บทที่ 6

Inductor Two-terminal element consisting of a winding of N turns for introducing inductance into an electric circuit.

Initial Condition The capacitor voltage or inductor current at the initial time.

Initial Time The time, t_o, when a new action is initiated in a circuit; usually, $t_o = 0$.

Integrator A circuit that performs the mathematical operation of integration.

Switched Circuit Electric circuit with one or more switches that open or close at time t_o.

Problems

Section 7.3 Capacitors and Their v-i Equation

P 7.3-1 A 15-μF capacitor has a voltage of 5 V across it at $t = 0$. If a constant current of 25 mA flows through the capacitor, how long will it take for the capacitor to charge up to 150 μC?

Answer: $t = 3$ ms

P 7.3-2 The voltage, $v(t)$, across a capacitor and current, $i(t)$, in that capacitor adhere to the passive convention. Determine the current, $i(t)$, when the capacitance is $C = 0.125$ F and the voltage is $v(t) = 12\cos(2t + 30°)$ V.

Hint:
$$\frac{d}{dt}A\cos(\omega t + \theta) = -A\sin(\omega t + \theta)\cdot\frac{d}{dt}(\omega t + \theta) = -A\omega\sin(\omega t + \theta) = A\omega\cos\left(\omega t + \left(\theta + \frac{\pi}{2}\right)\right)$$

Answer: $i(t) = 3\cos(2t + 120°)$ A

P 7.3-3 The voltage, $v(t)$, across a capacitor and current, $i(t)$, in that capacitor adhere to the passive convention. Determine the capacitance when the voltage is $v(t) = 12\cos(500t - 45°)$ V and the current is $i(t) = 3\cos(500t + 45°)$ mA.

Answer: $C = 0.5\ \mu$F

P 7.3-4 Determine $v(t)$ for the circuit shown in Figure P 7.3-4a when the $i_s(t)$ is as shown in Figure P 7.3-4b and $v_o(0^-) = -1$mV.

P 7.3-5 Determine $v(t)$ for the circuit of Figure P 7.3-5 when $v(0) = -4$mV.

P 7.3-6 A current source, i_s, as shown in Figure P 7.3-6, is connected to an uncharged capacitor at $t_0 = 0$. Determine the voltage waveform from $t = 0$ s to $t = 2.5$, and sketch the waveform when $C = 1$ mF.

P 7.3-7 The voltage across a 40-μF capacitor is 25 V at $t_0 = 0$. If the current through the capacitor as a function of time is given by $i(t) = 6e^{-6t}$ mA for $t > 0$, find $v(t)$ for $t > 0$.

Answer: $v(t) = 50 - 25e^{-6t}$ V

P 7.3-8 Find i for the circuit of Figure P 7.3-8 if $v = 5(1 - 2e^{-2t})$ V.

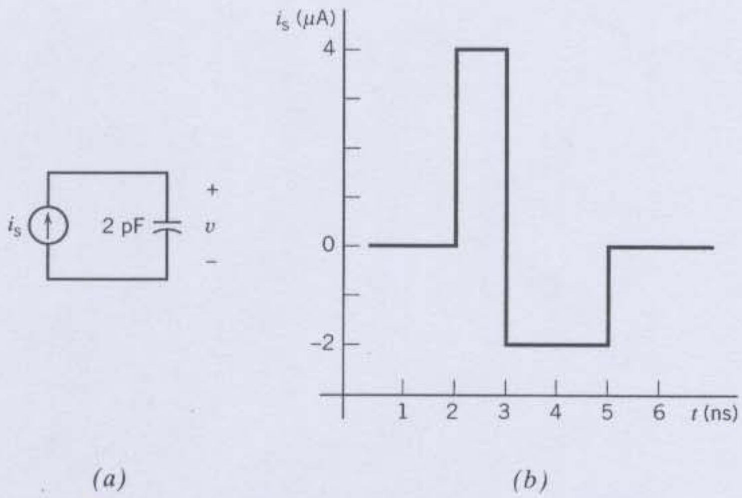
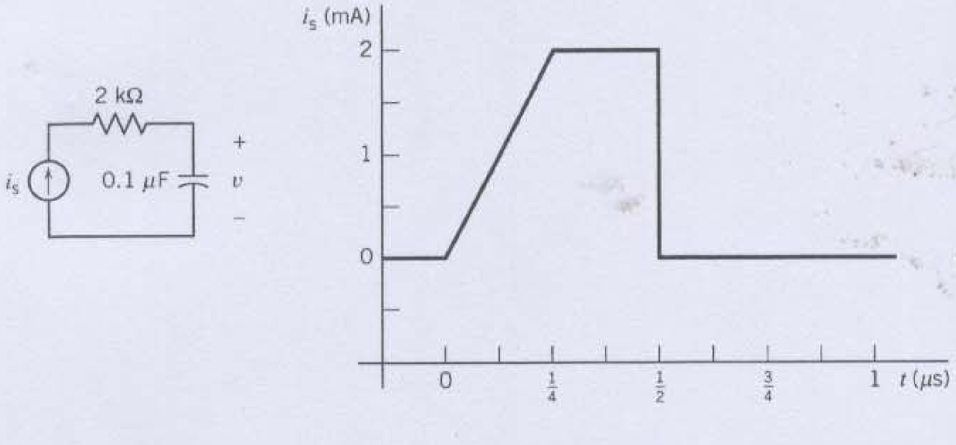


Figure P 7.3-4 (a) Circuit and (b) waveform of current source.



(*a*) (*b*)

Figure P 7.3-5 (*a*) Circuit and (*b*) waveform of current source.

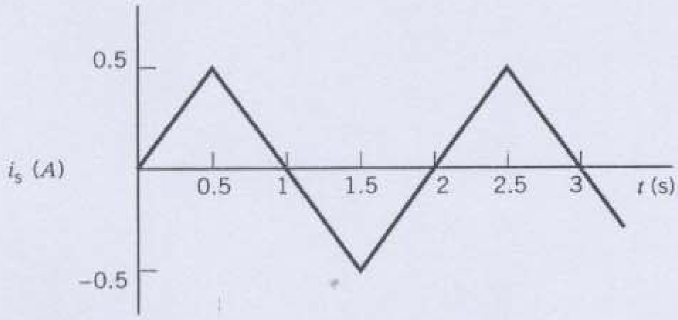


Figure P 7.3-6 Waveform of current source i_s.

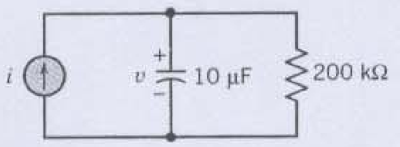


Figure P 7.3-8

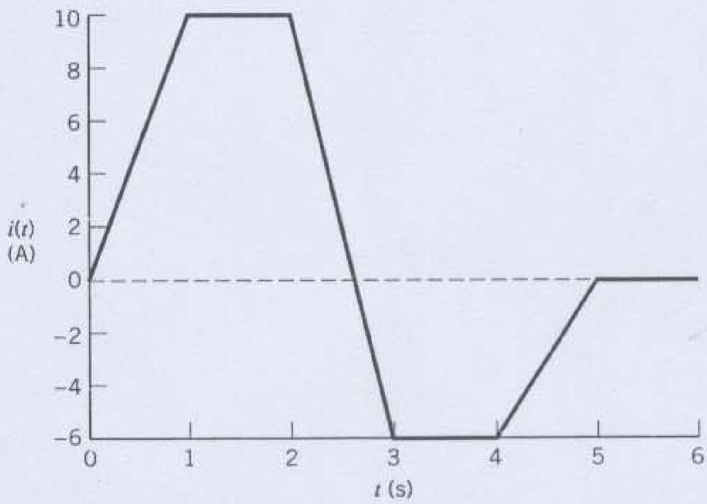


Figure P 7.4-1

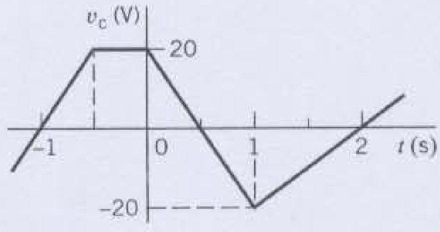


Figure P 7.4-3

Section 7.4 Energy Storage in a Capacitor

P 7.4-1 The current, i, through a capacitor is shown in Figure P 7.4-1. When $v(0) = 0$ and $C = 0.5$ F, determine and plot $v(t)$, $p(t)$, and $w(t)$ for $0 \text{ s} < t < 6$ s.

P 7.4-2 In a pulse power circuit the voltage of a 10-μF capacitor is zero for $t < 0$ and

$$v = 5(1 - e^{-4000t}) \text{ V} \qquad t \geq 0$$

Determine i_c and the energy stored in the capacitor at $t = 0$ ms and $t = 10$ ms.

P 7.4-3 If $v_c(t)$ is given by the waveform shown in Figure P 7.4-3, sketch the capacitor current for $-1 \text{ s} \leq t \leq 2$ s. Sketch the power and the energy for the capacitor over the same time interval when $C = 1$ mF.

P 7.4-4 The current through a 2-μF capacitor is 50 cos $(10t + \pi/6)$ μA for all time. The average voltage across the capacitor is zero. What is the maximum value of the energy stored in the capacitor? What is the first nonnegative value of t at which the maximum energy is stored?

P 7.4-5 The energy stored by a 1-mF capacitor used in a laser power supply is given as $w = 4e^{-10t}$ J for $t \geq 0$. Find the capacitor voltage and current at $t = 0.1$ s.

P 7.4-6 A capacitor is used in the electronic flash unit of a camera. A small battery with a constant voltage of 6 V is used to charge a capacitor with a constant current of 10 μA. How long does it take to charge the capacitor when $C = 10\ \mu$F? What is the stored energy?

P 7.4-7 If a capacitor can store energy, as does a battery, could not a capacitor be used to power an electric train? Such a capacitor (electrolytic) would be about 1 cm^3 per 100 μF, for the voltage level required. Suppose we start with an initial voltage of 500 V and can extract all the energy from the capacitor to drive the car for one hour. The auto drive train requires 1.5 kW. How big would the capacitor have to be? Is this practical?

Section 7.5 Series and Parallel Capacitors

P 7.5-1 Find the current $i(t)$ for the circuit of Figure P 7.5-1.
Answer: $i(t) = -1.2 \sin 100t$ mA

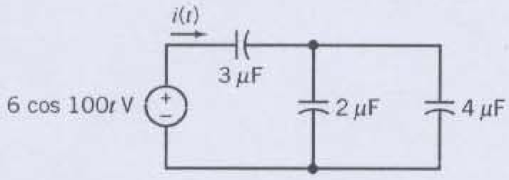


Figure P 7.5-1

P 7.5-2 Find the current $i(t)$ for the circuit of Figure P 7.5-2.
Answer: $i(t) = -1.5\, e^{-250\,t}$ mA

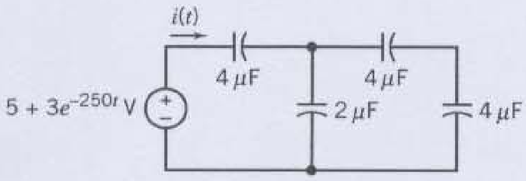


Figure P 7.5-2

P 7.5-3 The circuit of Figure P 7.5-3 contains five identical capacitors. Find the value of the capacitance C.
Answer: $C = 10\ \mu$F

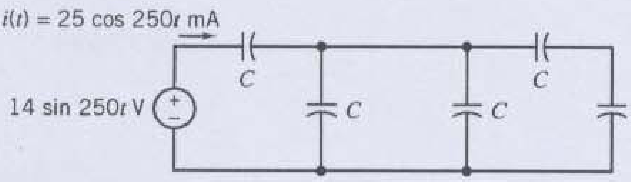


Figure P 7.5-3

Section 7.6 Inductors

P 7.6-1 Nikola Tesla (1857–1943) was an American electrical engineer who experimented with electric induction. Tesla built a large coil with a very large inductance, shown in Figure P 7.6-1. The coil was connected to a source current

$$i_s = 100 \sin 400t \text{ A}$$

so that the inductor current $i_L = i_s$. Find the voltage across the inductor and explain the discharge in the air shown in the figure. Assume that $L = 200$ H, and the average discharge distance is 2 m. Note that the dielectric strength of air is 3×10^6 V/m.

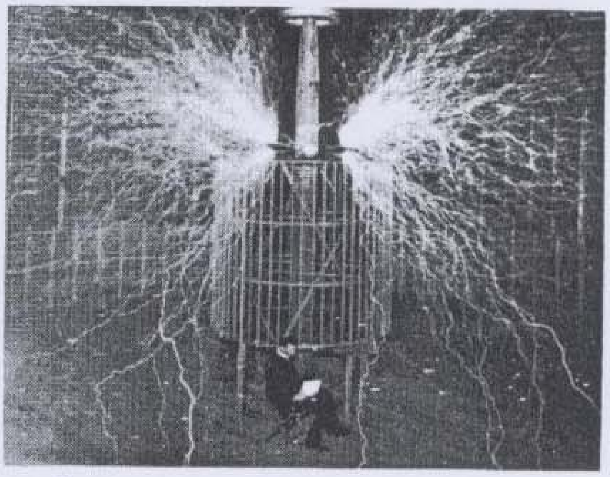


Figure P 7.6-1 Nikola Tesla sits impassively as alternating current induction coils discharge millions of volts with a roar audible 10 miles away (about 1910). Courtesy of Burndy Library.

P 7.6-2 The model of an electric motor consists of a series combination of a resistor and inductor. A current $i(t) = 4te^{-t}$ A flows through the series combination of a 10 Ω resistor and 0.1-henry inductor. Find the voltage across the combination.
Answer: $v(t) = 0.4e^{-t} + 39.6te^{-t}$ V

P 7.6-3 The current through a 20-mH inductor is shown in Figure P 7.6-3. Find the inductor voltage at $t = 1$ ms and $t = 6$ ms.

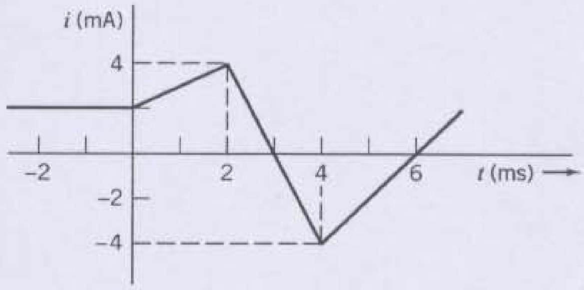


Figure P 7.6-3

P 7.6-4 The voltage, $v(t)$, across an inductor and current, $i(t)$, in that inductor adhere to the passive convention. Determine the voltage, $v(t)$, when the inductance is $L = 250$ mH and the current is $i(t) = 120 \sin(500\,t - 30°)$ mA.

Hint:
$$\frac{d}{dt} A \sin(\omega t + \theta) = A \cos(\omega t + \theta) \cdot \frac{d}{dt}(\omega t + \theta) = A\omega \cos(\omega t + \theta) = A\omega \sin\left(\omega t + \left(\theta + \frac{\pi}{2}\right)\right)$$

Answer: $v(t) = 15 \sin(500\,t + 60°)$ V

P 7.6-5 Determine $i_L(t)$ for $t > 0$ when $i_L(0) = -2\ \mu$A for the circuit of Figure P 7.6-5*a* when $v_s(t)$ is as shown in Figure P 7.6-5*b*.

P 7.6-6 Determine $v(t)$ for $t > 0$ for the circuit of Figure P 7.6-6*a* when $i_L(0) = 0$ and i_s is as shown in Figure P 7.6-6*b*.

P 7.6-7 Find R of the circuit shown in Figure P 7.6-7 if $v_1 = 1e^{-200t}$ V for $t \geq 0$.
Answer: $R = 80\Omega$

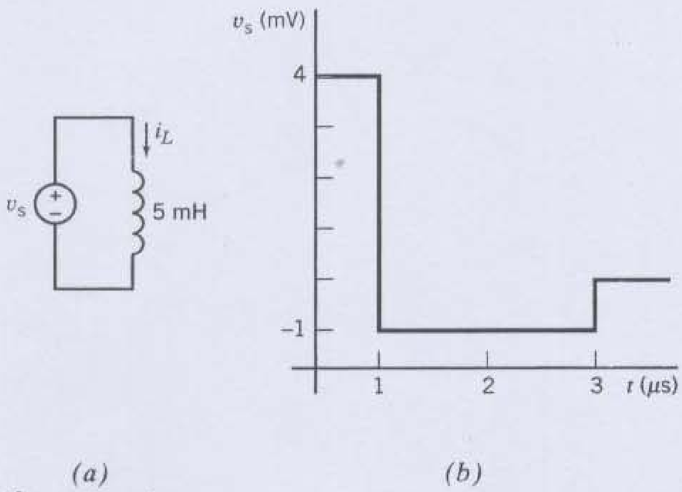


Figure P 7.6-5

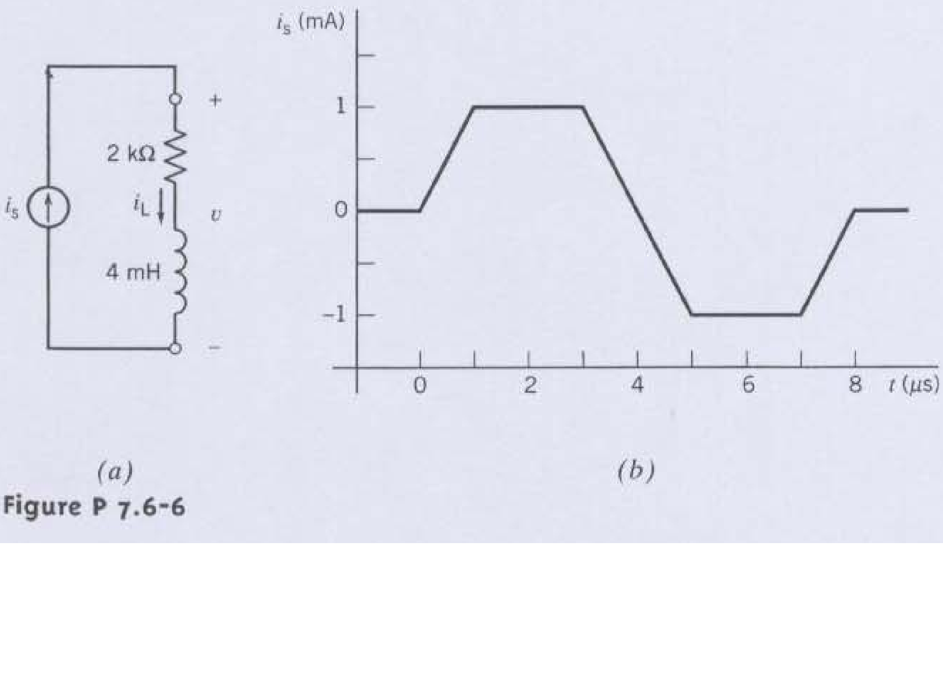


Figure P 7.6-6

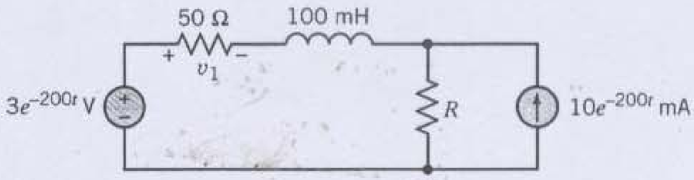


Figure P 7.6-7

Section 7.7 Energy Storage in an Inductor

P 7.7-1 The current, $i(t)$, in a 100-mH inductor connected in a telephone circuit changes according to

$$i(t) = \begin{cases} 0 & t \leq 0 \\ 4\,t & 0 < t < 1 \\ 4 & t \geq 1 \end{cases}$$

where the units of time are ms and the units of current are mA. Determine the power, $p(t)$, absorbed by the inductor and the energy, $w(t)$, stored in the inductor.

Answer:
$$p(t) = \begin{cases} 0 & t \leq 0 \\ 1.6\,t & 0 < t < 1 \\ 0 & t \geq 1 \end{cases}$$

and
$$w(t) = \begin{cases} 0 & t \leq 0 \\ 0.8\,t^2 & 0 < t < 1 \\ 0.8 & t \geq 1 \end{cases}$$

The units of $p(t)$ are W and the units of $w(t)$ are J.

P 7.7-2 The current, $i(t)$, in a 5 H inductor is

$$i(t) = \begin{cases} 0 & t \leq 0 \\ 4 \sin 2t & t \geq 0 \end{cases}$$

where the units of time are s and the units of current are A. Determine the power, $p(t)$, absorbed by the inductor and the energy, $w(t)$, stored in the inductor.
Hint: 2 (cos A) (sin B) = sin(A+B) + sin (A-B)

P 7.7-3 The voltage, $v(t)$, across a 25-mH inductor used in a fusion power experiment is

$$v(t) = \begin{cases} 0 & t \le 0 \\ 6 \cos 100\,t & t \ge 0 \end{cases}$$

where the units of time are s and the units of voltage are V. The current in this inductor is zero before the voltage changes at $t = 0$. Determine the power, $p(t)$, absorbed by the inductor and the energy, $w(t)$, stored in the inductor.
Hint: 2 (cos A) (sin B) = sin(A+B) + sin (A-B)

Answer: $p(t) = 7.2 \sin 200\,t$ W and $w(t) = 3.6\,[1 - \cos 200\,t]$ mJ

Section 7.8 Series and Parallel Inductors

P 7.8-1 Find the current $i(t)$ for the circuit of Figure P 7.8-1.
Answer: $i(t) = 15 \sin 100t$ mA

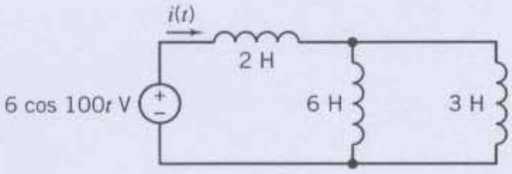


Figure P 7.8-1

P 7.8-2 Find the voltage $v(t)$ for the circuit of Figure P 7.8-2.
Answer: $v(t) = -6\,e^{-250\,t}$ mV

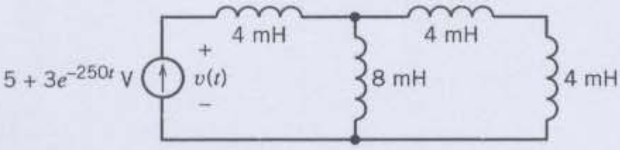


Figure P 7.8-2

P 7.8-3 The circuit of Figure P 7.8-3 contains four identical inductors. Find the value of the inductance L.
Answer: $L = 2.86$ H

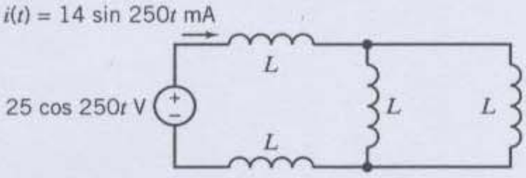


Figure P 7.8-3

Section 7.9 Initial Conditions of Switched Circuits

P 7.9-1 The switch in Figure 7.9-1 has been open for a long time before closing at time $t = 0$. Find $v_C(0+)$ and $i_L(0+)$, the values of the capacitor voltage and inductor current immediately after the switch closes. Let $v_C(\infty)$ and $i_L(\infty)$ denote the values of the capacitor voltage and inductor current after the switch has been closed for a long time. Find $v_C(\infty)$ and $i_L(\infty)$.
Answer: $v_C(0+) = 12$ V, $i_L(0+) = 0$, $v_C(\infty) = 4$ V and $i_L(\infty) = 1$ mA

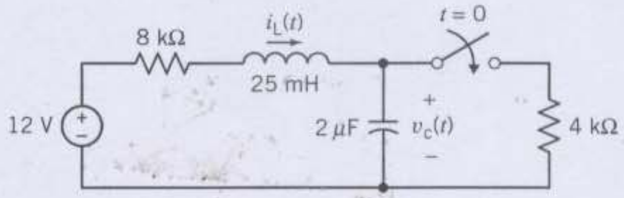


Figure P 7.9-1

P 7.9-2 The switch in Figure 7.9-2 has been open for a long time before closing at time $t = 0$. Find $v_C(0+)$ and $i_L(0+)$, the values of the capacitor voltage and inductor current immediately after the switch closes. Let $v_C(\infty)$ and $i_L(\infty)$ denote the values of the capacitor voltage and inductor current after the switch has been closed for a long time. Find $v_C(\infty)$ and $i_L(\infty)$.
Answer: $v_C(0+) = 6$ V, $i_L(0+) = 1$ mA, $v_C(\infty) = 3$ V and $i_L(\infty) = 1.5$ mA

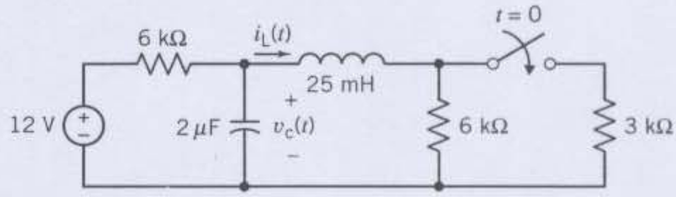


Figure P 7.9-2

P 7.9-3 The switch in Figure 7.9-3 has been open for a long time before closing at time $t = 0$. Find $v_C(0+)$ and $i_L(0+)$, the values of the capacitor voltage and inductor current immediately after the switch closes. Let $v_C(\infty)$ and $i_L(\infty)$ denote the values of the capacitor voltage and inductor current after the switch has been closed for a long time. Find $v_C(\infty)$ and $i_L(\infty)$.
Answer: $v_C(0+) = 0$ V, $i_L(0+) = 0$, $v_C(\infty) = 8$ V and $i_L(\infty) = 0.5$ mA

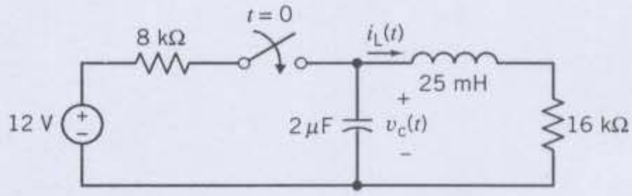


Figure P 7.9-3

P 7.9-4 Find $v_c(0^+)$ and $dv_c(0^+)/dt$ if $v(0^-) = 15$ V for the circuit of Figure P 7.9-4.

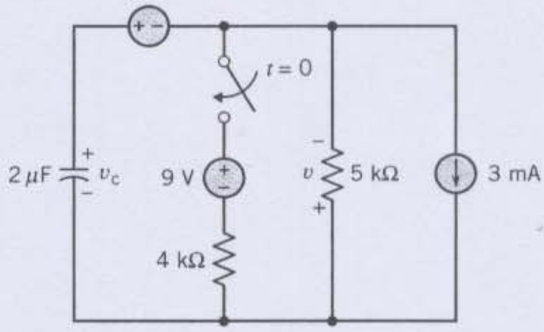


Figure P 7.9-4

P 7.9-5 For the circuit shown in Figure P 7.9-5, find $dv_c(0^+)/dt$, $di_L(0^+)/dt$, and $i(0^+)$ if $v(0^-) = 16$ V. Assume that the switch was closed for a long time prior to $t = 0$.

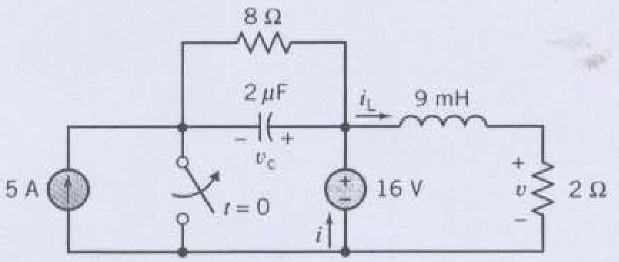


Figure P 7.9-5

P 7.9-6 For the circuit of Figure P 7.9-6, determine the current and voltage of each passive element at $t = 0^-$ and $t = 0^+$. The current source is $i_s = 0$ for $t < 0$ and $i_s = 4$ A for $t \geq 0$.

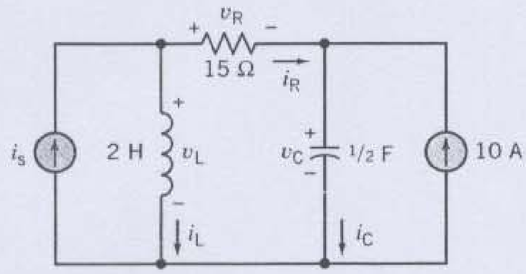


Figure P 7.9-6

Section 7.10 The Operational Amplifier and *RC* Circuits

P 7.10-1 Find the output voltage $v_o(t)$ for the circuit of Figure P 7.10-1 when

$$v_s(t) = \begin{cases} 0 & t < 0 \\ 12 \cos 100\,t \quad V & t \geq 0 \end{cases}$$

and the initial condition is $v_o(0) = 0$.
Answer: $v_o(t) = -3 \sin 100\,\mathrm{t}$ V

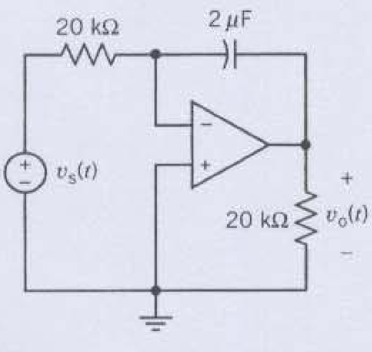


Figure P 7.10-1

P 7.10-2 Find the output voltage $v_o(t)$ for the circuit of Figure P 7.10-2 when

$$v_s(t) = \begin{cases} 0 & t < 0 \\ -4V & 0 \leq t < 3 \text{ ms} \\ 0 & t \geq \text{ ms} \end{cases}$$

and the initial condition is $v_o(0) = 0$.

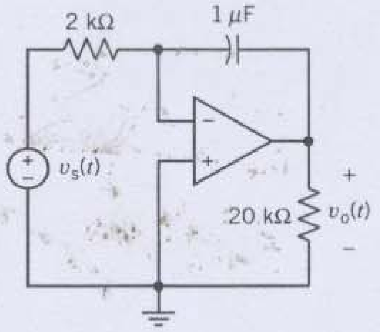


Figure P 7.10-2

P 7.10-3 Determine values of R and C for the circuit of Figure P 7.10-3 so that

$$v_o(t) = \begin{cases} 0 & t < 0 \\ 250\,t & 0 \leq t < 20 \text{ ms} \\ 5 & t \geq 20 \text{ ms} \end{cases}$$

$$\text{when } v_s(t) = \begin{cases} 0 & t < 0 \\ -5 & 0 \leq t < 20 \text{ ms} \\ 0 & t \geq 20 \text{ ms} \end{cases}$$

Both $v_s(t)$ and $v_o(t)$ have units of V.
Answer: $R = 20$ kΩ and $C = 1$ μF

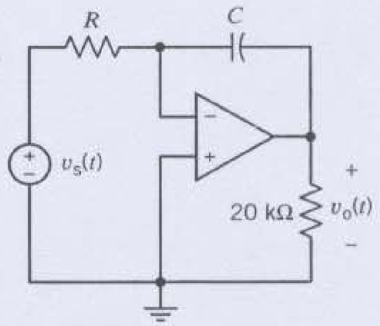


Figure P 7.10-3

P 7.10-4 Find the output voltage $v_o(t)$ for the circuit of Figure P 7.10-4 when

$$v_s(t) = \begin{cases} 0 & t < 0 \\ -6 \sin 100t \quad V & t \geq 0 \end{cases}$$

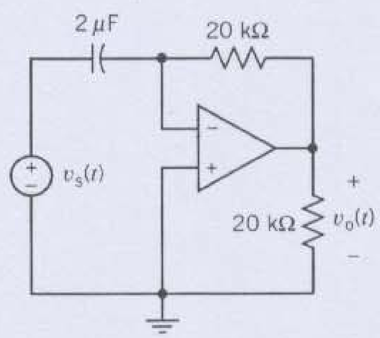


Figure P 7.10-4

Verification Problems

VP 7-1 A homework solution indicates that the current and voltage of a 100-H inductor are

$$i(t) = \begin{cases} .025 & t < 1 \\ -\dfrac{t}{25} + 0.065 & 1 < t < 3 \\ \dfrac{t}{50} - 0.115 & 3 < t < 9 \\ 0.065 & t > 9 \end{cases}$$

$$\text{and } v(t) = \begin{cases} 0 & t < 1 \\ -4 & 1 < t < 3 \\ 2 & 3 < t < 9 \\ 0 & t > 9 \end{cases}$$

where the units of current are A, the units of voltage are V, and the units of time are s. Verify that the inductor current does not change instantaneously.

VP 7-2 A homework solution indicates that the current and voltage of a 100-H inductor are

$$i(t) = \begin{cases} -\dfrac{t}{200} + 0.025 & t < 1 \\ -\dfrac{t}{100} + 0.03 & 1 < t < 4 \\ \dfrac{t}{100} - 0.03 & 4 < t < 9 \\ 0.015 & t > 9 \end{cases}$$

$$\text{and } v(t) = \begin{cases} -1 & t < 1 \\ -2 & 1 < t < 4 \\ 1 & 4 < t < 9 \\ 0 & t > 9 \end{cases}$$

where the units of current are A, the units of voltage are V, and the units of time are s. Is this homework solution correct? Justify your answer.

Design Problems

DP 7.1 Select the resistance R for the circuit shown in Figure DP 7.1 so that $v(0) = 20$ V and $i(0) = 5$ A. Assume that the switch has been closed for a long time.

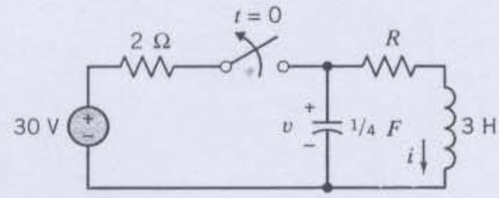


Figure DP 7.1

DP 7.2 A laser pulse power circuit is shown in Figure DP 7.2. It is required that $v(0) = 7.4$ V and $i(0) = 3.7$ A. Determine the required resistance R. Assume that the switch has been closed for a long time before it is opened at $t = 0$.

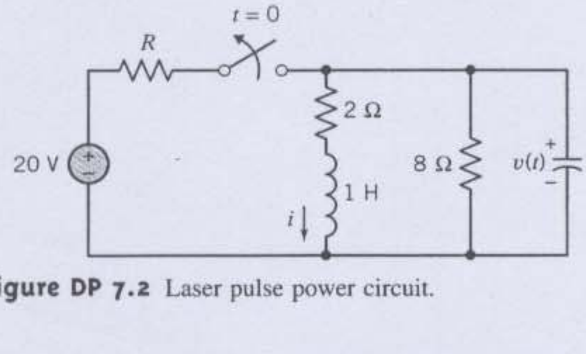


Figure DP 7.2 Laser pulse power circuit.

DP 7.3 A high-speed flash unit for sports photography requires a flash voltage $v(0^+) = 3$ V and

$$\left.\frac{dv(t)}{dt}\right|_{t=0^+} = 24 \text{ V/s}$$

The flash unit uses the circuit shown in Figure DP 7.3. Switch 1 has been closed a long time, and switch 2 has been open a long time at $t = 0$. Actually, the long time in this case is 3 s. Determine the required battery voltage, V_B, when $C = 1/8$ F.

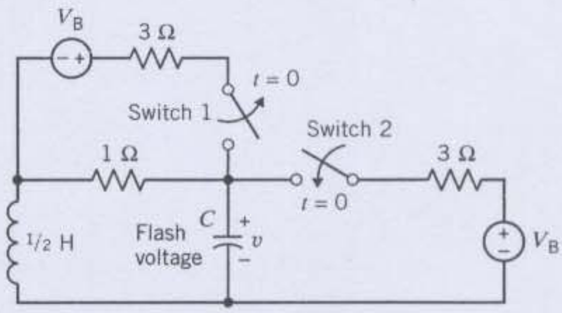


Figure DP 7.3 High-speed flash unit circuit.

DP 7.4 For the circuit shown in Figure DP 7.4, select a value of R so that the energy stored in the inductor is equal to the energy stored in the capacitor at steady state.

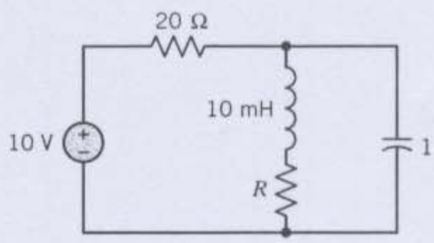


Figure DP 7.4