Binomial Probability Distribution (cont.)

Example 1

Let us consider the purchase decision of the next three customers who enter the Martin Clothing Store. On the basis of past experience, the store manager estimates that probability that any one customer will make a purchase is 0.30. 1) Check the four requirements for a binomial experiment. 2) What is the probability that two of the next three customers will make a purchase?

Mathematically, the number of experimental outcomes resulting in exactly *x* success in *n* trials can be computed using the following formula:

$$\binom{n}{x} = \frac{n!}{x!\,(n-x)!}$$

where *n! = n(n-1)(n-2)…..(2)(1),* and by definition 0! = 1

Use the above formula to find the number of ways of obtaining 2 purchases in 3 trials.

(Solutions)

The probability of purchases by the first two customers and no purchase by the third customer, denoted (S, S, F), is given by

$$p\, p\, (1 - p)$$

As referred to the above question, with 0.30 probability of a purchase on any one trial, the probability of a purchase on the (S, S, F) outcome is:

(Solution)

Two other experimental outcomes also result in two successes and one failure. The probabilities for all three experimental outcomes involving two successes follow:

1st Customer	**2nd Customer**	**3rd Customer**	**Experimental Outcome**	**Prob of Experimental Outcome**
Purchase	Purchase	No purchase	(S, S, F)	pp(1-p) = 0.063
Purchase	No purchase	Purchase	(S, F, S)	p(1-p)p =
No purchase	Purchase	Purchase	(F, S, S)	=

We can see that in binomial experiment, all sequences of trial outcomes yielding *x* successes and *n* trials have the same probability of occurrence:

Prob of a particular sequence of trail outcomes with x successes and n trails = $p^x (1-p)^{(n-x)}$

Therefore, the **binomial probability function** is denoted:

$$f(x) = \binom{n}{x} p^x (1-p)^{(n-x)}$$

According the information from Example 1, fill in the following table and draw the graph representing the probability distribution for the number of customers who make purchase.

x	**f(x)**
0	
1	
2	
3	

Expected Value and Variance for the Binomial Distribution

In case where the random variable has a binomial distribution with a known number of trails *n* and a known probability of success *p*, the general formulas for the expected value and variance can be simplified.

$$E(x) = \mu = np$$

$$Var(x) = \sigma^2 = np(1-p)$$

$$\sigma = \sqrt{\sigma^2}$$

Exercise

1. Using the information from example 1, find the expected number of customer, variance, and standard deviation.

2. Suppose that for the next month, the Martin Clothing Store forecasts 1000 customers will enter the store. Find the expected number of customer and standard deviation.

3. Consider a binomial experiment with two tails and $p = 0.4$.
    a. Draw a tree diagram for this experiment.
    b. Compute the probability of one success, $f(1)$
    c. Compute $f(0)$
    d. Compute $f(2)$
    e. Compute the probability of at least one success.
    f. Compute the expected value, variance, and standard deviation.

4. When a new machine is functioning properly, only 3% of the items produced are defective. Assume that we will randomly select two parts produced on the machine and that we are interested in the number of defective parts found.
    a. Describe the conditions under which this situation would be binomial experiment.
    b. Draw a tree diagram similar showing this problem as a two-trail experiment.
    c. How many experimental outcomes result in exactly one defect being found?
    d. Compute the probabilities associated with finding no defects, exactly on effect, and two defects.