Population Mean: σ Unknown

Interval estimate of a population mean: σ Unknown

Confidence Interval: $\bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$

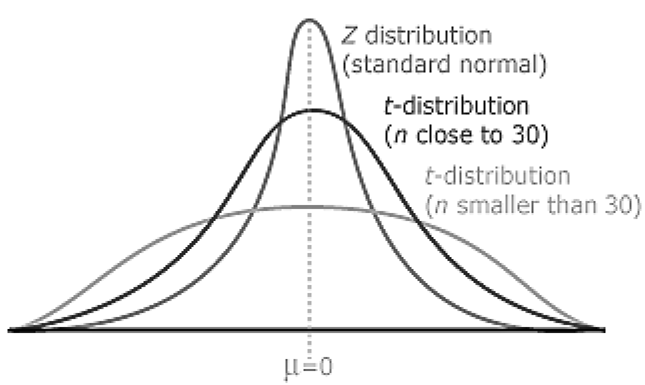
When σ is unknown, we use **t distribution** to find the margin error.

A t distribution depends on the **degree of freedom (n – 1)**.

S is the sample standard deviation, which is an estimate of the population standard deviation σ.

$$s = \sqrt{\frac{\sum(x - \bar{x})^2}{n - 1}}$$

A t distribution with more degree of freedom exhibits less variability and more closely resembles the standard normal distribution. Additionally, the mean of the t distribution is zero.



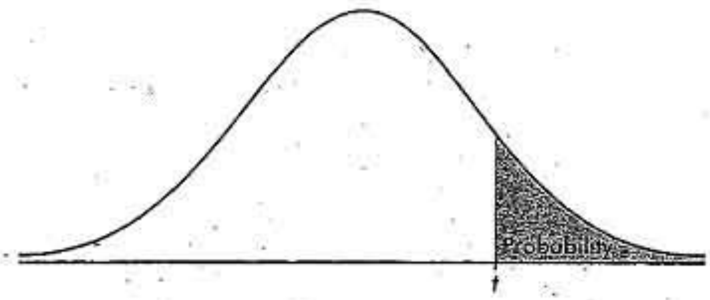


TABLE B: *t*-DISTRIBUTION CRITICAL VALUES

	Tail probability p											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	.765	.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	.741	.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	.727	.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	.718	.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	.711	.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	.706	.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	.703	.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	.700	.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	.697	.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	.695	.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	.694	.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	.692	.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	.691	.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	.690	.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	.689	.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	.688	.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	.688	.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	.687	.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	.686	.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	.686	.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	.685	.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	.685	.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	.684	.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	.684	.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	.684	.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	.683	.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	.683	.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	.683	.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	.681	.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	.679	.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	.679	.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	.678	.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	.677	.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	.675	.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
∞	.674	.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
	Confidence level C											

If the population follows a normal distribution, the confidence interval $\bar{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}}$ can be used for any sample size. If the population does not follow a normal distribution, this confidence interval will be approximate.

In most applications, a sample size of ≥ 30 is adequate to develop an interval estimate of a population mean. However, if the population distribution is highly skewed, most statisticians would recommend increasing the sample size to 50 or more.

Examples

1. Scheer Industries is considering a new computer-assisted program to train maintenance employees to do machine repairs. In order to fully evaluate the program, the director of manufacturing requested an estimate of the population mean time required for maintenance employees to complete the computer-assisted training. A sample of 20 employees is selected, with each employee in the sample completing the training program. Using the information below, find the 95% confidence interval.

52	59	54	42	44	50	42	48	55	54
60	55	44	62	62	57	45	46	43	56

2. Find the t value(s) for each of the following cases.
    a. Upper tail area of 0.025 with12 degrees of freedom
    b. Lower tail area of 0.05 with 50 degrees of freedom
    c. Upper tail area of 0.01 with 30 degrees of freedom
    d. Where 90% of the area falls between these two t values with 25 degrees of freedom
    e. Where 95% of the area falls between these two t values with 45 degrees of freedom

3. The following sample data are from a normal population: 10, 8, 12, 15, 13, 11, 6, 5.
    a. What is the point estimate of the population mean?
    b. What is the point estimate of the population standard deviation?
    c. With 95% confidence, what is the margin of error for the estimation for the population mean?
    d. What is the 95% confidence interval for the population mean?.

4. A simple random sample with n = 54 provided a sample mean of 22.5 and a sample standard deviation of 4.4.
    a. Develop a 90% confidence interval for the population mean.
    b. Develop a 95% confidence interval for the population mean.
    c. Develop a 99% confidence interval for the population mean.
    d. What happens to the margin of error and the confidence interval as the confidence level increased.

5. The mean number of hours of flying time for pilots at Continental Airlines is 49 hours per month. Assume that this mean was based on actual flying times for a sample of 100 Continental pilots and that the sample standard deviation was 8.5 hours.
    a. At 95% confidence, what is the margin of error?
    b. What is the 95% confidence interval estimate of the population mean flying time for the pilots?

Determining the Sample Size

Sample size for an interval estimate of a population mean:

$$\sqrt{n} = \frac{(z_{\alpha/2})^2}{E^2}$$

Where E = the desired margin of error.

(Notes)

Research Application: To find the value of σ, we may employ the following strategies:

1. Use the estimate of the population standard deviation computed from data of previous studies as the planning value for σ.
2. Use a pilot study to select a preliminary sample. The sample standard deviation from the preliminary sample can be used as the planning value for σ.
3. Use judgment of a "best guess" for the value of σ. For example, we might begin by estimating the largest and smallest data values in the population. The difference between the largest and smallest values provides an estimate of the range for the data. Finally, the range divided by 4 is often suggested as a rough approximation of the standard deviation and thus an acceptable planning value for σ.

Examples

1. How large a sample should be selected to provide a 95% confidence interval with a margin of error of 10? Assume that the population standard deviation is 40.

2. Bride's magazine reported that the mean cost of a wedding is $19,000. Assume that the population standard deviation is $9,400. Bride's plans to use an annual survey to monitor the cost of wedding. Use 95% confidence.
    a. What is the recommended sample size if the desired margin of error is $1,000?
    b. What is the recommended sample size if the desired margin of error is $500?
    c. What is the recommended sample size if the desired margin of error is $200?

3. Smith Travel Research provided information on the one-night cost of hotel rooms throughout the United States. Use $2 as the desired margin of error and $22.50 as the planning value for the population standard deviation to find the sample size recommended in (a), (b), and (c).
    a. A 90% confidence interval estimate of the population mean cost of hotel rooms.
    b. A 95% confidence interval estimate of the population mean cost of hotel rooms.
    c. A 99% confidence interval estimate of the population mean cost of hotel rooms.
    d. When the desired margin of error is fixed, what happens to the sample size as the confidence level is increased? Would you recommend a 99% confidence level be used by Smith Travel Research? Discuss.