = 2202$ kJ/kgK, $\rho = 1.129 \text{ kg/m}^3$; For liquid phase at 100^{0}C: $\rho_L = 958 \text{ kg/m}^3$, $c_p = 4129$ J/kgK, $\mu_L = 0.279 \text{X} 10^{-3}$ kg/m s and Pr = 1.73.

8. Air at 300 K and at atmospheric pressure flows at a mean velocity of 50 m/s over a flat plate 1 m long. Assuming the concentration of vapour in air to be negligible, calculate the mass transfer coefficient of water vapour from the plate into the air. The diffusion of water vapour into air is 0.5 X 10^{-4} m^2/s. The Colburn j-factor for heat transfer coefficient is given by j_H=0.0296 Re $^{-0.2}$. (Solution)

9. An oil cooler has to cool oil flowing at 20 kg/min from 100^{0}C to 50^{0}C. The specific heat of the oil is 2000 J/kg K. Water with similar flow rate at an ambient temperature of 35^{0}C is used to cool the oil. Should we use a parallel flow or a counter flow heat exchanger? Calculate the surface area of the heat exchanger if the external heat transfer coefficient is 100 W/m^{2}K. (Solution)